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RESEARCH NOTES

An Explicit Solution for Thermal Calculation and Synthesis of Superstructure Heat Exchanger Networks*

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Abstract For the optimal design of a heat exchanger network, the inlet and outlet stream temperatures of each heat exchanger in the network should be known. An explicit analytical solution of stream temperatures of an arbitrary connected heat exchanger network was introduced, which is suitable for the thermal calculation of heat exchanger networks. For the heat exchanger network synthesis, this solution was further developed and coupled with the stage-wise superstructure heat exchanger networks. The new calculation procedure reduced the computer memory requirement dramatically. On the basis of this solution, a mathematical model for synthesis of heat exchanger networks with genetic algorithm was formulated, which is always feasible and no iteration is needed. Two examples were calculated with the proposed approach and better results were obtained.
Keywords heat recovery system, stage-wise superstructure, heat exchanger network synthesis, genetic algorithm, optimization

1 INTRODUCTION

In the past three decades, extensive efforts have been made in the fields of energy integration and energy recovery technologies because of the steadily increasing energy cost and CO_2 discharge. A heat recovery system consisting of a set of heat exchangers can be treated as a heat exchanger network (HEN), which is widely used in process industries such as gas processing and petrochemical industries, to exchange heat energy among several process streams with different supply temperatures. By the use of HENs, a large amount of utility costs such as the costs of steam and cooling water, as well as the costs of heaters and coolers, can be saved. However, it would increase the investment for the additional heat exchangers, and therefore a balance between the capital costs and running costs should be established. The task of the synthesis of HENs is to find a HEN which has the minimum total annualized cost of the process. Because of its significant benefits in saving energy consumption and equipment costs, the HEN synthesis has been considered as one of the most important research subjects in process engineering. The well-known procedures of HEN synthesis are the pinch method proposed by Linnhoff[1—4], the mathematical programming procedures developed by Grossmann and his coworkers[5—7], and the stochastic or heuristic algorithms such as genetic algorithm[8,9], genetic/simulated annealing algorithm[10—14], and Tabu search procedure[15].

In the above procedures, the heat transfer areas of heat exchangers are determined from the calculated stream temperatures of each heat exchanger. In a mathematical programming procedure the stream temperatures of a heat exchanger network will be simultaneously determined if a feasible initial point can be found, but this is often a difficult and tedious task. In other procedures, an iteration approach for stream temperatures of HENs should be used. Strelow[16,17] proposed a matrix form of solution, which is suitable for arbitrary heat exchanger networks. Luo *et al*.[18] and Roetzel and Luo[19] suggested a similar explicit solution of stream temperatures and applied it to HEN synthesis. The disadvantage of the matrix algorithm is that it needs a large size of computer memory, and therefore, it is not suitable for the synthesis of large scale HENs.

In the present work, a new mathematical model for the temperature calculation of HENs with the stage-wise superstructure proposed by Yee *et al*.[20] was formulated, and solved analytically and explicitly. The solution was applied to the calculation and synthesis of HENs and good results were obtained.

2 A GENERAL SOLUTION

Consider a heat exchanger network having M_E heat exchangers, N' stream entrances, and N'' stream exits. In each heat exchanger there are two fluid channels for hot and cold streams, respectively. Therefore, there are M channels in the HEN with $M=2M_E$. A general solution for such a HEN was given by Roetzel and Luo[19] and is described here with some improvements.

For each counterflow heat exchanger the outlet stream temperatures can be expressed as:

$$\begin{bmatrix} t_{h,k}(x'') \\ t_{c,k}(x'') \end{bmatrix} = \boldsymbol{V}_k \begin{bmatrix} t_{h,k}(x') \\ t_{c,k}(x') \end{bmatrix}, \text{ or } \boldsymbol{T}_k(x'') = \boldsymbol{V}_k \boldsymbol{T}_k(x') \quad (k = 1,\ 2,\ \cdots,\ M_E) \tag{1}$$

in which

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$$\boldsymbol{V}_k=\begin{bmatrix} v_{\mathrm{hh},k} & v_{\mathrm{hc},k} \\ v_{\mathrm{ch},k} & v_{\mathrm{cc},k}\end{bmatrix} =\begin{bmatrix} \dfrac{(1-R_k)\mathrm{e}^{-NTU_k(1-R_k)}}{1-R_k\mathrm{e}^{-NTU_k(1-R_k)}} & \dfrac{1-\mathrm{e}^{-NTU_k(1-R_k)}}{1-R_k\mathrm{e}^{-NTU_k(1-R_k)}} \\ \dfrac{R_k-R_k\mathrm{e}^{-NTU_k(1-R_k)}}{1-R_k\mathrm{e}^{-NTU_k(1-R_k)}} & \dfrac{1-R_k}{1-R_k\mathrm{e}^{-NTU_k(1-R_k)}}\end{bmatrix} \quad (2)$$

