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Preliminary report

THE MASS OF (1) CERES OBTAINED FROM FOUR CLOSE ENCOUNTERS

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SUMMARY: The technique of asteroid mass determination from perturbation during close approach requires as many as possible different close approaches in order to derive reliable mass of a perturber. Here are presented some results for the mass of (1) Ceres obtained using newly found close approaches.

Key words. Celestial mechanics

1. INTRODUCTION

In the absence of any satellite and until Dawn Discovery Mission (Russel et al. 2002) will take place, the direct determination of the mass of (1) Ceres must rely upon its gravitational effects on other asteroids.

Usually, mass determinations of (1) Ceres performed by other authors were based on the use of perturbed asteroids numbered under 5000. Consequently, the number of close encounters and of perturbed asteroids suitable for its mass determination was limited.

Galad and Gray (2002) were first to propose using close encounters with high numbered asteroids as a tool for mass determination of large asteroids. We have already tested bodies strongly perturbed by (4) Vesta (Kovačević 2005a) and the same work has been done for the largest asteroid (1) Ceres. We found four additional close encounters of (1) Ceres that were not listed by Galad and Gray.

In the present paper we give report about these pilot tests. As the next step of our research these results will be combined with masses of (1) Ceres obtained from the close encounters listed by the other authors.

2. PROCEDURE OF MASS DETERMINATION

Among the first 50000 numbered minor planets we found four perturbed asteroids, suitable for mass determination of (1) Ceres : (2051) Chang, (6010) Lyzenga, (6594) Tasman and (34755) 2001QW120.

Geometrical and kinematical conditions (Table 1) as well as expected gravitational effects (Fig. 1) revealed potentially good efficiency of these close approaches. A good selection of the suitable perturbed asteroids is an important step in the mass determination process. Many different criteria were used for this purpose in the past. Usually, the deflection angle between the trajectory of an asteroid before and after the close encounter with a more massive body was used as a selection parameter (Hilton et al. 1996). In this work, the suitable asteroids were found in a different way: combining traditional approach and procedure introduced by Kuzmanoski (1992). The outcome of this procedure was the list of dates of the closest encounters of (1) Ceres with suitable perturbed asteroids as well as the absolute value of the maximum difference in right ascension and declination between two trajectories of perturbed body: the first one takes into account perturbation of (1) Ceres, whereas the second does not. If the difference was large (typically, larger than 15 arcsec in right ascension) and if the available observations covered long enough period before and after the encounter, the perturbed asteroid was selected as a good candidate for the mass determination.

Bearing in mind that some other minor planets could perturb the motion of the chosen perturbed asteroids, the 9 largest asteroids have been included in the dynamical model, as well as all major planets. The masses of perturbing asteroids used are given in Table 2. The gravitational influence of the perturbed asteroids on the perturber is negligible due to their small masses.

Table 1. Geometrical and kinematical parameters of close encounters: ρ is minimum distance, V_r is relative velocity and θ is deflection angle of perturbed asteroid.

Perturbed asteroid	date d.m.y	ρ [AU]	V_r [km s^{-1}]	θ [arcsec]
2051 Chang	22.10.1943	0.012	4.48	0.72
6010 Lyzenga	29.04.1973	0.011	8.03	0.24
6594 Tasman	15.05.1982	0.013	6.71	0.30
34755 2001QW120	27.11.1967	0.005	8.35	0.49

Table 2. Masses of perturbing minor planets.

