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Influence diagnostics in exponentiated-Weibull regression models with censored data

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Abstract

Diagnostic methods have been an important tool in regression analysis to detect anomalies, such as departures from the error assumptions and the presence of outliers and influential observations with the fitted models. The literature provides plenty of approaches for detecting outlying or influential observations in data sets. In this paper, we follow the local influence approach (Cook 1986) in detecting influential observations with exponentiated-Weibull regression models. The relevance of the approach is illustrated with a real data set, where it is shown that by removing the most influential observations, there is a change in the decision about which model fits the data better.

MSC: 62H10, 62J20, 62N01

Keywords: Exponentiated-Weibull distribution; censored data; local influence; influence diagnostic; survival data.

1 Introduction

In this paper we consider data sets representing the elapsed time until the occurrence of an event of interest such as the recurrence of a disease, death of a patient, failure of equipment, performance of a task, and so on.

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This elapsed time is generally termed *survival time* or life time. To simplify notation, the units under study are called individuals, and the data set is called survival data. It is common, in such kind of data, to have censored observations, that is, for some individuals the exact time of death is not known. It is only known that it lies beyond a certain value (censoring time). In this paper, we consider that the censoring times are random and noninformative. Moreover, in most situations, survival times can be affected by covariates (explanatory variables), such as the age at disease onset, blood pressure, cholesterol level, treatment type and many other important factors.

In this paper, we consider the exponentiated Weibull model, which includes as special cases the Weibull and exponential models. As considered by Mudhokar *et al.* (1995), it can be used in adjusting survival data with bathtub-type risk functions. Cancho *et al.* (1999) conducted a Bayesian study for exponentiated-Weibull regression models and Bolfarine and Cancho (2001) considered an exponentiated-Weibull survival model with a survival fraction.

An important step in regression analysis is to conduct a robustness study to detect influential or extreme observations that can cause important distortions on the results of the analysis. Numerous approaches have been proposed in the literature with a view to detect influential or outlying observations that can seriously affect parameter estimates. Studies of case deletion have were started with Cook (1977). Important reviews on the main approaches to detect influential observations are considered in Cook and Weisberg (1982) and Chaterjee and Hadi (1988).

A general framework to detect influence of observations was proposed by Cook (1986) and has often been applied with regression models. The method basically indicates how sensitive the analysis is when small perturbations are made to the data or the model. For instance, under the normal error, Lawrance (1988) investigated local influence applications in linear models with a response transformation parameter, Beckman *et al.* (1987) presented influence influence studies in mixed effects analysis of variance, Tsai and Wu (1992) considered first-order autoregressive models with nonconstant variances, and Paula (1993) used local influence methods with linear regression models when there are inequality constraints on the parameters. Moving away from normal models, Petit and Bin Daud (1989) investigated local influence with proportional hazard regression models, Escobar and Meeker (1992) adapted local influence methods to regression analysis with censoring, and O'Hara *et al.* (1992) and Kim (1995) applied local influence methods with multivariate regression. More recently, Galea *et al.* (1997) and Liu (2000) used local influence with elliptical linear regression models; Kwan and Fung (1998) applied the methodology to factor analysis and Gu and Fung (1998) discussed local influence in canonical correlation analysis. An interesting discussion and comparison with other influence measures is considered in Fung and Kwan (1997). An important extension of the method to assess the local influence of observations on the predictions from the fitted model was proposed by Thomas and Cook (1990).

In Sections 2 and 3, we review the exponentiated-Weibull regression model considered in Bolfarine *et al.* (2001). In Sections 4 and 5, we discuss the local influence method and local influence on predictions. Likelihood displacement is used to evaluate the influence of observations on the maximum likelihood estimators. Section 6 presents the results of an analysis with a real data set, including a residual analysis.

