

传热学

Heat transfer

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## 第五章

# 单相流体对流换热

# 5-5 自然对流换热

## 一、概述

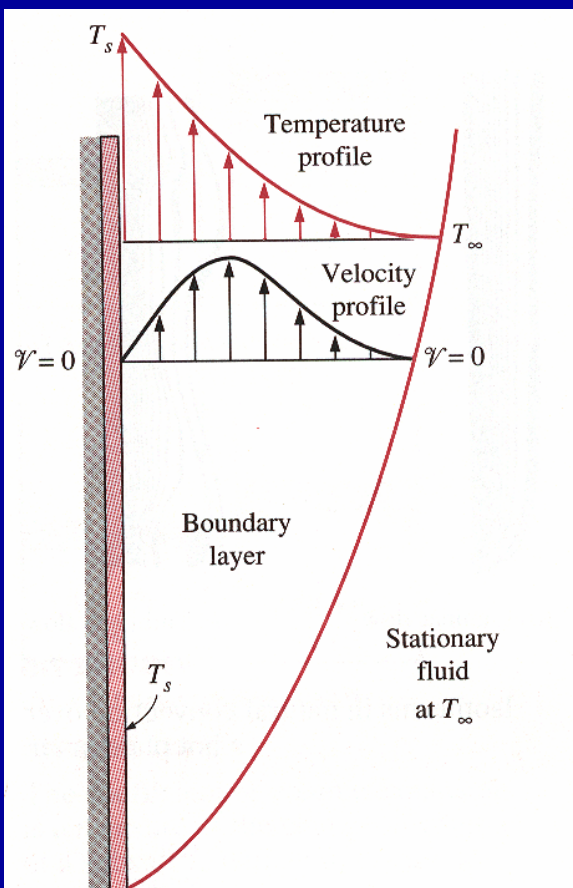


FIGURE 7-7

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature  $T_s$  inserted in a fluid at temperature  $T_\infty$ .

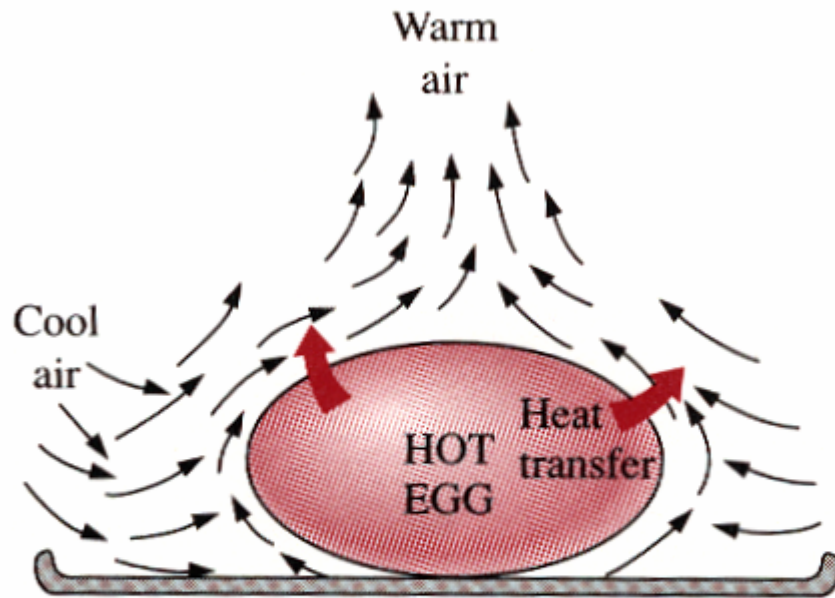
### 1、自然对流的起因

静止的流体，与不同温度的固体壁面相接触，热边界层内、外的密度差形成浮升力（或沉降力）

$$f_B = (\rho_f - \rho)g = \rho g \beta \Delta t$$

导致流动

固体壁面与流体的温差是自然对流的根本原因

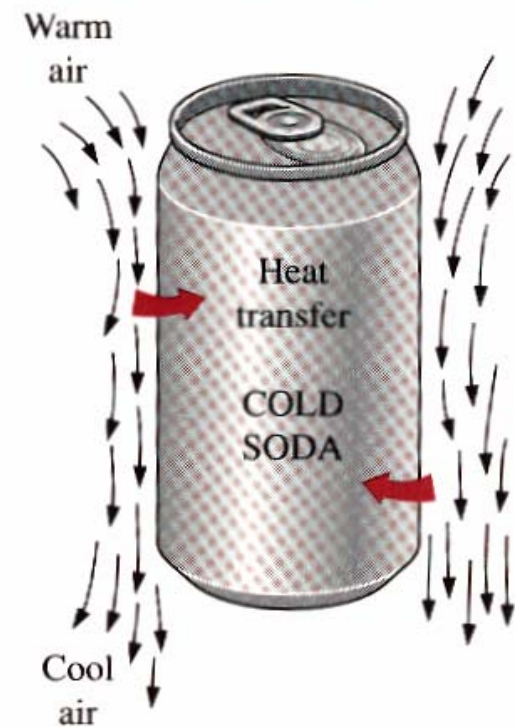


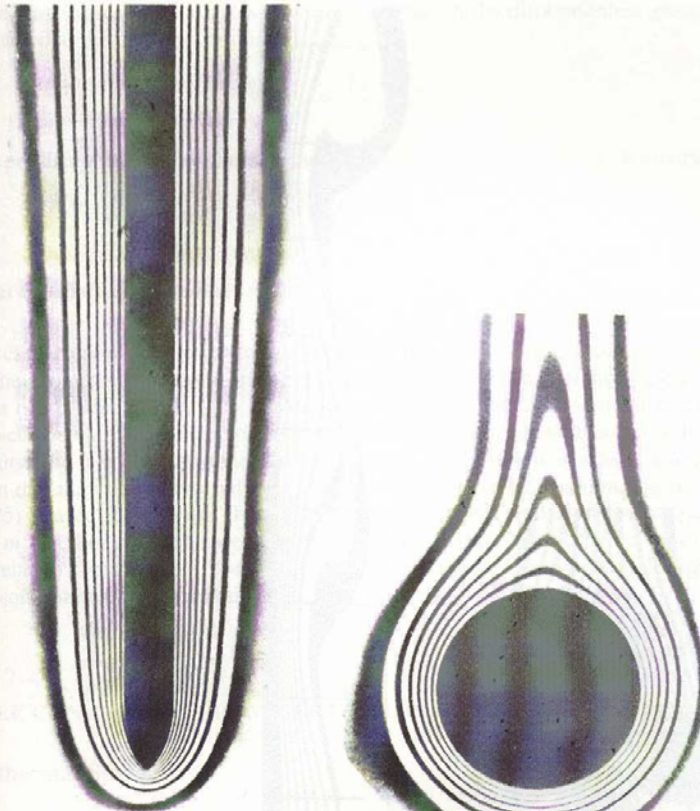
**FIGURE 7-1**

The cooling of a boiled egg in a cooler environment by natural convection.

**FIGURE 7-2**

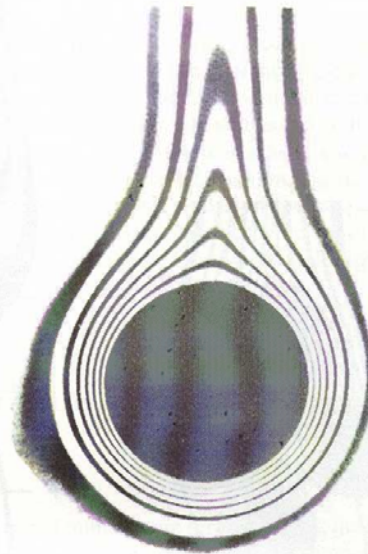
The warming up of a cold drink in a warmer environment by natural convection.