with $R_k=\dot{W}_{2k-1}/\dot{W}_{2k}$ and $NTU_k=(UA)_{2k-1,2k}/\dot{W}_{2k-1}$. Thus, the outlet fluid temperatures of all channels of the HEN can be expressed in a matrix form as

$$\boldsymbol{T}(x'')=\boldsymbol{VT}(x') \quad (3)$$

in which

$$\boldsymbol{T}=\left[t_{\mathrm{h},1},t_{\mathrm{c},1},t_{\mathrm{h},2},t_{\mathrm{c},2},\cdots,t_{\mathrm{h},M_{\mathrm{E}}},t_{\mathrm{c},M_{\mathrm{E}}}\right]^{\mathrm{T}} \quad (4)$$

$$\boldsymbol{V}=\begin{bmatrix}\boldsymbol{V}_1 & & \boldsymbol{0}\\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{V}_{M_{\mathrm{E}}}\end{bmatrix} \quad (5)$$

To illustrate the interconnections among the channels, entrances, and exits, the following four matching matrices were introduced.

Interconnection matrix $\boldsymbol{G}$:

$M\times M$ matrix whose elements g_{ij} are defined as the ratio of the thermal flow rate flowing from channel j into channel i to that flowing through channel i;

Entrance matching matrix $\boldsymbol{G}'$:

$M\times N'$ matrix whose elements g'_{ik} are defined as the ratio of the thermal flow rate flowing from the entrance k to channel i to that flowing through channel i;

Exit matching matrix $\boldsymbol{G}''$:

$N''\times M$ matrix whose elements g''_{li} are defined as the ratio of the thermal flow rate flowing from channel i to the exit l to that flowing out of exit l;

Bypass matrix $\boldsymbol{G}'''$:

$N''\times N'$ matrix whose elements g'''_{lk} are defined as the ratio of the thermal flow rate flowing from entrance k to exit l to that flowing out of exit l.

In a heat exchanger network, there might be such a knot at which the streams converge and split again, which can be defined as a mixer. If there is a mixer before channel i or exit l, then, in the above definitions of the matrices, the thermal flow rate flowing through the mixer should be taken as the denominator. With these matrices, the energy balances at the inlets of M channels and at the exits of N'' streams yield,

$$t_i(x'_i)=\sum_{k=1}^{N'}g'_{ik}t'_k+\sum_{j=1}^{M}g_{ij}t_j(x''_j) \quad (i=1,\ 2,\ \cdots,\ M) \quad (6)$$

$$t''_l=\sum_{k=1}^{N'}g'''_{lk}t'_k+\sum_{i=1}^{M}g''_{li}\ t_i(x''_i) \quad (l=1,\ 2,\ \cdots,\ N'') \quad (7)$$

respectively, or in a matrix form as

$$\boldsymbol{T}(x')=\boldsymbol{G}'\boldsymbol{T}'+\boldsymbol{GT}(x'') \quad (8)$$

$$\boldsymbol{T}''=\boldsymbol{G}'''\boldsymbol{T}'+\boldsymbol{G}''\boldsymbol{T}(x'') \quad (9)$$

By Substitution of Eq.(3) into Eqs.(8), (9), the inlet and outlet fluid temperatures of individual heat exchangers and the exit stream temperatures of the network were obtained as follows,

$$\boldsymbol{T}(x')=(\boldsymbol{I}-\boldsymbol{GV})^{-1}\boldsymbol{G}'\boldsymbol{T}' \quad (10)$$

$$\boldsymbol{T}(x'')=\boldsymbol{V}(\boldsymbol{I}-\boldsymbol{GV})^{-1}\boldsymbol{G}'\boldsymbol{T}' \quad (11)$$

$$\boldsymbol{T}''=[\boldsymbol{G}'''+\boldsymbol{G}''\boldsymbol{V}(\boldsymbol{I}-\boldsymbol{GV})^{-1}\boldsymbol{G}']\boldsymbol{T}' \quad (12)$$

in which $\boldsymbol{I}$ is a unit matrix.