2 The exponentiated-Weibull distribution

The Weibull family of distributions has been widely used in the analysis of survival data specially in medical and engineering application. This family is suitable in situations where the risk function is constant or monotone. It is not, however, suitable in situations where the risk function is unimodal or presents a bathtub shape. Many parametric families have been considered for modeling survival data with a more general shape for the risk function. For example, Prentice (1974) considered the generalized F distribution; Stacy (1962) proposed the generalized gamma distribution while Mudhokar *et al.* (1995) presented an extension of the Weibull distribution, which is called the exponentiated Weibull family of distributions, and can adequately fit data sets presenting unimodal, monotone and bathtub shaped risk functions.

The exponentiated-Weibull distribution considered in Mudhokar et al. (1995) with parameters α, θ and σ considers that life time T has a density function given by

$$f(t;\alpha,\theta,\sigma) = \frac{\alpha\theta}{\sigma}\left[1-\exp\left(-\left(\frac{\mathrm{t}}{\sigma}\right)^{\alpha}\right)\right]\exp\left[-\left(\frac{\mathrm{t}}{\sigma}\right)^{\alpha}\right]\left(\frac{\mathrm{t}}{\sigma}\right)^{\alpha-1}, \quad \forall \mathrm{t}>0 \tag{1}$$

where $\alpha > 0$, $\theta > 0$ are shape parameters and $\sigma > 0$ is a scale parameter. As a special cases, there is the Weibull distribution when $\theta = 1$ and the exponential distribution when $\alpha = 1$, $\theta = 1$. The survival function corresponding to random variable T with exponentiated-Weibull density is given by

$$S(t;\alpha,\theta,\sigma) = P(T \geq t) = 1-\left[1-\exp\left(-\left(\frac{\mathrm{t_i}}{\sigma}\right)^{\alpha}\right)\right]^{\theta}. \tag{2}$$

The great flexibility of this model in fitting survival data can be depicted from its risk function, which can be monotonically decreasing if $\alpha \leq 1$ and $\alpha\theta \leq 1$, monotonically increasing if $\alpha \geq 1$ and $\alpha\theta \geq 1$ and present a bathtub shape if $\alpha > 1$ and $\alpha\theta < 1$.

Let $t_1, t_2, \ldots, t_n$ be a random sample of random variate T with exponentiated-Weibull distribution. The likelihood function corresponding to the observed sample is given by

$$L(t;\alpha,\theta,\sigma) = \alpha^r\theta^r\sigma^{-r\alpha}\exp\left[-\sum_{\mathrm{i}\in\mathrm{F}}\left(\frac{\mathrm{t_i}}{\sigma}\right)^{\sigma}\right]\prod_{\mathrm{i}\in\mathrm{F}}\mathrm{t_i^{\alpha-1}}\left(1-\exp\left[-\left(\frac{\mathrm{t_i}}{\sigma}\right)^{\alpha}\right]\right)^{\theta-1} \prod_{i\in C}\left[1-\left(1-\exp\left[-\left(\frac{\mathrm{t_i}}{\sigma}\right)^{\alpha}\right]\right)^{\theta},\right] \quad (3)$$

where r is the observed number of failures, F denotes the set of uncensored observations and C denotes the set of censored observations. No explicit expressions are available for the maximum likelihood estimators of α, σ and θ, which are obtained by maximizing the log-likelihood numerically. One approach that can be used is the Newton-Raphson algorithm.

3 Exponentiated-Weibull Regression models

In many practical applications, lifetimes are affected by covariates such as cholesterol level, blood pressure and many others. The covariate vector is denoted by $\mathbf{x} = (x_1, x_2, \ldots, x_p)^T$ which is related to responses $Y = \log(T)$ through a regression model. It is also considered that the scale parameter σ of the exponentiated-Weibull model depends on the matrix of explanatory variables X. Considering the transformation $\sigma = \exp(\mu)$ and $\alpha = 1/\delta$, it follows that the density function of Y can be written as

$$f(y) = \frac{\theta}{\delta}\left\{1-\exp\left[-\exp\left(\frac{\mathrm{y}-\mu}{\delta}\right)\right]\right\}^{\theta-1}\exp\left\{\left(\frac{\mathrm{y}-\mu}{\delta}\right)-\exp\left(\frac{\mathrm{y}-\mu}{\delta}\right)\right\} \quad (4)$$