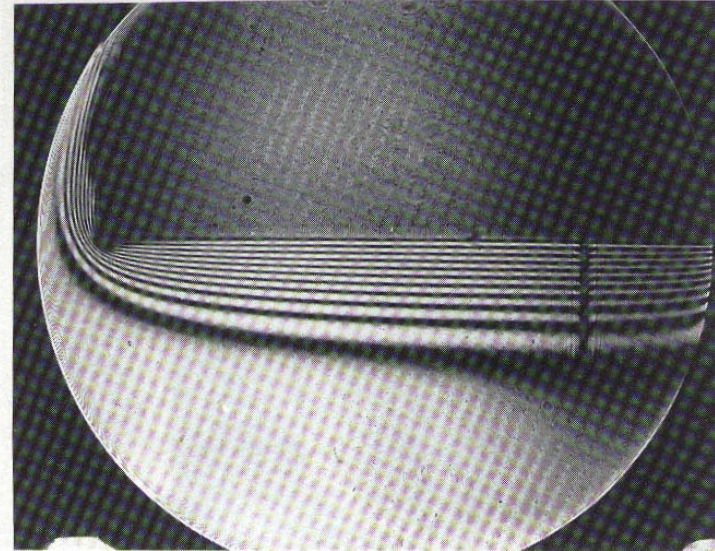
**FIG. 7-3**

Interferometer photograph showing lines of constant temperature around a heated vertical flat plate in free convection. (Photograph courtesy E. Soehngen.)



**FIG. 7-4**

Interferometer photograph showing lines of constant temperature around a heated horizontal cylinder in free convection. (Photograph courtesy E. Soehngen.)



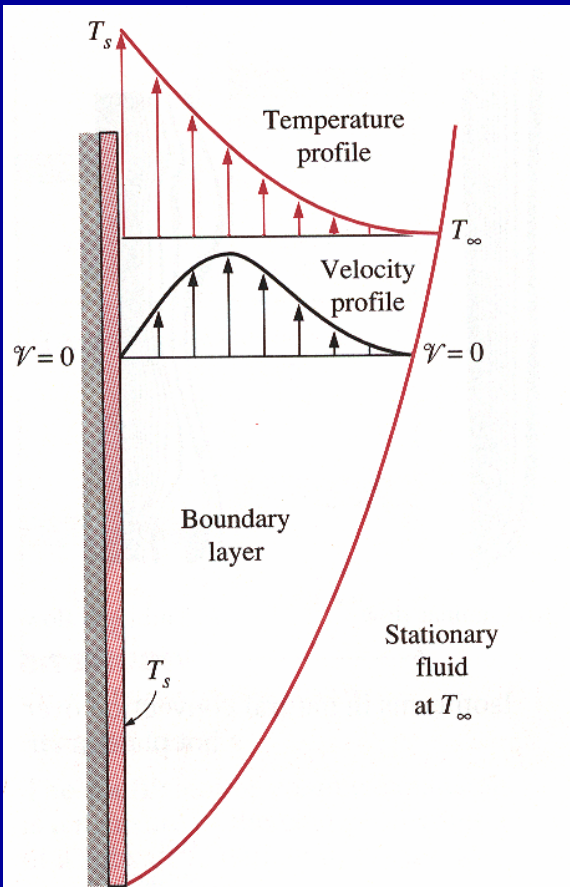
**Figure 4.32** The thermal boundary layer below a 10 cm × 30 cm heated horizontal plate:  $Ra_L = 3.33 \times 10^6$ ,  $Nu_L = 15.0$ . The pin marks the plate center. Air rises from some distance below the plate until, in the vicinity of the plate, it flows sideways and then around the corner, carrying the heat away in a vertical plume. There are 10 isotherms visible; these are 3°C apart adjacent to the plate where the air is hot (and index of refraction is lower), decreasing to 2° apart as the ambient air is approached. (Photograph courtesy of Professor D. K. Edwards, University of California, Irvine.)

## 2、边界层的特征

(1) 速度边界层的厚度与热边界层的厚度相等

(2) 速度边界层内速度分布在内部呈现峰值

(3) 在边界层边缘处，依然具有强迫对流边界层的特点



**FIGURE 7-7**

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature  $T_s$  inserted in a fluid at temperature  $T_\infty$ .

$$\frac{\partial u}{\partial y} = 0 \quad \frac{\partial T}{\partial y} = 0 \quad \frac{\partial p}{\partial y} = 0$$

### 3、流态判据

层流:

$$Gr Pr < 10^8$$

湍流:

$$Gr Pr > 10^{10}$$

过渡区:

$$10^8 < Gr Pr < 10^{10}$$

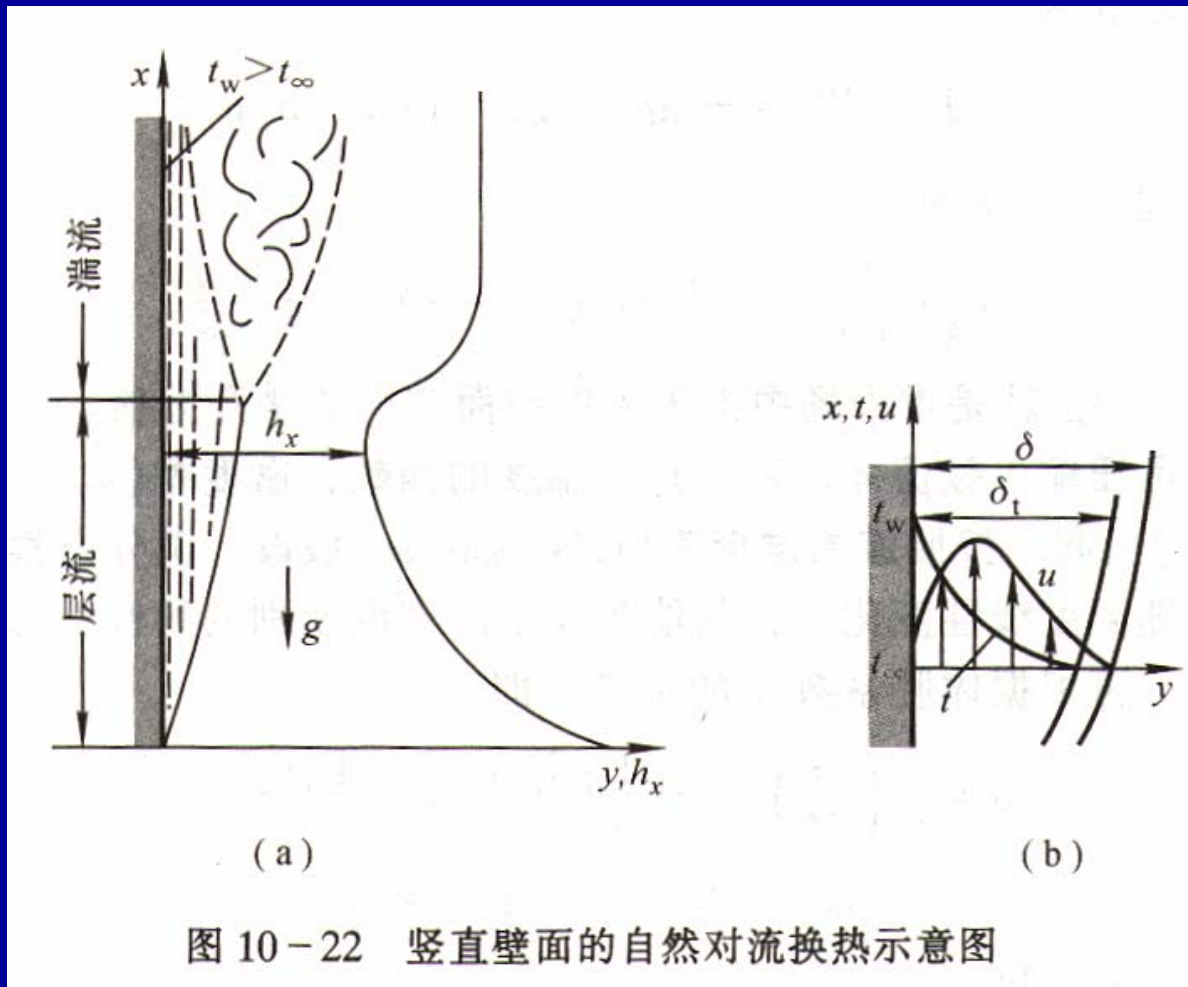
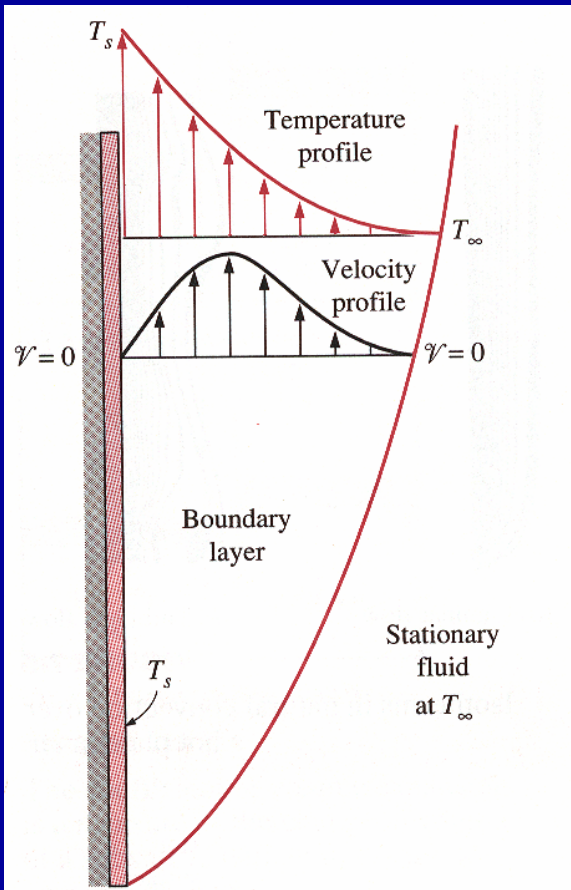