The above temperature solution is general and explicit, and can be applied to the temperature calculations of HENs with arbitrary connections among them. Roetzel and Luo[19] illustrated some applications of the solution. However, this temperature solution is not suitable for the synthesis of large scale HENs because the number of possible heat exchangers in the HEN might be very large. For example, if the superstructure is used, as shown in Fig.1, for a synthesis task with 50 hot process streams and 50 cold process streams, then the possible number of heat exchangers is 125×10^3. The computer memory even for one $M\times M$ matrix would be 250GB. Fortunately, in the stage-wise superstructure HEN shown in Fig.1, the Interconnection matrix $\boldsymbol{G}$ and the bypass matrix $\boldsymbol{G}'''$ are zero matrices, therefore, Eqs.(10)—(12) reduce to,

$$\boldsymbol{T}(x')=\boldsymbol{G}'\boldsymbol{T}' \quad (13)$$

$$\boldsymbol{T}(x'')=\boldsymbol{VG}'\boldsymbol{T}' \quad (14)$$

$$\boldsymbol{T}''=\boldsymbol{G}''\boldsymbol{VG}'\boldsymbol{T}' \quad (15)$$

As the matrices $\boldsymbol{V}$, $\boldsymbol{G}'$, and $\boldsymbol{G}''$ are sparse matrices, the computer memory can be reduced by special algorithms.

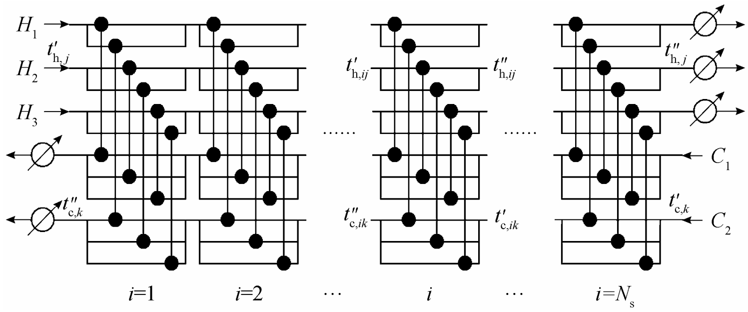

Figure 1 The stage-wise superstructure of a heat exchanger network

3 SOLUTION FOR SUPERSTRUCTURE HENS

In the present work a procedure for the thermal calculation of the stage-wise superstructure HENs shown in Fig.1 is proposed. Let N_h be the number of the hot process streams, N_c that of the cold ones, and N_s the number of stages of the structure. We use the subscripts i, j, and k to denote the stage, hot stream, and cold stream, respectively, and define $\boldsymbol{T}'$ and $\boldsymbol{T}''$ as the entrance and exit temperature vectors of the whole HEN, $\boldsymbol{T}_i'$ and $\boldsymbol{T}_i'$ the inlet and outlet temperature vectors of stage i, and $t''_{h,ijk}$ and $t''_{c,ijk}$ the outlet temperatures of the heat exchanger for hot stream j and cold stream k in stage i, respectively. Stage 1 lies at the left end of the HEN, and stage N_s lies at the right end. Then, Eq.(14) can be written for outlet stage temperatures of hot and cold streams as,

$$t''_{h,ij}=\sum_{k=1}^{N_c}\frac{\dot{W}_{h,ijk}}{\dot{W}_{h,j}}t''_{h,ijk}\quad (i=1,\ 2,\ \cdots,\ N_s;\ j=1,\ 2,\ \cdots,\ N_h)\tag{16}$$

$$t''_{c,ik}=\sum_{j=1}^{N_h}\frac{\dot{W}_{c,ijk}}{\dot{W}_{c,k}}t''_{c,ijk}\quad (i=1,\ 2,\ \cdots,\ N_s;\ k=1,\ 2,\ \cdots,\ N_c)\tag{17}$$

The outlet branch temperatures of hot and cold streams are given by Eq.(14) and can be written as,

$$t''_{h,ijk}=v_{hh,ijk}t'_{h,ij}+v_{hc,ijk}t'_{c,ik}$$
$$(i=1,\ 2,\ \cdots,\ N_s;\ j=1,\ 2,\ \cdots,\ N_h;\ k=1,\ 2,\ \cdots,\ N_c)\tag{18}$$

$$t''_{c,ijk}=v_{ch,ijk}t'_{h,ij}+v_{cc,ijk}t'_{c,ik}$$
$$(i=1,\ 2,\ \cdots,\ N_s;\ j=1,\ 2,\ \cdots,\ N_h;\ k=1,\ 2,\ \cdots,\ N_c)\tag{19}$$

in which $v_{hh,ijk}$, $v_{hc,ijk}$, $v_{ch,ijk}$, and $v_{cc,ijk}$ are the elements of $\boldsymbol{V}_{ijk}$ given by Eq.(2) for the heat exchanger for hot stream j and cold stream k in stage i. The substitution of Eqs.(18), (19) into Eqs.(16), (17) yields