$y > 0$, where $\alpha > 0$, $\theta > 0$, and $-\infty < \mu < \infty$. Using (4), we can write the above model as a log-linear model

$$Y = \mu + \delta Z \quad (5)$$

where variable Z follows the density

$$f(z) = \theta\{1-\exp[-\exp(\mathrm{z})]\}^{\theta-1}\exp[\mathrm{z}-\exp(\mathrm{z})], \quad \forall -\infty < \mathrm{z} < \infty \quad (6)$$

with survival function given by

$$S(y) = 1-\left\{1-\exp\left[-\exp\left(\frac{\mathrm{y}-\mu}{\delta}\right)\right]\right\}^{\theta}. \quad (7)$$

We consider now the regression model based on the log-exponentiated-Weibull given in (5), relating response Y and covariate vector $\mathbf{x}$, so that the conditional distribution $Y|\mathbf{x}$ can be represents as

$$Y_i = \mathbf{x}_i^T\beta + \delta Z_i, \quad i = 1, \ldots, n, \tag{8}$$

where $\beta = (\beta_1, \ldots, \beta_p)^T$, $\delta > 0$ and $\theta > 0$ are unknown parameters, $\mathbf{x}_i^T = (x_{i1}, x_{i2}, \ldots, x_{ip})$ is the explanatory vector and Z follows the distribution in (7).

In this case, the survival function of $Y|\mathbf{x}$ is given by

$$S(y) = 1 - \left\{1 - \exp\left[-\exp\left(\frac{y - \mathbf{x}^T\beta}{\delta}\right)\right]\right\}^{\theta}. \tag{9}$$

Moreover, corresponding to sample $(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)$ of n observations from distribution (4), where y_i represents the logarithm of the survival time and $\mathbf{x}_i$ the covariate vector associated with the i-th individual, the log-likelihood function can be written as

$$\begin{aligned} l(\gamma) &= r\log(\theta) - r\log(\delta) + (\theta - 1)\sum_{i\in F}\log\left\{1 - \exp[-\exp(z_i)]\right\} + \\ &\quad + \sum_{i\in F}\left[z_i - \exp(z_i)\right] + \sum_{i\in C}\log\left\{1 - [1 - \exp(-\exp(z_i))]^{\theta}\right\}, \end{aligned} \tag{10}$$

where r is the number of uncensored observations (failures) and $z_i = \frac{y_i - \mathbf{x}_i^T\beta}{\delta}$. Maximum likelihood estimates for the parameter vector $\gamma = (\theta, \delta, \beta^T)^T$ can be obtained by maximizing the likelihood function while Bayesian estimation is discussed by Cancho *et al.* (1999). In this paper, software Ox (MAXBFGS subroutine) (see Doornik, 1996) was used to compute maximum likelihood estimates (MLE). Covariance estimates for the maximum likelihood estimators $\widehat{\gamma}$ can also be obtained using the Hessian matrix. Confidence intervals and hypothesis testing can be conducted by using the large sample distribution of MLE which is a normal distribution with the covariance matrix as the inverse of the Fisher information as long as regularity conditions are satisfied. More specifically, the asymptotic covariance matrix is given by $\mathbf{I}^{-1}(\gamma)$ with $\mathbf{I}(\gamma) = -E[\ddot{\mathbf{L}}(\gamma)]$ such that $\ddot{\mathbf{L}}(\gamma) = \left\{\frac{\partial^2 l(\gamma)}{\partial\gamma\partial\gamma^T}\right\}$.