图 10-22 竖直壁面的自然对流换热示意图

■ **自模化现象:** 在常壁温或常热流边界条件下, 达到旺盛湍流时,  $h_x$  将保持不变, 与壁面高度无关

## 二、自然对流换热的数学描述

### 1. 竖壁上自然对流换热边界层微分方程组



**FIGURE 7-7**

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature  $T_s$  inserted in a fluid at temperature  $T_\infty$ .

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\rho g - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}$$



■ 流体物性除浮升力项中的密度外均为常量 —— **Boussinesq**假设

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\rho g - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

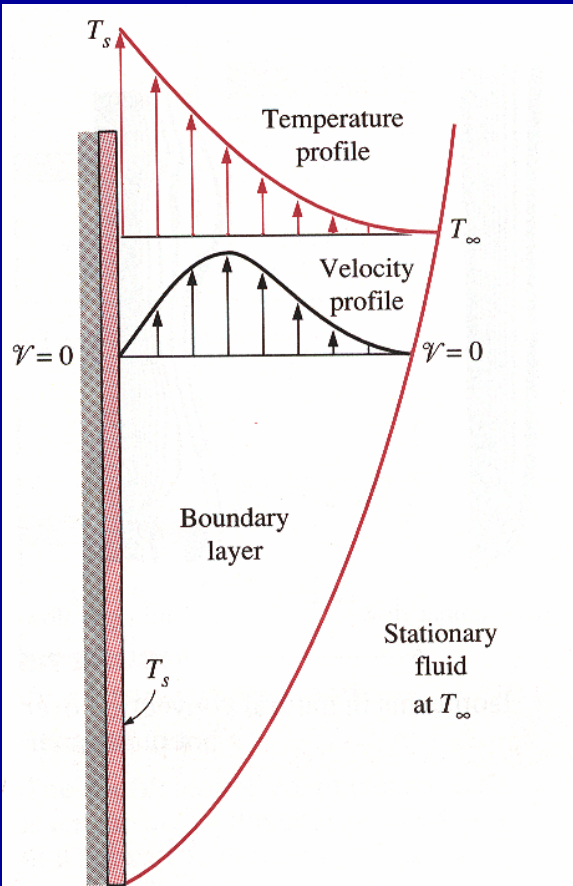
$$y = \delta, \quad u \rightarrow 0, \quad v \rightarrow 0;$$

$$T \rightarrow T_\infty, \quad \rho \rightarrow \rho_\infty$$

$$\frac{\partial p}{\partial y} = 0$$

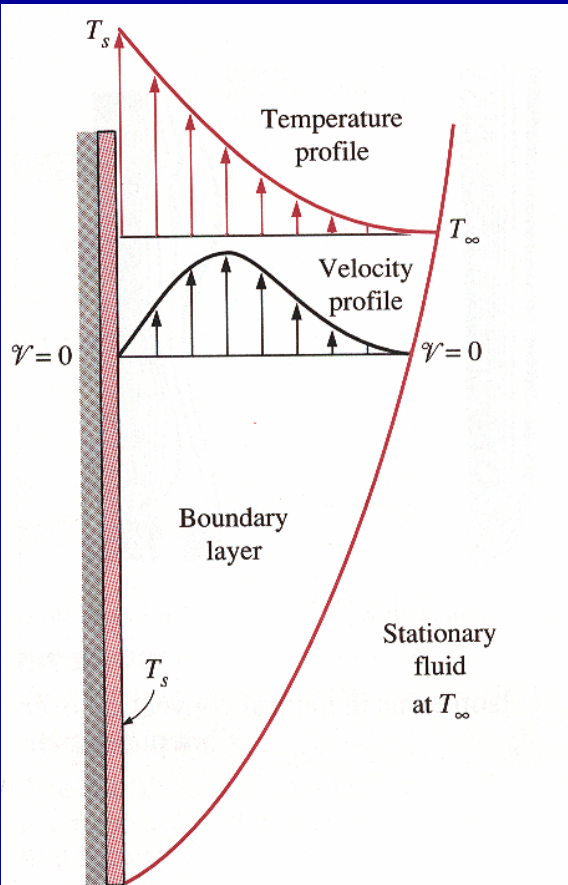
$$\frac{dp}{dx} = -\rho_\infty g$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = (\rho_\infty - \rho)g + \mu \frac{\partial^2 u}{\partial y^2}$$



**FIGURE 7-7**

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■ 假设：密度与温度保持线性关系  
体积膨胀系数：

$$\beta = \frac{(dV/V)}{dT} = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p$$

$$\rho_\infty - \rho = -\rho\beta(T_\infty - T)$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \rho g \beta (T - T_\infty) + \mu \frac{\partial^2 u}{\partial y^2}$$

## 2. 描述自然对流换热的相似特征数

引入无量纲参数-相似特征数推导的重要方法:

$$x' = \frac{x}{l}, y' = \frac{y}{l},$$

$$u' = \frac{u}{u_0}, v' = \frac{v}{u_0}, \quad u_0 \text{-----参考速度}$$

$$\theta' = \frac{T - T_f}{T_w - T_f}$$

$$h_x = \frac{\lambda}{l} \left( \frac{\partial \theta'}{\partial y'} \right)_{w,x}$$

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0$$

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = \frac{g\beta(T_w - T_f)l}{u_0^2} + \frac{1}{\text{Re}} \frac{\partial^2 u'}{\partial y'^2}$$

$$u' \frac{\partial \theta'}{\partial x'} + v' \frac{\partial \theta'}{\partial y'} = \frac{a}{u_0 l} \frac{\partial^2 \theta'}{\partial y'^2} = \frac{1}{\text{Re Pr}} \frac{\partial^2 \theta'}{\partial y'^2}$$

### 3. 葛拉晓夫数

$$Gr = \frac{g\beta(T_w - T_f)l}{u_0^2} \cdot \left(\frac{u_0 l}{\nu}\right)^2 = \frac{g\beta(T_w - T_f)l^3}{\nu^2}$$

物理意义----表征浮升力与粘性力比值的一种度量

■ 在自然对流中，**Re**不是一个独立的准则，而是**Gr**的函数

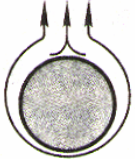

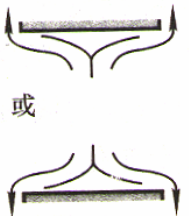
$$\mathbf{Nu} = f(\mathbf{Gr}, \mathbf{Pr})$$