$$t''_{h,ij}=\sum_{k=1}^{N_c}\frac{\dot{W}_{h,ijk}}{\dot{W}_{h,j}}v_{hh,ijk}t'_{h,ij}+\sum_{k=1}^{N_c}\frac{\dot{W}_{h,ijk}}{\dot{W}_{h,j}}v_{hc,ijk}t'_{c,ik}$$
$$(i=1,\ 2,\ \cdots,\ N_s;\ j=1,\ 2,\ \cdots,\ N_h)\tag{20}$$

$$t''_{c,ik}=\sum_{j=1}^{N_h}\frac{\dot{W}_{c,ijk}}{\dot{W}_{c,k}}v_{ch,ijk}t'_{h,ij}+\sum_{j=1}^{N_h}\frac{\dot{W}_{c,ijk}}{\dot{W}_{c,k}}v_{cc,ijk}t'_{c,ik}$$
$$(i=1,\ 2,\ \cdots,\ N_s;\ k=1,\ 2,\ \cdots,\ N_c)\tag{21}$$

The matrix form of Eqs.(20), (21) reads,

$$\boldsymbol{T}''_{h,i}=\boldsymbol{V}_{hh,i}\boldsymbol{T}'_{h,i}+\boldsymbol{V}_{hc,i}\boldsymbol{T}'_{c,i}\quad (i=1,\ 2,\ \cdots,\ N_s)\tag{22}$$

$$\boldsymbol{T}''_{c,i}=\boldsymbol{V}_{ch,i}\boldsymbol{T}'_{h,i}+\boldsymbol{V}_{cc,i}\boldsymbol{T}'_{c,i}\quad (i=1,\ 2,\ \cdots,\ N_s)\tag{23}$$

Eqs.(20), (21) [or Eqs.(22), (23)] give only the relation between the inlet and outlet temperature vectors in stage i. Unless N_s=1, the temperatures cannot be obtained directly because there are unknown inlet stage temperature vectors. The known inlet stage temperature vector of the hot streams is given at stage 1 and that of the cold streams is given at stage N_s. Let

$$\boldsymbol{T}''_{h,i}=\boldsymbol{V}^*_{hh,i}\boldsymbol{T}'_h+\boldsymbol{V}^*_{hc,i}\boldsymbol{T}'_{c,i}\quad (i=1,\ 2,\ \cdots,\ N_s)\tag{24}$$

$$\boldsymbol{T}''_c=\boldsymbol{V}^*_{ch,i}\boldsymbol{T}'_h+\boldsymbol{V}^*_{cc,i}\boldsymbol{T}'_{c,i}\quad (i=1,\ 2,\ \cdots,\ N_s)\tag{25}$$

which leads to

$$\boldsymbol{V}^*_{hh,1}=\boldsymbol{V}_{hh,1},\ \boldsymbol{V}^*_{hc,1}=\boldsymbol{V}_{hc,1},\ \boldsymbol{V}^*_{ch,1}=\boldsymbol{V}_{ch,1},\ \boldsymbol{V}^*_{cc,1}=\boldsymbol{V}_{cc,1}\tag{26}$$

Substituting Eq.(23) into Eq.(24) for stage $i-1$ and using the relation $\boldsymbol{T}'_{h,i}=\boldsymbol{T}''_{h,i-1}$, one obtains $\boldsymbol{T}'_{h,i}$ expressed by $\boldsymbol{T}'_h$ and $\boldsymbol{T}'_{c,i}$. This expression, together with Eq.(23) and the relation $\boldsymbol{T}''_{c,i}=\boldsymbol{T}'_{c,i-1}$, is further substituted into Eq.(25) for stage $i-1$ and Eq.(22), respectively, which yields $\boldsymbol{T}'_{h,i}$ and $\boldsymbol{T}''_c$ expressed by $\boldsymbol{T}'_h$ and $\boldsymbol{T}'_{c,i}$. Comparing them with Eqs.(24), (25) for stage i yields the recursion relations as,

$$\boldsymbol{V}^*_{hh,i}=\boldsymbol{V}_{hh,i}(\boldsymbol{I}-\boldsymbol{V}^*_{hc,i-1}\boldsymbol{V}_{ch,i})^{-1}\boldsymbol{V}^*_{hh,i-1}\quad (i=2,\ 3,\ \cdots,\ N_s)\tag{27}$$