Since it is not possible to compute the Fisher information matrix $\mathbf{I}(\gamma)$ due to the censored observations (censoring is random and noninformative), it is possible to use in its place the matrix of second derivatives of the log likelihood, $-\ddot{\mathbf{L}}(\gamma)$, evaluated at the MLE $\gamma = \widehat{\gamma}$, which is consistent. Then

$$\ddot{\mathbf{L}}(\gamma) = \begin{pmatrix} \mathbf{L}_{\theta\theta} & \mathbf{L}_{\theta\delta} & \mathbf{L}_{\theta\beta} \\ . & \mathbf{L}_{\delta\delta} & \mathbf{L}_{\delta\beta} \\ . & . & \mathbf{L}_{\beta\beta} \end{pmatrix}$$

with the submatrices in appendix A.

4 Influence diagnostics

Let $l(\gamma)$ denote the log-likelihood function from the postulated model, where $\gamma = (\theta, \delta, \beta^T)^T$, and let ω be a $n \times 1$ vector of perturbations restricted to some open subset $\mathbf{\Omega} \subset \mathbb{R}^n$. The perturbations are made on the log-likelihood function. We will assume, in particular, the case-weights perturbation scheme such that the log-likelihood function takes the form

$$l(\gamma|\omega) = \sum_{i \in F} \omega_i \log f(y_i; \gamma) + \sum_{i \in C} \omega_i \log S(y_i; \gamma),$$

where $0 \leq \omega_i \leq 1$ and $\omega_0 = (1, 1, \ldots, 1)^T$ is the vector of no perturbation. Note that $l(\gamma|\omega_0) = l(\gamma)$. To assess the influence of the perturbations on the maximum likelihood estimate $\hat{\gamma}$, we consider the likelihood displacement

$$LD(\omega) = 2\{l(\hat{\gamma}) - l(\hat{\gamma}_\omega)\},$$

where $\hat{\gamma}_\omega$ denotes the maximum likelihood estimate under model $l(\gamma|\omega)$. The $LD(\omega)$ measures distance between $\hat{\gamma}$ and $\hat{\gamma}_\omega$ in terms of the log-likelihood difference. It is a nonnegative function with a global minimum at ω_0.

The idea of local influence (Cook, 1986) is concerned about characterizing the behaviour of $LD(\omega)$ around ω_0. The procedure consists in selecting a unit direction $\mathbf{d}$, $||\mathbf{d}|| = 1$, and then to consider the plot of $LD(\omega_0 + a\mathbf{d})$ against a, where $a \in \mathbb{R}$. This plot is called *lifted line*. Note that, since $LD(\omega_0) = 0$, $LD(\omega_0 + a\mathbf{d})$ has a local minimum at $a = 0$. Each lifted line can be characterized by considering the normal curvature $C_{\mathbf{d}}(\gamma)$ around $a = 0$. This curvature is interpreted as the inverse radius of the best fitting circle at $a = 0$. The suggestion is to consider direction $\mathbf{d}_{max}$ corresponding to the largest curvature $C_{\mathbf{d}_{max}}(\gamma)$. The index plot of $\mathbf{d}_{max}$ may reveal those observations that, under small perturbations, exercise notable influence on $LD(\omega)$. Cook(1986) showed that normal curvature at direction $\mathbf{d}$ takes the form $C_{\mathbf{d}}(\gamma) = 2|\mathbf{d}^T \mathbf{\Delta}^T (\ddot{\mathbf{L}})^{-1} \mathbf{\Delta} \mathbf{d}|$ where $-\ddot{\mathbf{L}}$ is the observed Fisher information matrix for the postulated model ($\omega = \omega_0$) and $\mathbf{\Delta}$ is the $(p+1) \times n$ matrix with elements $\Delta_{ji} = \partial^2 L(\gamma|\omega)/\partial\theta_i \partial\omega_j$, evaluated at $\gamma = \hat{\gamma}$ and $\omega = \omega_0$, $j = 1, \ldots, p+2$ and $i = 1, \ldots, n$. Then, $C_{\mathbf{d}_{max}}$ is the largest eigenvalue of the matrix $\mathbf{B} = \mathbf{\Delta}^T (\ddot{\mathbf{L}})^{-1} \mathbf{\Delta}$, and $\mathbf{d}_{max}$ is the corresponding eigenvector. The index plot of $\mathbf{d}_{max}$ for matrix $\mathbf{\Delta}^T (\ddot{\mathbf{L}})^{-1} \mathbf{\Delta}$ may show how to perturb the log-likelihood function to obtain larger changes in the estimate of γ. We find, after some algebraic manipulation, the following expressions for the weighted log-likelihood function and for the elements of matrix $\mathbf{\Delta}$:

In this case the log-likelihood function takes the form

$$l(\gamma|\omega) = \Big[\mathrm{r}\log(\theta) - \mathrm{r}\log(\delta) \Big] \sum_{i \in F} w_i + (\theta - 1) \sum_{i \in F} w_i \log \Big\{ 1 - \exp\Big[- \exp\Big(\frac{\mathrm{y_i} - \mathbf{x}_i^{\mathrm{T}} \beta}{\delta} \Big) \Big] \Big\} \tag{11}$$

$$+\sum_{i\in F} w_i\left[\frac{y_i-\mathbf{x}_i^T\beta}{\delta}\right]-\sum_{i\in F} w_i\exp\left[\frac{\mathrm{y_i}-\mathbf{x}_\mathrm{i}^\mathrm{T}\beta}{\delta}\right]+$$
$$+\sum_{i\in C} w_i\log\left\{1-\left[1-\exp\left(-\exp\left(\frac{\mathrm{y_i}-\mathbf{x}_\mathrm{i}^\mathrm{T}\beta}{\delta}\right)\right)\right]^{\theta}\right\}$$

Let us denote $\mathbf{\Delta}=(\mathbf{\Delta}_1,\ldots,\mathbf{\Delta}_{p+2})^T$.
Then the elements of vector $\mathbf{\Delta}_1$ take the form

$$\Delta_{1i}=\begin{cases}\dfrac{r}{\widehat{\theta}}+\log(\widehat{g_i}) & \text{if} \quad i\in F\\[2ex] -\dfrac{(\widehat{g_i})^{\widehat{\theta}}}{\left[1-(\widehat{g_i})^{\widehat{\theta}}\right]}\log\left[(\widehat{g_i})^{\widehat{\theta}}\right] & \text{if} \quad i\in C\end{cases}$$

On the other hand, the elements of vector $\mathbf{\Delta}_2$ can be shown to be given by

$$\Delta_{2i}=\begin{cases}\dfrac{1}{\widehat{\delta}}\left\{-r-\dfrac{[\widehat{\theta}-1]\,\widehat{h_i}\,\widehat{z_i}}{\widehat{g_i}}-\widehat{z_i}\,[1-\exp\{\widehat{\mathrm{z_i}}\}]\right\} & \text{if} \quad i\in F\\[2ex] \dfrac{\widehat{\theta}\widehat{z_i}\,\widehat{h_i}\,(\widehat{g_i})^{\widehat{\theta}-1}}{\widehat{\delta}\,[1-(\widehat{g_i})]^{\widehat{\theta}}} & \text{if} \quad i\in C\end{cases}$$