### 三、大空间自然对流换热

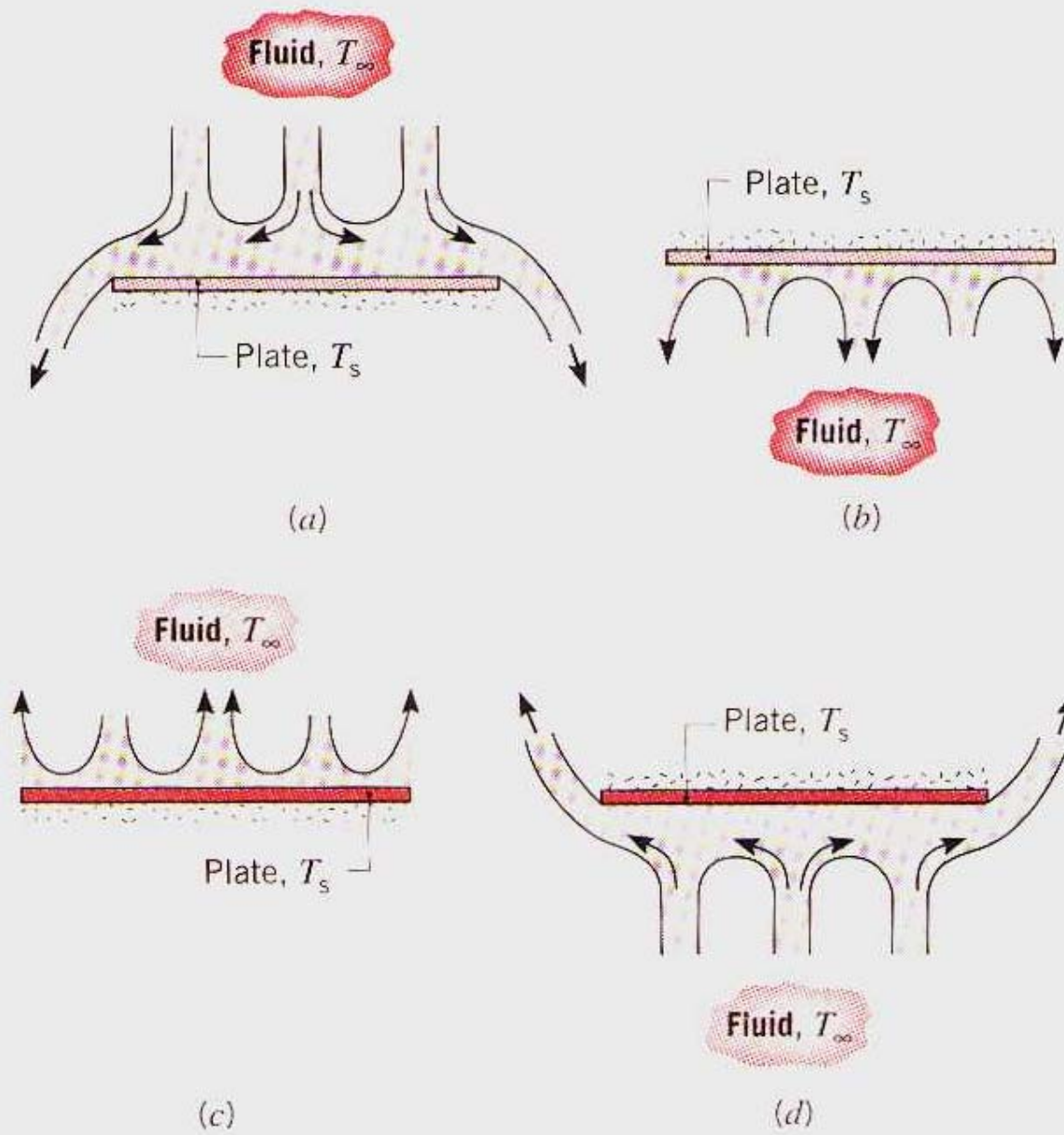
自然对流换热准则关联式常采用幂函数形式：

$$Nu = C(Gr \cdot Pr)^n = C Ra^n; \quad T_w = \text{const}$$

表 10-5 (完)

壁面形状 与位置	流动情况	特征长度	C	n	GrPr 适用范围
水平圆柱		圆柱外径 $d$	0.85 0.48 0.125	0.188 1/4 1/3	$10^2 \sim 10^4$ $10^4 \sim 10^7$ $10^7 \sim 10^{12}$
水平热面朝上或 水平冷面朝下 <sup>①</sup>		平壁面积与周 长之比 $A/U$ , 圆盘取 $0.9d$	0.54 0.15	1/4 1/3	$10^4 \sim 10^7$ $10^7 \sim 10^{11}$
水平热面朝下或 水平冷面朝上		平壁面积与周 长之比 $A/U$ 圆盘取 $0.9d$	0.27	1/4	$10^5 \sim 10^{11}$

① 热壁指  $t_w > t_\infty$ , 冷壁指  $t_w < t_\infty$



**FIGURE 9.8**  
 Buoyancy-driven flows on horizontal cold ( $T_s < T_\infty$ ) and hot ( $T_s > T_\infty$ ) plates: (a) top surface of cold plate, (b) bottom surface of cold plate, (c) top surface of hot plate, and (d) bottom surface of hot plate.