$$\boldsymbol{V}^*_{hc,i}=\boldsymbol{V}_{hh,i}(\boldsymbol{I}-\boldsymbol{V}^*_{hc,i-1}\boldsymbol{V}_{ch,i})^{-1}\boldsymbol{V}^*_{hc,i-1}\boldsymbol{V}_{cc,i}+\boldsymbol{V}_{hc,i}$$
$$(i=2,\ 3,\ \cdots,\ N_s)\tag{28}$$

$$\boldsymbol{V}^*_{ch,i}=\boldsymbol{V}^*_{cc,i-1}\boldsymbol{V}_{ch,i}(\boldsymbol{I}-\boldsymbol{V}^*_{hc,i-1}\boldsymbol{V}_{ch,i})^{-1}\boldsymbol{V}^*_{hh,i-1}+\boldsymbol{V}^*_{ch,i-1}$$
$$(i=2,\ 3,\ \cdots,\ N_s)\tag{29}$$

$$\boldsymbol{V}^*_{cc,i}=\boldsymbol{V}^*_{cc,i-1}[\boldsymbol{V}_{ch,i}(\boldsymbol{I}-\boldsymbol{V}^*_{hc,i-1}\boldsymbol{V}_{ch,i})^{-1}\boldsymbol{V}^*_{hc,i-1}\boldsymbol{V}_{cc,i}+\boldsymbol{V}_{cc,i}]$$
$$(i=2,\ 3,\ \cdots,\ N_s)\tag{30}$$

For $i=N_s$, the following can finally be obtained,

$$\boldsymbol{T}''_h=\boldsymbol{V}^*_{hh,N_s}\boldsymbol{T}'_h+\boldsymbol{V}^*_{hc,N_s}\boldsymbol{T}'_c\tag{31}$$

$$\boldsymbol{T}''_c=\boldsymbol{V}^*_{ch,N_s}\boldsymbol{T}'_h+\boldsymbol{V}^*_{cc,N_s}\boldsymbol{T}'_c\tag{32}$$

By rewriting Eq.(22) as

$$\boldsymbol{T}'_{h,i}=\boldsymbol{V}^{-1}_{hh,i}(\boldsymbol{T}''_{h,i}-\boldsymbol{V}_{hc,i}\boldsymbol{T}'_{c,i})\quad (i=2,\ 3,\ \cdots,\ N_s)\tag{33}$$

and using Eq.(23), the stage temperature vectors $\boldsymbol{T}'_{h,i}$ and $\boldsymbol{T}''_{c,i}$ from stage N_s to stage 2 can be calculated. The branch temperatures can then be obtained with Eqs.(18), (19).

The above solution is more suitable for $N_h \leqslant N_c$ because in the above calculations the matrices to be inversed have a size of $N_h \times N_h$. If $N_h > N_c$, a similar solution can be obtained in which the matrices to be inversed have a size of $N_c \times N_c$.

4 MATHEMATICAL MODEL FOR HEN SYNTHESIS

A simple synthesis problem of HENs can be stated as follows: Given are N_h hot process streams, N_c cold process streams, a hot utility (HU), and a cold utility (CU). Specified are thermal capacity flow rates and the supply and target temperatures of each stream. Given also are the temperature levels and costs of the hot and cold utilities, the overall heat transfer coefficients of heat exchangers, heaters and coolers, and their costs. The objective is to determine the configuration of the HEN and the values of heat transfer areas and thermal flow rates of each heat exchanger in the HEN, which bring the total annualized cost of the HEN to the minimum. If the temperature solution is applied to the stage-wise superstructure HEN and the strategy of excessive use of utilities[8] are used, the objective function has a relatively simple form,

$$\min\left\{\sum_{n=1}^{N_\mathrm{h}+N_\mathrm{c}}\left\{C_\mathrm{CU}\max\left[\dot{W}_n(t''_n-\overset{+}{t}_{\mathrm{OUT},n}),\ 0\right]+C_\mathrm{HU}\max\left[\dot{W}_n(\overset{-}{t}_{\mathrm{OUT},n}-t''_n),\ 0\right]\right\}+\sum_{i=1}^{N_\mathrm{s}}\sum_{j=1}^{N_\mathrm{h}}\sum_{k=1}^{N_\mathrm{c}}z_{ijk}(a+bA_{ijk}^c)+\sum_{n=1}^{N_\mathrm{h}+N_\mathrm{c}}[z_{\mathrm{CU},n}(a+bA_{\mathrm{CU},n}^c)+z_{\mathrm{HU},n}(a+bA_{\mathrm{HU},n}^c)]\right\} \tag{34}$$

s.t.
$$\sum_{k=1}^{N_\mathrm{c}}\dot{W}_{\mathrm{h},ijk}=\dot{W}_{\mathrm{h},j}\quad(i=1,\ 2,\ \cdots,\ N_\mathrm{s};\ j=1,\ 2,\ \cdots,\ N_\mathrm{h}) \tag{35}$$