Asteroid	Mass $10^{-10}\,M_\odot$	
(1) Ceres	4.76	adopted
(2) Pallas	1.08	adopted
(4) Vesta	1.35	adopted
(10) Hygiea	0.47	Scholl et al. 1987
(11) Parthenope	0.0256	Viateau & Rapaport 2001
(16) Psyche	0.34	Kuzmanoski & Kovačević 2002
(52) Europa	0.011	IRAS
(511) Davida	0.014	IRAS
(704) Interamnia	0.013	IRAS

The numerical integration of differential equations of motion of perturbed bodies is carried out by Adams-Bashforth-Moulton predictor-corrector method (Moshier 1992). The initial osculating orbital elements for the epoch JD 2452600.5 were taken from the Edward Bowell database (`http://www.lowell.edu/users/elgb/`). In order to analyze the motion of perturbed asteroids, sets of observations were downloaded from the public database AstDys (`http://hamilton.dm.unipi.it/astdys`). Calculations of the mass of (1) Ceres were done by means of initial osculating elements of the perturbed asteroid for the epoch JD 2453200.5 used for backward integration.

The classical least-squares method was applied for the mass determination of (1) Ceres. Since the observational errors should have a Gaussian (normal) distribution, the selection criterion had to retain observations with (O-C) residuals having normal distribution and reject those with large (O-C) values. In our calculations we applied kσ-type criteria ($k = 3$), commonly used by other authors.

3. RESULTS AND DISCUSSION

The close encounters that occured between the largest asteroid in the main asteroid belt and (2051) Chang, (6010) Lyzenga, (6594) Tasman and (34755) 2001QW120 have some common characteristics. The relative velocities are greater, about 60% than the average value of asteroid encounter velocity (except in the case of (2051) Chang). This fact implies that these close encounters are not long lasting. The evolution of the distance between (1) Ceres and the perturbed asteroids in time is given in Fig. 1. As can be seen from Table 1, the minimum distances between (1) Ceres and four perturbed asteroids are small. Consequently, the gravitational influence of (1) Ceres on these bodies is in the range [10 arcsec, 20 arcsec] and could be compared with those seen in the case of close encounter with low numbered asteroid like (488) Kreusa (Kovačević 2005b).

It is well known that the mass of an asteroid, calculated from the gravitational effects, can be biased because of small number, inhomogeneous distribution or short time span covered by the observations of perturbed asteroid. In each case we had small number of observations and they had the inhomogeneous distribution. Inspection of Table 3 shows that only one observation belongs to preencounter part of orbit in each case. Having in mind that the range of values for Ceres mass, determined by other authors, is $(4.6-5.0)\,10^{-10}M_\odot$, we expected that our results could exceed this range because of the influence of distribution anomalies. However this did not happen.

Analyzing the gravitational effects in the case of the close encounter with (2051) Chang, it can be noticed that they are 10 arcsec. Also pre and postencounter time spans covered by observations are long enough to provide a good perturbed orbit and perturbing mass. Obtained value for the mass of (1) Ceres from this close encounter $(4.63 \pm 0.26)\,10^{-10} M_{\odot}$ is very similar to the result obtained by using (6010) Lyzenga as a perturbed body. Despite the fact that kinematical characteristics of these close encounters (Table 1) are different, the absolute value of difference of obtained masses (as well as their formal errors) is only $0.03 \times 10^{-10} M_{\odot}$ (Table 3). Large number of good postencounter observations of (2051) Chang, covering long part of perturbed orbit, improved the obtained result for the mass of (1) Ceres.

In the case of the close encounter with (6010) Lyzenga time spans covered by pre(post)encounter observations are similar. Despite the fact that the distribution of observations is uneven, preencounter observations were made long enough before the close encounter, providing the possibility of taking into account non negligible gravitational effects. The value of the mass of (1) Ceres is $(4.60 \pm 0.29)\,10^{-10} M_{\odot}$. This result for the mass of (1) Ceres differs from the adopted by no more than $3\sigma_i$ (where $3\sigma_i$ is its own formal error). Obtained formal error could be affected by large O-C residual of the used preencounter observation (-10 arcsec in right ascension). Also, we noticed that the same magnitude of formal error appeared during recalculation of the mass of Ceres using low numbered asteroids ((488) Kreussa,(548) Kressida,(621) Werdandi,(792) Metcalfia, (1847) Stobbe, (3344) Modena) (Kovačević 2005b). These close encounters are characterized by gravitational effects around 20 arcsec in right ascension. Some of them have greater number of the used preencounter observations.