注意：对于自然对流湍流换热，准则关联式为：

$t_w = const$  时（第一类边界条件）：

$$Nu_x = \frac{h_x x}{\lambda} = 0.10(Gr_x Pr)^{1/3}$$

展开关联式后，等号两边的定型尺寸可以消去。表明：  
自然对流湍流换热的表面传热系数与定型尺寸无关

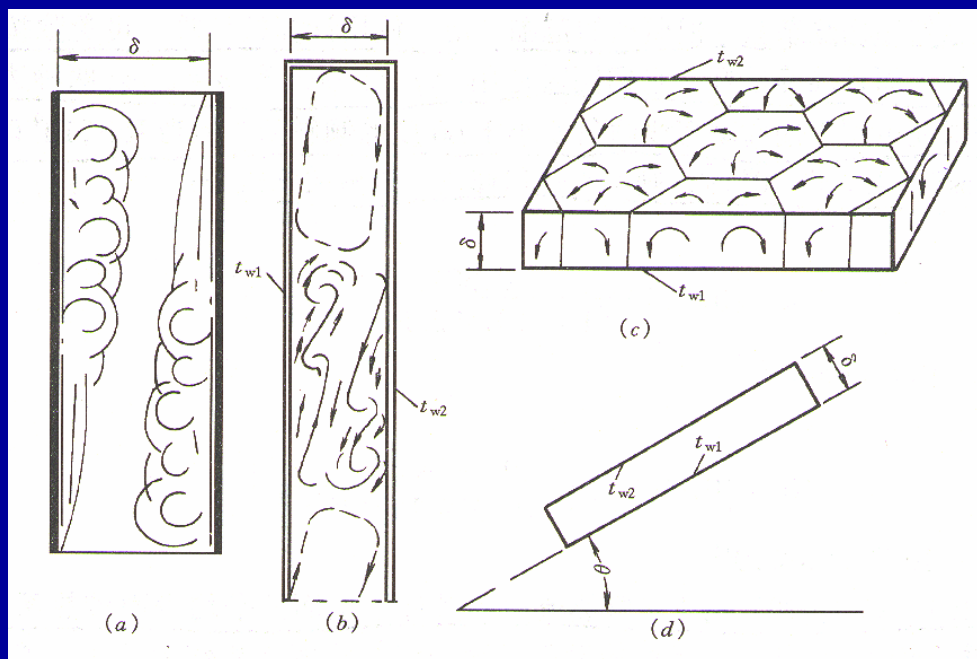
### ■ 自模化现象

利用该特性，湍流自然对流换热的实验研究可以采用较小尺寸的物体进行，只要求实验现象的  $GrPr$  值处于湍流范围



## 四、有限空间中的自然对流换热

有限空间自然对流换热：热由封闭的有限空间高温壁传到它的低温壁的换热过程

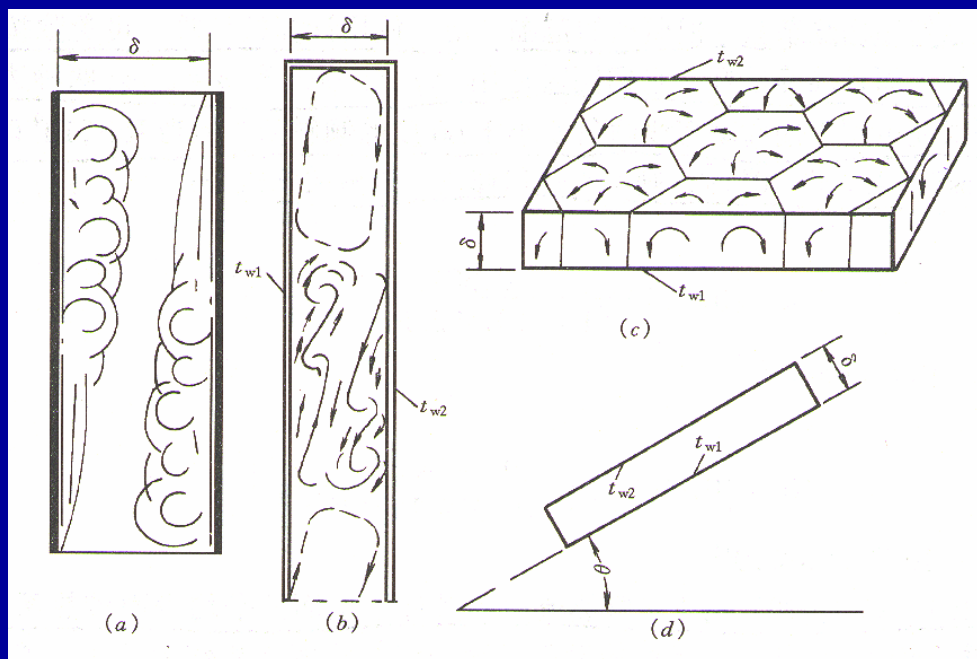


靠近热壁的流体因浮升力而向上运动，靠近冷壁的流体则向下运动

有限空间中的自然对流换热是热壁与冷壁间两个自然对流过程的组合

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# 1、竖直夹层



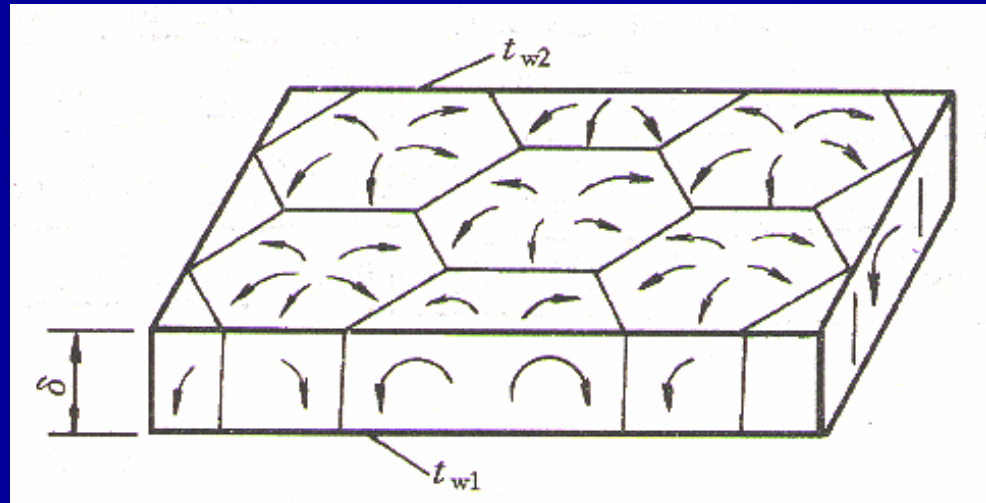
■ 夹层厚度与高度比较大(大于0.3)

冷热两壁的自然对流边界层不会互相干扰。可按无限空间自然对流换热规律分别计算冷、热两壁的自然对流换热及夹层总热阻

■ 夹层厚度与高度比较小(小于0.3)

夹层内冷、热壁上两股流动边界层相互结合和影响，出现行程较短的环流；夹层中可能有若干个环流

## 2、水平夹层



### ■ 热面在上

冷热面之间无流动发生；若无外界扰动，则应按导热问题分析

### ■ 热面在下

$Gr_{\delta} < 1700$ ，自然对流难以形成，可按导热问题分析  
 $Gr_{\delta} > 1700$ ，形成相互交替上升和下降的对流，呈现有序的蜂窝状分布的环流；

$Gr_{\delta} > 5000$ ，蜂窝状分布的环流消失，出现湍流流动

## 有限空间中的自然对流换热的计算

通常把两侧的换热用一个当量表面传热系数来表示：

$$q = h_e (T_{w1} - T_{w2})$$

$q$  — 通过夹层的热流密度 [W/m<sup>2</sup>];

$t_{w1}$ 、 $t_{w2}$  — 夹层的热壁和冷壁温度 °C

$h_e$  — 当量表面传热系数 [W/m<sup>2</sup>K]

封闭夹层空间自然对流换热准则关联式用下式表示：

$$Nu_{\delta} = \frac{h_e \delta}{\lambda} = C (\text{Gr}_{\delta} \text{Pr})^m \left( \frac{\delta}{H} \right)^n$$

定性温度：

$$t_m = \frac{t_{w1} + t_{w2}}{2}$$

定型尺寸： $\delta$        $H$  — 垂直夹层高度 [m]

夹层位置	$Nu_{\delta}$ 准则关联式	适用范围
竖壁夹层 (气体) <sup>①</sup>	= 1 (导热)	$Gr_{\delta} < 2000$
	$= 0.18 Gr_{\delta}^{1/4} \left( \frac{\delta}{H} \right)^{1/9}$ (层流)	$2000 < Gr_{\delta} < 2 \times 10^5$ ;
	$= 0.065 Gr_{\delta}^{1/3} \left( \frac{\delta}{H} \right)^{1/9}$ (紊流)	$2 \times 10^5 < Gr_{\delta} < 2 \times 10^7$
水平夹层 (热面在下) <sup>②</sup> (气体)	$= 0.059 (Gr_{\delta} \cdot Pr)^{0.4}$	$1700 < (Gr_{\delta} \cdot Pr) < 7000$
	$= 0.212 (Gr_{\delta} \cdot Pr)^{1/4}$	$7000 < (Gr_{\delta} \cdot Pr) < 3.2 \times 10^5$
	$= 0.061 (Gr_{\delta} \cdot Pr)^{1/3}$	$(Gr_{\delta} \cdot Pr) > 3.2 \times 10^5$
倾斜夹层 (热面在下与 水平夹角为 $\theta$ ) <sup>③④</sup> (气 体)	$= 1 + 1.446 \left( 1 - \frac{1708}{Gr_{\delta} \cdot Pr \cdot \cos\theta} \right)$	$1708 < (Gr_{\delta} \cdot Pr \cdot \cos\theta) < 5900$
	$= 0.229 (Gr_{\delta} \cdot Pr \cdot \cos\theta)^{0.252}$	$5900 < (Gr_{\delta} \cdot Pr \cdot \cos\theta) < 9.23 \times 10^4$
	$= 0.157 (Gr_{\delta} \cdot Pr \cdot \cos\theta)^{0.285}$	$9.23 \times 10^4 < (Gr_{\delta} \cdot Pr \cdot \cos\theta) < \times 10^6$

$$q = h_e (T_{w1} - T_{w2}) = \frac{h_e \delta}{\lambda} \frac{\lambda}{\delta} (T_{w1} - T_{w2}) = Nu_{\delta} \frac{\lambda}{\delta} (T_{w1} - T_{w2})$$

文献中封闭夹层的换热强弱也用当量导热系数表示:

$$q = \frac{\lambda_e}{\delta} (T_{w1} - T_{w2}); \quad Nu_{\delta} = \frac{\lambda_e}{\lambda}; \quad h_e = \frac{\lambda_e}{\delta}; \quad \lambda_e = h_e \delta$$

## 5-6 混合对流换热

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在重力场（或离心力）中，在任何非定温的受迫对流过程中，由于流体各部分温度的差异而出现密度差——引起不同程度的自然对流

- 在受迫对流换热中，若流速和动量转移率很大，则自然对流换热的影响可以忽略
- 若密度差很大，则浮升力（或离心力）引起的自然对流的影响可能大到无需考虑受迫对流的程度

对一个具体的对流换热问题进行分析计算时，首先应明确在什么条件下属于纯受迫对流换热、自然对流换热、混合对流换热

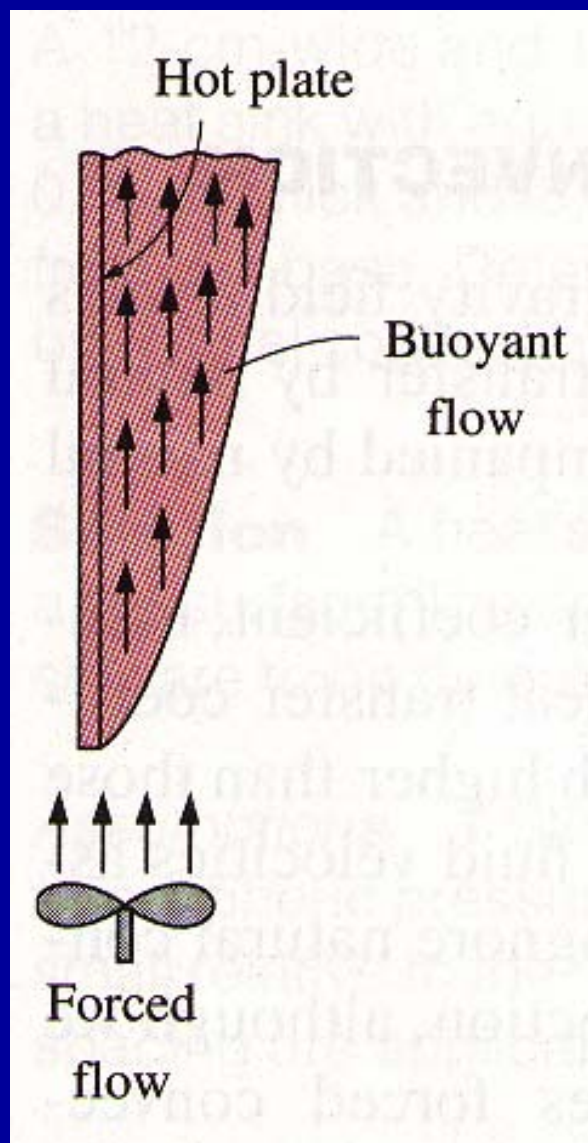
浮升力与惯性力之比

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\text{Gr}}{\text{Re}_0^2} \Theta + \frac{1}{\text{Re}_0} \frac{\partial^2 U}{\partial Y^2}$$