$$\sum_{j=1}^{N_\mathrm{h}}\dot{W}_{\mathrm{c},ijk}=\dot{W}_{\mathrm{c},k}\quad(i=1,\ 2,\ \cdots,\ N_\mathrm{s};\ k=1,\ 2,\ \cdots,\ N_\mathrm{c}) \tag{36}$$

where the binary variables z_{ijk}, $z_{\mathrm{CU},n}$ and $z_{\mathrm{HU},n}$ are determined by

$$z_{ijk}=\begin{cases}1, & A_{ijk}>0\\ 0, & A_{ijk}\leqslant 0\end{cases}\quad(i=1,\ 2,\ \cdots,\ N_\mathrm{s};\ j=1,\ 2,\ \cdots,\ N_\mathrm{h};\ k=1,\ 2,\ \cdots,\ N_\mathrm{c}) \tag{37}$$

$$z_{\mathrm{CU},n}=\begin{cases}1, & t''_n-\overset{+}{t}_{\mathrm{OUT},n}>0\\ 0, & t''_n-\overset{+}{t}_{\mathrm{OUT},n}\leqslant 0\end{cases}\quad(n=1,\ 2,\ \cdots,\ N_\mathrm{h}+N_\mathrm{c}) \tag{38}$$

$$z_{\mathrm{HU},n}=\begin{cases}1, & \overset{-}{t}_{\mathrm{OUT},n}-t''_n>0\\ 0, & \overset{-}{t}_{\mathrm{OUT},n}-t''_n\leqslant 0\end{cases}\quad(n=1,\ 2,\ \cdots,\ N_\mathrm{h}+N_\mathrm{c}) \tag{39}$$

$$A_{\mathrm{CU},n}=\begin{cases}\dfrac{\dot{W}_n(t''_n-\overset{+}{t}_{\mathrm{OUT},n})}{U_{\mathrm{CU},n}\Delta t_{\mathrm{mCU},n}}, & t''_n-\overset{+}{t}_{\mathrm{OUT},n}>0\\ 0, & t''_n-\overset{+}{t}_{\mathrm{OUT},n}\leqslant 0\end{cases}\quad(n=1,\ 2,\ \cdots,\ N_\mathrm{h}+N_\mathrm{c}) \tag{40}$$

$$A_{\mathrm{HU},n}=\begin{cases}\dfrac{\dot{W}_n(\overset{-}{t}_{\mathrm{OUT},n}-t''_n)}{U_{\mathrm{HU},n}\Delta t_{\mathrm{mHU},n}}, & \overset{-}{t}_{\mathrm{OUT},n}-t''_n>0\\ 0, & \overset{-}{t}_{\mathrm{OUT},n}-t''_n\leqslant 0\end{cases}\quad(n=1,\ 2,\ \cdots,\ N_\mathrm{h}+N_\mathrm{c}) \tag{41}$$

The exponent c over A_{ijk}, $A_{\mathrm{cu},n}$ and $A_{\mathrm{HU},n}$ is the cost parameter of heat exchangers, its value is between 0 and 1; and the temperature vector $\boldsymbol{T}=\left[t_{\mathrm{h},1},t_{\mathrm{h},2},\cdots,\ t_{\mathrm{h},N_\mathrm{h}},t_{\mathrm{c},1},t_{\mathrm{c},2},\cdots,t_{\mathrm{c},N_\mathrm{c}}\right]^\mathrm{T}$. If the heat transfer areas A_{ijk} and the thermal flow rates in branches $\dot{W}_{\mathrm{h},ijk}$ and $\dot{W}_{\mathrm{c},ijk}$ are given, the above model is explicit and is always feasible. Eqs.(35), (36) are the only remained constraints which can be easily treated.

A genetic algorithm is used to solve this HEN synthesis problem. The genetic algorithm simulates the evolution phenomena in nature. It deals with a group of individuals evolving generation by generation. Here a HEN is taken as an individual where the heat transfer areas, thermal capacity flow rates of hot streams, and those of cold streams of all possible heat exchangers are considered to be the genes of the individual. The genetic algorithm consists of three main operators: selection, crossover, and mutation. The individual with a higher value of fitness (lower value of the objective function) has a greater chance to be selected to produce his offspring by crossover, or to return directly to the next generation. By using a crossover operation, two selected parents are combined to form their offspring. A mutation operation will introduces new genes into the population to avoid the evolution converging into a local optimum. In the present work, the genetic/simulated annealing algorithm developed by Yu *et al.*[10] was used. The elite strategy was applied to the evolution of each population in the group. The hill-climbing algorithm was used in genetic algorithm (GA) to modify the individuals.