The elements of vector $\mathbf{\Delta}_j$, for $j=3,\ldots,p+2$, may be expressed as

$$\Delta_{ji}=\begin{cases}\dfrac{x_{ij}}{\widehat{\delta}}\left\{-\dfrac{(\widehat{\theta}-1)\,\widehat{h}-i}{\widehat{g_i}}+\exp\{\widehat{\mathrm{z_i}}\}-1\right\} & \text{if} \quad i\in F\\[2ex] \dfrac{x_{ij}\,\widehat{\theta}\,\widehat{h_i}\,(\widehat{g_i})^{\widehat{\theta}-1}}{\widehat{\delta}\left[1-(\widehat{g_i})^{\widehat{\theta}}\right]} & \text{if} \quad i\in C\end{cases}$$

where

$$\widehat{h_i}=\exp\left[\widehat{\mathrm{z_i}}-\exp\{\widehat{\mathrm{z_i}}\}\right],\quad \widehat{\mathrm{g_i}}=1-\exp\left[-\exp\{\widehat{\mathrm{z_i}}\}\right]\quad \mathrm{e}\quad \widehat{\mathrm{z_i}}=\frac{\mathrm{y_i}-\mathbf{x}_\mathrm{i}^\mathrm{T}\widehat{\beta}}{\widehat{\delta}}$$

However, if the interest is only in vector β, the normal curvature in direction $\mathbf{d}$ is given by $C_{\mathbf{d}}(\beta)=2|\mathbf{d}^T\mathbf{\Delta}^T(\ddot{\mathbf{L}}^{-1}-\mathbf{B}_{22})\mathbf{\Delta}\mathbf{d}|$ (see Cook, 1986), where

$$\mathbf{B}_{22}=\begin{pmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \ddot{\mathbf{L}}_{22}^{-1}\end{pmatrix}$$

with $\ddot{\mathbf{L}}_{22}$ denoting the submatrix of $\ddot{\mathbf{L}}$ obtained according to partition

$$\ddot{\mathbf{L}}(\gamma) = \begin{pmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{pmatrix}$$

The index plot of the largest eigenvector of $\mathbf{\Delta}^T(\ddot{\mathbf{L}}^{-1} - \mathbf{B}_{22})\mathbf{\Delta}$ may reveal those observations most influential on $\hat{\beta}$.

On the other hand, considering the direction for the i-th individual the total local influence in that direction is given by

$$C_i = 2|\mathbf{\Delta}_i^T(\ddot{\mathbf{L}}^{-1} - \mathbf{B}_{22})\mathbf{\Delta}_i|. \quad (12)$$

5 Local influence on predictions

Let $\mathbf{z}$ a $p \times 1$ be a vector of values of the explanatory variables, for which we do not have necessarily an observed response. Then, the prediction at $\mathbf{z}$ is $\hat{\mu}(\mathbf{z}) = \sum_{j=1}^{p} z_j\hat{\beta}_j$. Analogously, the point prediction at $\mathbf{z}$ based on the perturbed model becomes $\hat{\mu}(\mathbf{z}, \omega) = \sum_{j=1}^{p} z_j\hat{\beta}_{j\omega}$, where $\hat{\beta}_\omega = (\hat{\beta}_{1\omega}, \ldots, \hat{\beta}_{p\omega})^T$ denotes the maximum likelihood estimate from the perturbed model. Thomas and Cook (1990) have investigated the effect of small perturbations on predictions at some particular point $\mathbf{z}$ in continuous generalized linear models and by assuming ϕ known or estimated separately from $\hat{\beta}$. ϕ^{-1} is defined as a dispersion parameter. For more details, see McCullagh and Nelder (1989). They defined three objective functions based on different residuals. Because the diagnostic calculations were identical for the proposed functions, they concentrated the application of the methodology on the objective function $f(\mathbf{z}, \omega) = \{\hat{\mu}(\mathbf{z}) - \hat{\mu}(\mathbf{z}, \omega)\}^2$.