自然对流与强迫对流的相对强弱可以用  $\text{Gr}/\text{Re}_0^2$  判断



# 一、沿竖平壁的混合对流换热



判断流动是纯受迫对流还是混合对流，可根据浮升力与惯性力的相对大小确定

自然对流边界层动量微分方程：

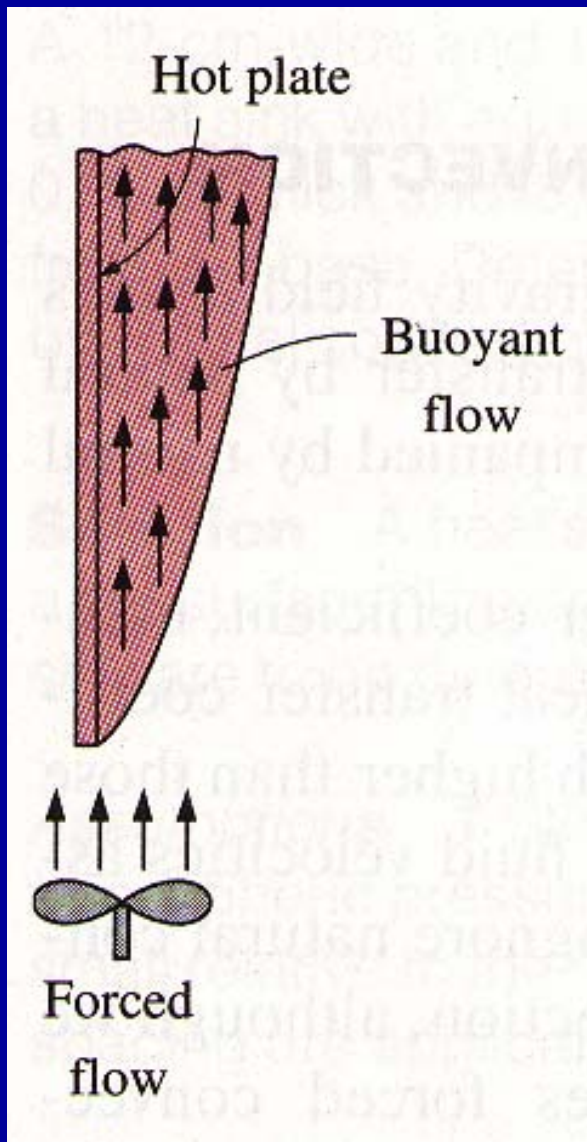
$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T - T_f) + \nu \frac{\partial^2 u}{\partial y^2}$$

■ 浮升力的数量级：

$$g\beta(T - T_f) \sim g\beta(T_w - T_f)$$

■ 惯性力的数量级：

$$u \frac{\partial u}{\partial x} \sim \frac{u_f^2}{l}$$



浮升力的数量级:

$$g\beta(T - T_f) \sim g\beta(T_w - T_f)$$

惯性力的数量级:

$$u \frac{\partial u}{\partial x} \sim \frac{u_f^2}{l}$$

两者之比:

$$\frac{g\beta\Delta T}{\frac{u_f^2}{l}} = \frac{g\beta\Delta T l}{u_f^2} = \frac{g\beta\Delta T l^3}{\nu^2} \frac{\nu^2}{u_f^2 l^2} = \frac{Gr}{Re^2}$$

纯受迫对流和纯自然对流换热准则关联式:

$$Nu'_m = 0.664 Re_m^{1/2} Pr_m^{1/3}$$

$$Nu''_m = 0.59 Gr_m^{1/4} Pr_m^{1/4}$$

纯受迫对流和纯自然对流传热准则  
关联式:

$$\text{Nu}'_m = 0.664 \text{Re}_m^{1/2} \text{Pr}_m^{1/3}$$

$$\text{Nu}''_m = 0.59 \text{Gr}_m^{1/4} \text{Pr}_m^{1/4}$$

二者相除:

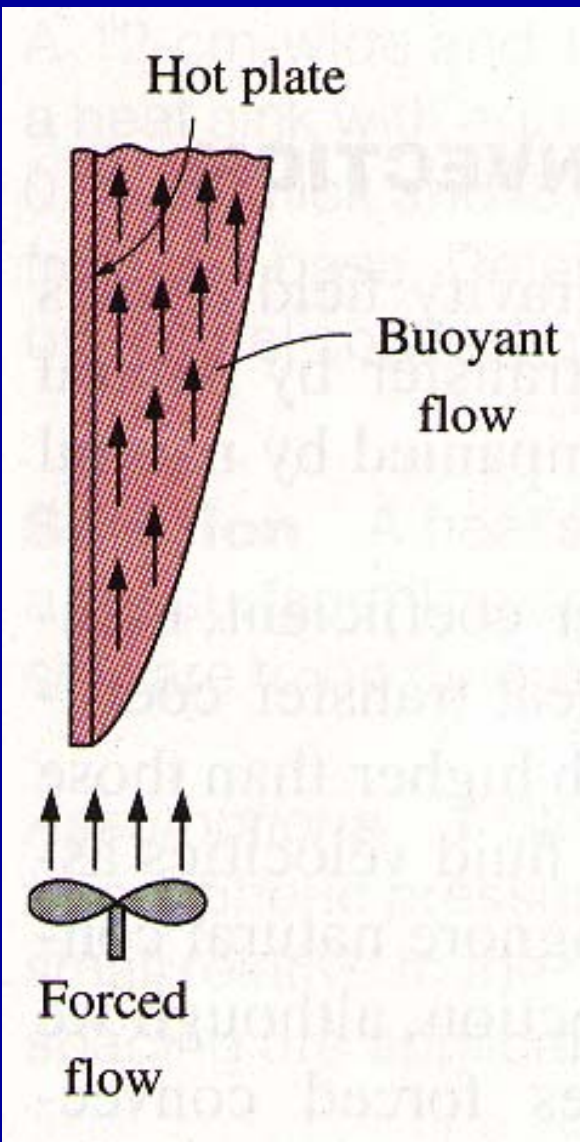
$$\frac{\text{Nu}''_m}{\text{Nu}'_m} = \frac{0.59 \text{Gr}_m^{1/4} \text{Pr}_m^{1/4}}{0.664 \text{Re}_m^{1/2} \text{Pr}_m^{1/3}} = \frac{0.889}{\text{Pr}_m^{0.083}} \frac{\text{Gr}_m^{1/4}}{\text{Re}_m^{1/2}}$$