The complete procedure can be summarized as follows:

(1) Generate a population.

(2) Choose the best individual as the elite of this population, and replace the worst individual with the elite (elite strategy).

(3) Apply the simulated annealing algorithm to the population to form its pseudo population scattered adequately.

(4) Let the population compete with its pseudo population and accept the individuals by the Metropolis acceptance rule.

(5) Apply the crossover operator to the population to create its offspring population.

(6) Apply the mutation operator to the offspring population.

(7) Replace the population with its offspring population.

(8) Return to step (2) for the next generation.

(9) Terminate the procedure if there is no evolution after 20 generations.

According to the above procedures, the software for synthesis of heat exchanger network with user interface was developed, so that different values of algorithm parameters and different combinations of strategies could be chosen for variety synthesis problems.

The first example is taken from Ciric and Floudas[21]. It involves four hot streams, three cold streams, one hot utility, and one cold utility. The problem data are listed in Table 1. The overall heat transfer coefficient U=0.8kW·m^{-2}·K^{-1} is used for all matches. The HEN given by Ciric and Floudas consists of 12 exchangers and yields a total cost of 114460$ per annum. Wei[11] found an optimal configuration of multistream heat exchanger network for this example using the parallel GA/simulated annealing algorithm (SA). In his configuration, if the two-stream heat exchangers were used instead of the multistream heat exchangers, nine heat exchangers were needed, and the total annualized cost would be 107981$ per annum. In the present work, a better configuration was found, which has eight heat exchangers and the total annualized cost is 105661$ per annum, as is shown in Fig.2.

Table 1 Problem data for example 1

Stream	t', ℃	t_{OUT}, ℃	$\dot{W}$, kW·℃$^{-1}$	Annual cost, $·kW^{-1}
H_1	160	110	7.032	—
H_2	249	138	8.44	—
H_3	227	106	11.816	—
H_4	271	146	7.0	—
C_1	96	160	9.144	—
C_2	115	217	7.296	—
C_3	140	250	18.0	—
S (steam)	300	300	—	80
CW (cooling water)	70	90	—	20

Note: Heat transfer coefficient U=0.8kW·m^{-2}·K^{-1} for all matches. Heat exchanger cost=1300$A^{0.6}$$ per annum.

The second example is taken from Ravagnani *et al.*[22]. The problem data are given in Table 2. Ravagnani *et al.* solved this problem by optimizing the minimum temperature difference with a genetic algorithm and obtain a HEN of six heat exchangers including one heater and one cooler. The total annualized cost is 117069$ per annum. With the present model and genetic algorithm, a better configuration with five heat exchangers is found, as is shown in Fig.3, and the total annualized cost is 109765$ per annum.

Table 2 Problem data for example 2

Stream	t', ℃	t_{OUT}, ℃	$\dot{W}$, kW·℃$^{-1}$	h, kW·m^{-2}·℃$^{-1}$	Annual cost, $·kW^{-1}
H_1	175	45	10	2.615	—
H_2	125	65	40	1.333	—
C_1	20	155	20	0.917	—
C_2	40	112	15	0.166	—
S	180	179	—	5.000	110
CW	15	25	—	2.500	10

Note: Heat exchanger cost=1200$A^{0.57}$$ per annum.

It should be pointed out that the present model does not include all possible configurations of HENs, for example, a series connection of two heat exchangers in a branch cannot be represented by the stage-wise superstructure. Applying the present algorithm to the 10SP1 problem was also attempted. Because of the special deceptive characteristics of the 10SP1 problem,