Table 3. The distribution of observations of perturbed asteroids. In the columns N_1, N_2, N_3, N_4, N_5 are given number of observations, number of discarded preenc. observations, number of used preenc. observations, number of discarded postenc. observations and number of used postenc. observations. T_1 and T_2 are time intervals covered by preencounter and postencounter observations.

Perturbed asteroid	N_1	N_2	N_3	T_1	N_4	N_5	T_2	Mass $[10^{-10}\,M_{\odot}]$
(2051) Chang	458	4	1	1933	51	402	1955-2002	4.63 ± 0.26
(6010) Lyzenga	173	1	1	1953	5	166	1974-2002	4.60 ± 0.29
(6594) Tasman	218	1	1	1954	15	201	1987-2002	5.02 ± 0.44
(34755) 2001QW120	99	2	1	1950	6	91	1985-2001	5.05 ± 0.76

Similar distribution of observations is seen in the case of close approach with (6594) Tasman. O-C residual of the used preencounter observation is around 6 arcsec in right ascension. The maximum variation in right ascension of (6594) Tasman due to the gravitational influence of (1) Ceres is 10 arcsec within the 32 year period before the closest approach. In this case, except the uneven distribution of observations, obtained value $(5.02 \pm 0.44)\,10^{-10} M_{\odot}$ for the mass of (1) Ceres was affected by small gravitational effects. However, collecting new observations could provide an extension of the time interval covered by postencounter observations in such a way that the mass of (1) Ceres could be determined by using forward integration. Also, if it is possible to find preencounter observations that were made far enough from the close approach, a more reliable mass determination of (1) Ceres could be expected.

In the case of the close approach with (34755) 2001QW120, the maximum variation in right ascension, as a consequence of gravitational perturbation of its orbit due to (1) Ceres, is about -17 arcsec after backward integration (Fig. 1). As it can be seen (Table 3), the set of used observations contains only 92 data, which implies that the obtained result for the mass of (1) Ceres ($(5.05 \pm 0.76)\,10^{-10} M_{\odot}$) could have been expected.

Much more reliable calculation of the corrections of perturbed orbit elements and the perturbing mass could be expected after expanding of the postencounter set of observations.