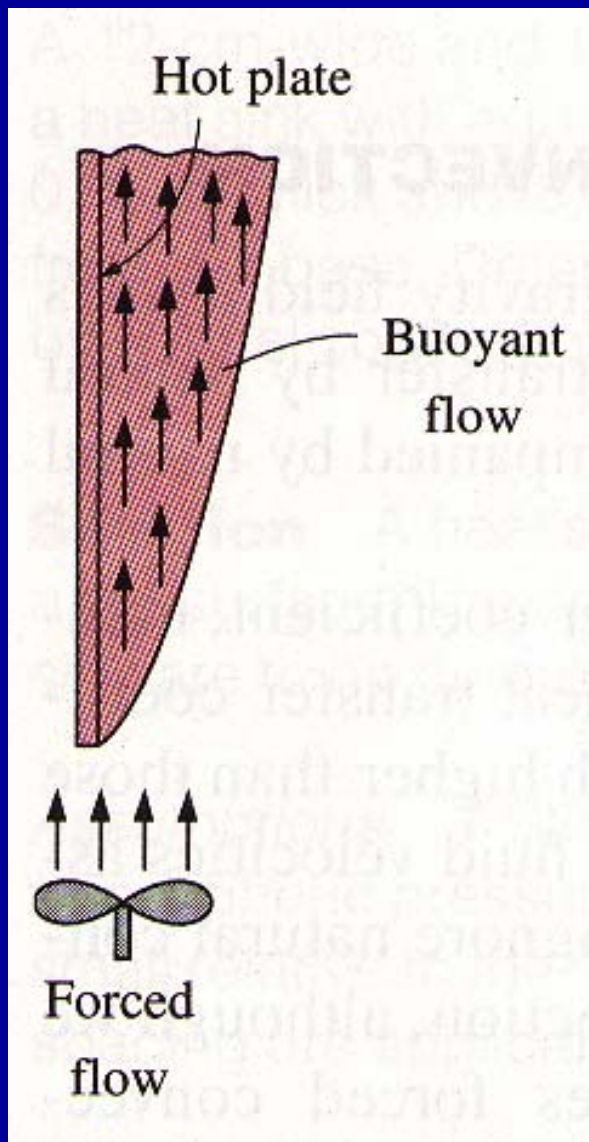
对于气体和  $\text{Pr}$  较小的水, 近似认为

$$\frac{0.889}{\text{Pr}_m^{0.083}} \approx 1$$

$$\frac{\text{Nu}''_m}{\text{Nu}'_m} \approx \frac{\text{Gr}_m^{1/4}}{\text{Re}_m^{1/2}}$$

故:





$$\frac{Nu_m''}{Nu_m'} \approx \frac{Gr_m^{1/4}}{Re_m^{1/2}}$$

受迫对流换热和自然对流换热沿竖平壁同时存在、并具有相同的换热效应时，必然是

$$Gr_m / Re_m^2 \approx 1$$

**Lloyd and Sparrow** 等：

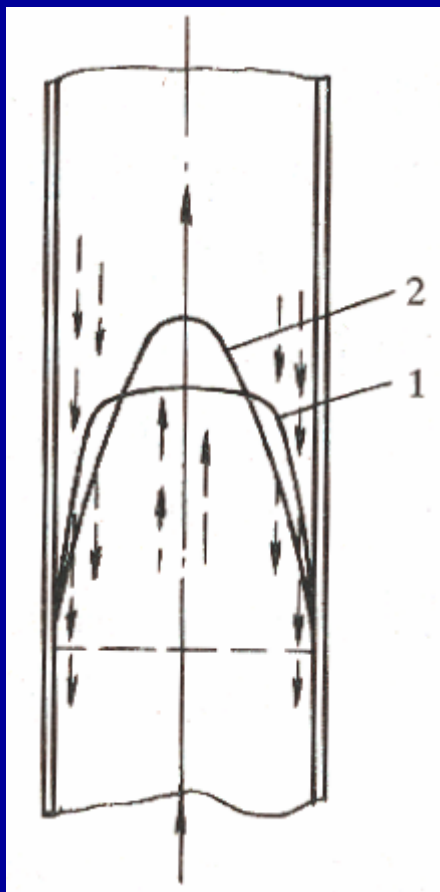
空气沿恒温竖平壁向上作层流受迫对流的换热：

$$\frac{Gr_m}{Re_m^2} \leq 0.02 \quad \text{— 纯受迫对流换热}$$

$$\frac{Gr_m}{Re_m^2} \geq 10 \quad \text{— 纯自然对流换热}$$

$$0.02 \leq \frac{Gr_m}{Re_m^2} \leq 10 \quad \text{— 混合对流换热(同时考虑两种对流)}$$

## 二、沿竖管内的混合对流换热



假设：流体向上流动并被冷却

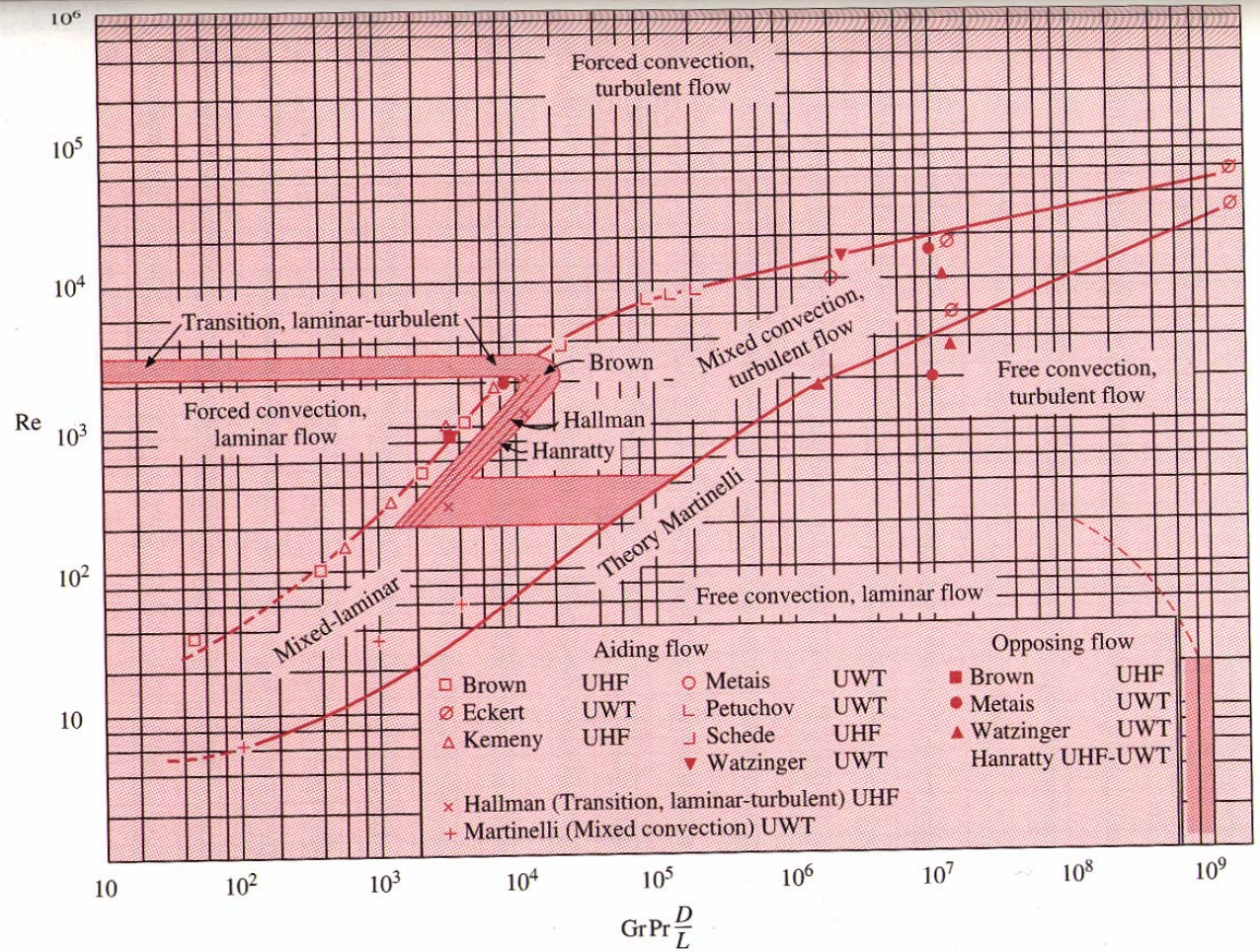
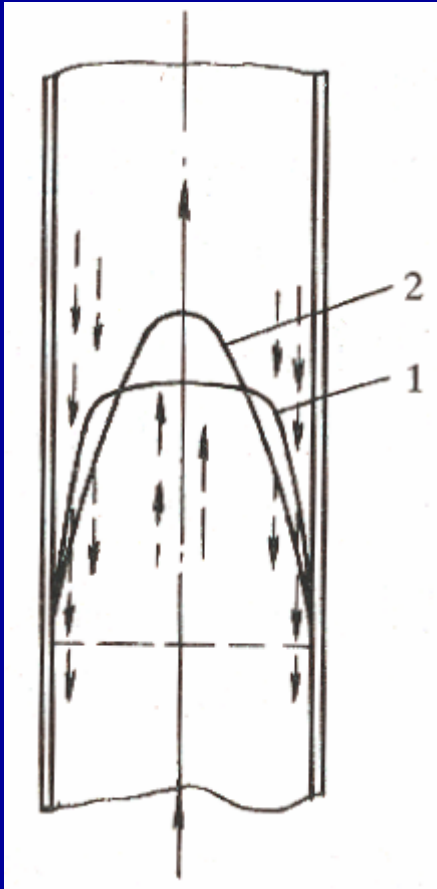
靠管壁处受迫对流与自然对流方向相反，靠管壁处流动速度降低；在流量一定时，管中心的速度将加快。