Similarly, we will concentrate our study on investigating the normal curvature of the surface formed by vector ω and function $f(\mathbf{z}, \omega)$, around ω_0. The normal curvature at unit direction $\mathbf{d}$ takes, in this case, form $C_{\mathbf{d}}(\mathbf{z}) = 2 \mid \mathbf{d}^T\ddot{\mathbf{f}}\mathbf{d} \mid$, where $\ddot{\mathbf{f}} = \partial^2 f/\partial\omega\partial\omega^T$ is evaluated at ω_0 and $\hat{\beta}$. From Thomas and Cook (1990) one has that

$$\ddot{\mathbf{f}} = \mathbf{\Delta}^T(\ddot{\mathbf{L}}_{\beta\beta}^{-1}\mathbf{z}\mathbf{z}^T\ddot{\mathbf{L}}_{\beta\beta}^{-1})\mathbf{\Delta},$$

where $\mathbf{\Delta} = \partial^2 l(\gamma \mid \omega)/\partial\beta\partial\omega^T$. Consequently

$$\mathbf{d}_{max}(\mathbf{z}) \propto -\mathbf{\Delta}^T\ddot{\mathbf{L}}_{\beta\beta}^{-1}\mathbf{z}.$$

In the sequel, we discuss the calculation of $\mathbf{d}_{max}(\mathbf{z})$ under additive perturbations for the response and for each continuous explanatory variable.

5.1 Response perturbation

Consider the regression model (8) by assuming now that each y_i is perturbed as $y_i \rightarrow y_i + \sigma\omega_i = y_i^*$, $i = 1, \ldots, n$, with σ playing a role of scale parameter. Below, we give the expressions for the log-likelihood function and for the elements of matrix $\mathbf{\Delta}$, with $z_i^* = (y_i^* - \mathbf{x}_i^T\beta)/\delta$, $i = 1, \ldots, n$.

Here, the perturbed log-likelihood function becomes expressed as

$$
\begin{aligned}
l(\gamma|\omega) \;=\; & \Big[\mathrm{rlog}(\theta) - \mathrm{rlog}(\delta)\Big] + (\theta - 1)\sum_{i\in F}\log\Big\{1 - \exp\Big[-\exp\Big(\frac{\mathrm{y}_\mathrm{i}^* - \mathbf{x}_\mathrm{i}^\mathrm{T}\beta}{\delta}\Big)\Big]\Big\} \qquad (13)\\
& + \sum_{i\in F}\Big[\frac{y_i^* - \mathbf{x}_i^T\beta}{\delta}\Big] - \sum_{i\in F}\exp\Big[\frac{\mathrm{y}_\mathrm{i}^* - \mathbf{x}_\mathrm{i}^\mathrm{T}\beta}{\delta}\Big] + \\
& + \sum_{i\in C}\log\Big\{1 - \Big[1 - \exp\Big(-\exp\big(\frac{\mathrm{y}_\mathrm{i}^* - \mathbf{x}_\mathrm{i}^\mathrm{T}\beta}{\delta}\big)\Big)\Big]^\theta\Big\}
\end{aligned}
$$

where $y_i^* = y_i + \sigma\omega_i$.

Matrix $\mathbf{\Delta} = (\mathbf{\Delta}_1, \ldots, \mathbf{\Delta}_{p+2})^T$ is given in appendix B.

Vector $\mathbf{d}_{max}(\mathbf{z})$ is constructed by taking $\mathbf{z} = \mathbf{x}_i$, which corresponds to the $n \times 1$ vector

$$\mathbf{d}_{max}(\mathbf{x}_i) \propto -\mathbf{\Delta}^T \ddot{\mathbf{L}}_{\beta\beta}^{-1}\mathbf{x}_i. \qquad (14)$$

A large value for the ith component of (15), $\mathbf{d}_{max_i}(\mathbf{x}_i)$, indicates that the ith observation should have substantial local influence on $\hat{y}_i$. Then, the suggestion is to take the index plot of the $n \times 1$ vector $(\mathbf{d}_{max_1}(\mathbf{x}_1), \ldots, \mathbf{d}_{max_n}(\mathbf{x}_n))^T$ in order to identify those observations with high influence on its own fitted value.