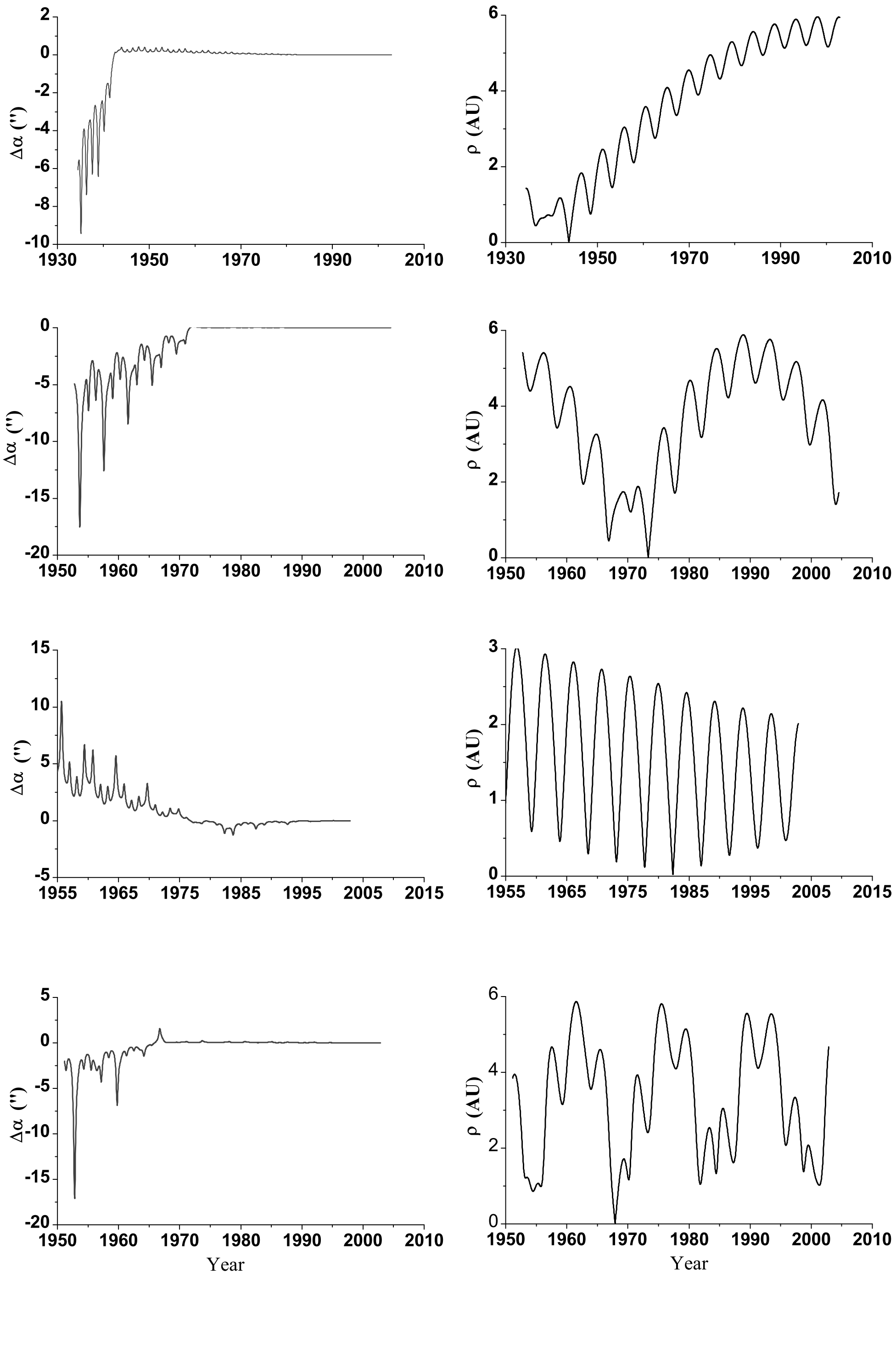

Fig. 1. *Differences of the geocentric right ascensions of perturbed bodies caused by (1) Ceres (left) and mutual distances between perturbed asteroids and (1) Ceres (right). Pairs of the plots from the top to the bottom correspond to close encounters with: (2051) Chang, (6010) Lyzenga, (6594) Tasman and (34755) 2001QW120, respectively.*

3. CONCLUDING REMARKS

Three of the four considered perturbed asteroids are relatively faint objects. Despite this, the obtained results are in the range of values obtained by other authors. The use of faint objects will provide an enlargement of the number of close encounters and perturbed asteroids for asteroid mass determination. Concerning the adopted value of the mass of (1) Ceres the best results are obtained from close encounters with (2051) Chang and (6010) Lyzenga, where postencounter parts of the orbit are long enough. However, the values of the mass of (1) Ceres obtained from its close approaches with (6594) Tasman and (34755) 2001QW120 are influenced by gravitational effects as well as by the number of used observations. It could be expected that after increasing the postencounter sets of observations of these perturbed asteroids, forward integration could provide more reliable results for the mass of (1) Ceres.

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МАСА АСТЕРОИДА (1) ЦЕРЕС ДОБИЈЕНА НА ОСНОВУ ЧЕТИРИ НОВА БЛИСКА ПРИЛАЗА

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Претходно саопштење

За одређивања маса астероида применом технике базиране на поремећајима током блиских прилаза, потребно је имати што већи број поремећених астероида, како би се што тачније одредила маса поремећајног тела. Овде ће бити представљени резултати одређивања масе (1) Цереса коришћењем новопронађених блиских прилаза.