—— 不利于换热

$$\frac{Gr_m}{Re_m^2} \leq 0.1 \quad \text{— 纯受迫对流换热}$$

$$\frac{Gr_m}{Re_m^2} \geq 10 \quad \text{— 纯自然对流换热}$$

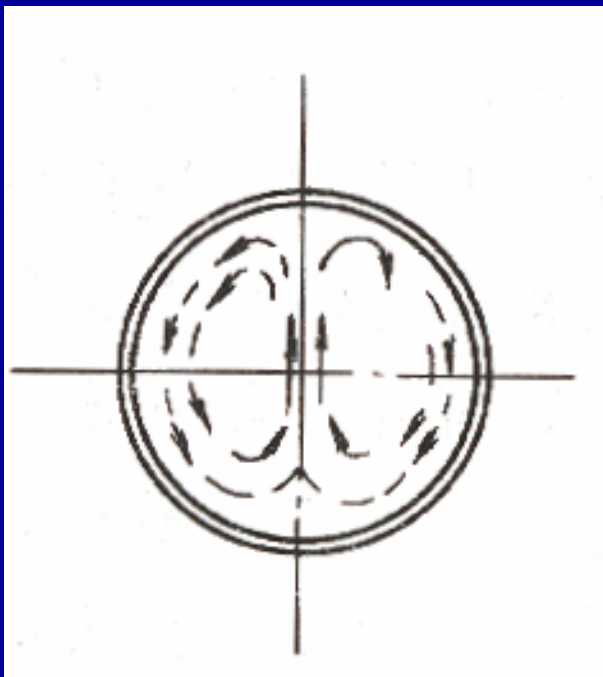
$$0.1 \leq \frac{Gr_m}{Re_m^2} \leq 10 \quad \text{— 混合对流换热(同时考虑两种对流)}$$



**FIG. 7-15**

Regimes of free, forced, and mixed convection for flow through vertical tubes, according to Ref. 10.

### 三、沿水平管内的混合对流换热



假设：流体被冷却

由于管内流体温度高于管壁温度，  
将形成沿管壁向下、由管芯向上的  
垂直于受迫流动方向的环流

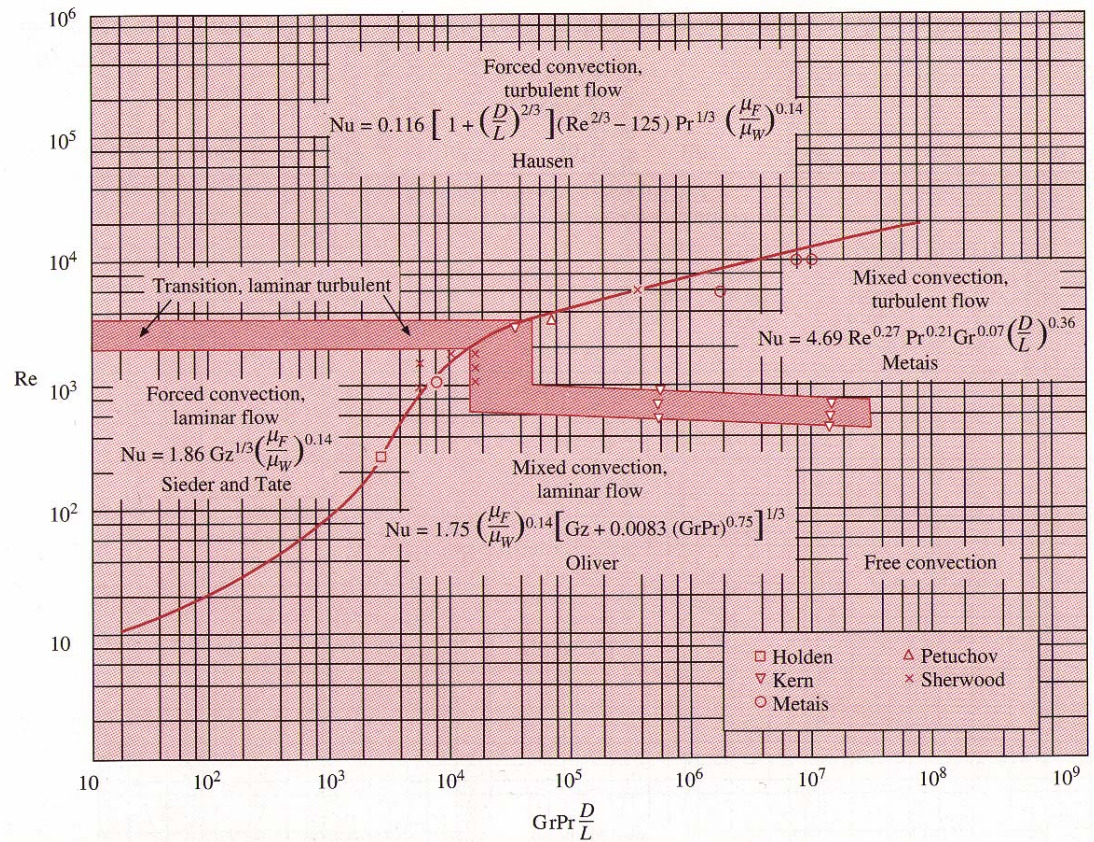
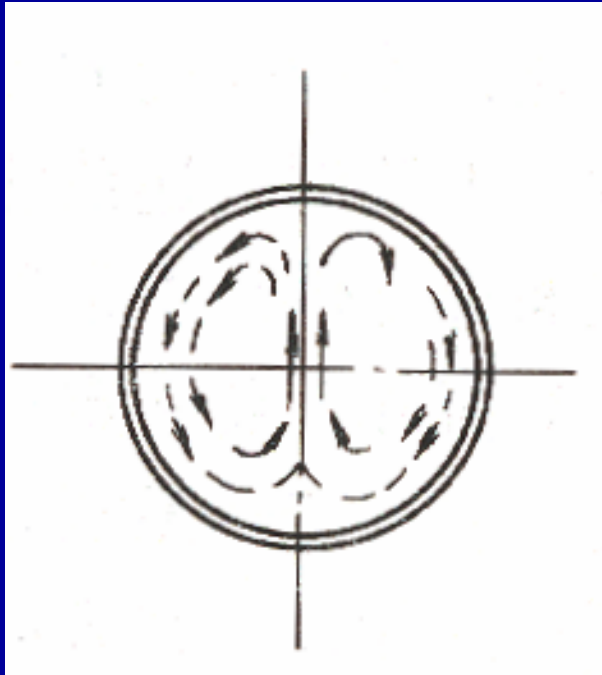
环流加强对流动的扰动—**换热增强**

对于混合对流换热过程 ( $0.02 \leq Gr_m / Re_m^2 \leq 10$ )

Metals and Eckert提出判断流态的准则：

$Gr_m Pr_m \frac{D}{L} \leq 2 \times 10^4$  混合对流换热中层流到紊流过渡  
的临界雷诺数为  $Re_{mc} \approx 2000$

$Gr_m Pr_m \frac{D}{L} \geq 2 \times 10^4$   $Re_{mc} \approx 800$



**FIG. 7-16**

Regimes of free, forced, and mixed convection for flow through horizontal tubes, according to Metais and Eckert [10].



**Brown and Gauvin**导出层流混合对流换热的计算式:

$$\text{Nu}_m = 1.75 \left[ \text{Gz}_m + 0.012 \left( \text{Gz}_m \text{Gr}_m^{1/3} \right)^{4/3} \right]^{1/3} \left( \frac{\mu_f}{\mu_w} \right)^{0.14}$$

$$\text{Gz}_m = \text{Re}_m \text{Pr}_m \frac{D}{L}$$

——**Graetz**准则

**Metaix and Eckert**提出湍流混合对流换热的计算式:

$$\text{Nu}_m = 4.69 \text{Re}_m^{0.27} \text{Pr}_m^{0.21} \text{Gr}_m^{0.07} \left( \frac{D}{L} \right)^{0.36}$$

定型尺度: 水平管内直径 **$D$**