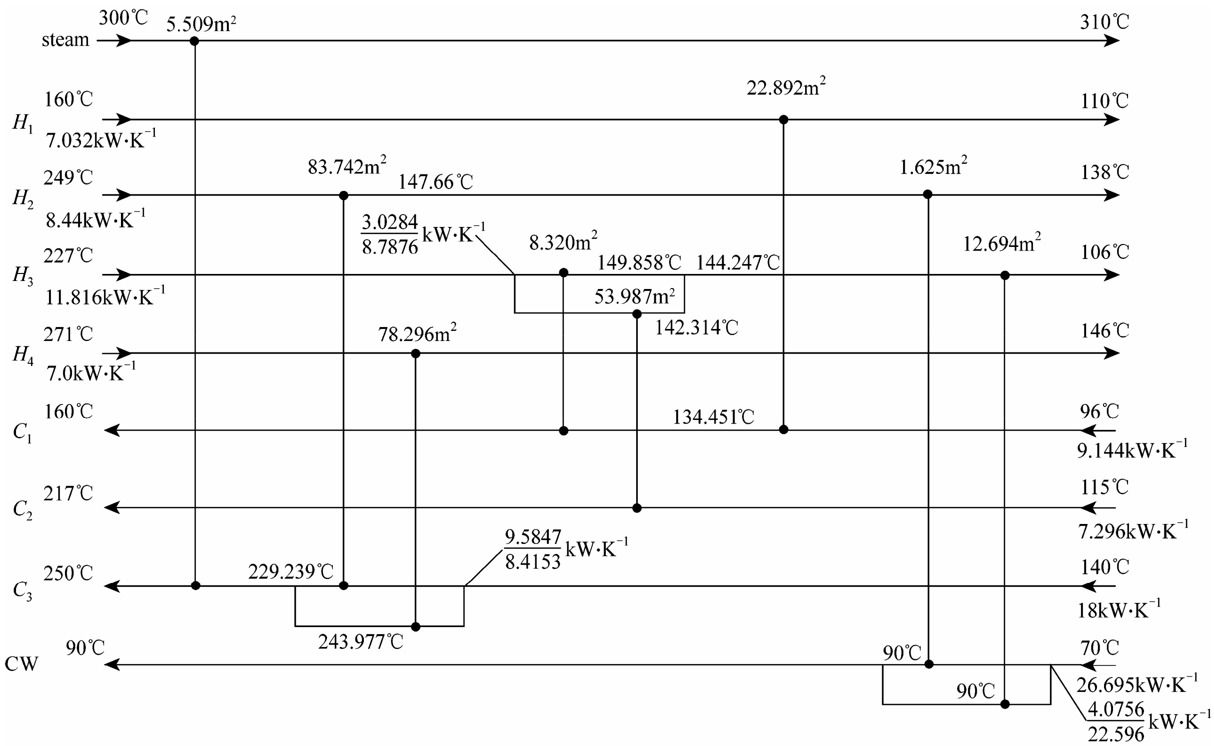

Figure 2 Optimal configuration of HEN for example 1

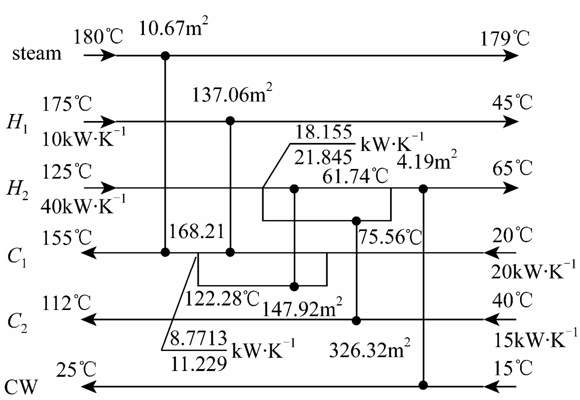

Figure 3 Optimal configuration of HEN for example 2

no results better than Lin and Miller[15] and Wei[11] could be found. The coding model of the genetic algorithm to enhance the structural search ability of the algorithm should be further developed.

NOMENCLATURE

A	heat transfer area of heat exchanger, m^2
a, b	parameters used for annual cost of heat exchangers, \$
c	parameter used for cost of heat exchangers
C_{CU}	annual cold utility cost, \$·kW^{-1}
C_{HU}	annual hot utility cost, \$·kW^{-1}
$\boldsymbol{G}$	interchannel matching matrix
$\boldsymbol{G}'$	entrance matching matrix
$\boldsymbol{G}''$	exit matching matrix
$\boldsymbol{G}'''$	bypass matrix
M	number of channels
M_{E}	number of heat exchangers
N'	number of stream entrances
N''	number of stream exits
N_{c}	number of cold process streams
N_{h}	number of hot process streams
N_{s}	number of stages of a stage-wise superstructure
$\boldsymbol{T}$	stream temperature vector, ℃
t'	supply temperature of stream, ℃
t''	outlet stream temperature of a network before the stream is heated or cooled by utilities, ℃
t_{OUT}^{+}, t_{OUT}^{-}	upper bound and lower bound of target temperature, ℃
U	overall heat transfer coefficient, kW·m^{-2}·K^{-1}
x'	inlet coordinate of heat exchanger, m
x''	outlet coordinate of heat exchanger, m
$\dot{W}$	thermal capacity flow rate, kW·K^{-1}
z	binary variable
Δt_{m}	logarithmic temperature difference, ℃

Subscripts

c	cold stream
CU	cold utility
h	hot stream
HU	hot utility

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