5.2 Explanatory variable perturbation

Consider now an additive perturbation on a particular continuous explanatory variable, namely X_t, by making $x_{it\omega} = x_{it} + \omega_i S_t$, where S_t is a scaled factor. This perturbation scheme leads to the following expressions for the log-likelihood function and for the elements of matrix $\mathbf{\Delta}$.

The perturbed log-likelihood function is, in this case, expressed as

$$
\begin{aligned}
l(\gamma|\omega) \;=\; & \Big[\mathrm{rlog}(\theta) - \mathrm{rlog}(\delta)\Big] + (\theta - 1)\sum_{i\in F}\log\Big\{1 - \exp\Big[-\exp\Big(\frac{\mathrm{y}_\mathrm{i} - \mathbf{x}_\mathrm{i}^{*\mathrm{T}}\beta}{\delta}\Big)\Big]\Big\} \qquad (15)\\
& + \sum_{i\in F}\Big[\frac{y_i - \mathbf{x}_i^{*T}\beta}{\delta}\Big] - \sum_{i\in F}\exp\Big[\frac{\mathrm{y}_\mathrm{i} - \mathbf{x}_\mathrm{i}^{*\mathrm{T}}\beta}{\delta}\Big] +
\end{aligned}
$$

$$+\sum_{i\in C}\log\Big\{1-\Big[1-\exp\Big(-\exp\big(\frac{\mathrm{y_i}-\mathbf{x}_\mathrm{i}^{*\mathrm{T}}\beta}{\delta}\big)\Big)\Big]^{\theta}\Big\}$$

where $\mathbf{x}_i^{*T} = \beta_1 + \beta_2 x_{i2} + \cdots + \beta_t(x_{it} + \omega_i S_t) + \cdots + \beta_p x_{ip}$.

Matrix $\mathbf{\Delta} = (\mathbf{\Delta}_1, \ldots, \mathbf{\Delta}_{p+2})^T$ is given in appendix C.

Similarly to the response perturbation case the suggestion here is to evaluate the largest curvature at $\mathbf{z} = \mathbf{x}_i$, which leads to

$$C_{max}(\mathbf{x}_i) = 2|\mathbf{d}_{max}^T \ddot{\mathbf{f}} \mathbf{d}_{max}|,$$

and consequently

$$\mathbf{d}_{max}(\mathbf{x}_i) \propto -\mathbf{\Delta}^T \ddot{\mathbf{L}}_{\beta\beta}^{-1} \mathbf{x}_i.$$

To see for which observed values of X_t the prediction is most sensitive under small changes in X_t, we can perform the plot of $C_{max}(\mathbf{x}_i)$ against x_{it}. The index plot of the $n\times 1$ vector $(\ell_{max_1}(\mathbf{x}_1), \ldots, \ell_{max_n}(\mathbf{x}_n))^T$ can indicate those observations for which a small perturbation in the value of X_t leads to a substantial change in the prediction.

6 Application

We provide an application of the results derived in the previous sections using simulated and real data. The required numerical evaluations were implemented using program Ox (see Doornik, 1996).

6.1 Simulation study

We conducted a simulation study to analyze the behaviour of the local influence on the exponentiated-Weibull model. The simulated data consisting of 30 uncensored observations generated from the exponentiated-Weibull distribution with $z_i = \frac{y_i - \beta_0 - \beta_1 x_i}{\delta}$. Parameter values considered were $\theta = 4$, $\delta = 2$, $\beta_0 = 4$ e $\beta_1 = 2$.

To illustrate the behaviour of the approach developed in the paper, we modified observation 26, that is, we changed $y_{26} \longrightarrow y_{26} + S_t$, where S_t corresponds to the standard deviation of response Y. Parameter estimates are presented in the Table 1.

Table 1: *Maximum likelihood estimates with standard error (SE) for simulated data.*

Parameter	Estimate	SE
θ	3.1879	4.6234
δ	1.7248	1.3133
β_0	4.2291	2.0272
β_1	1.9972	0.0288

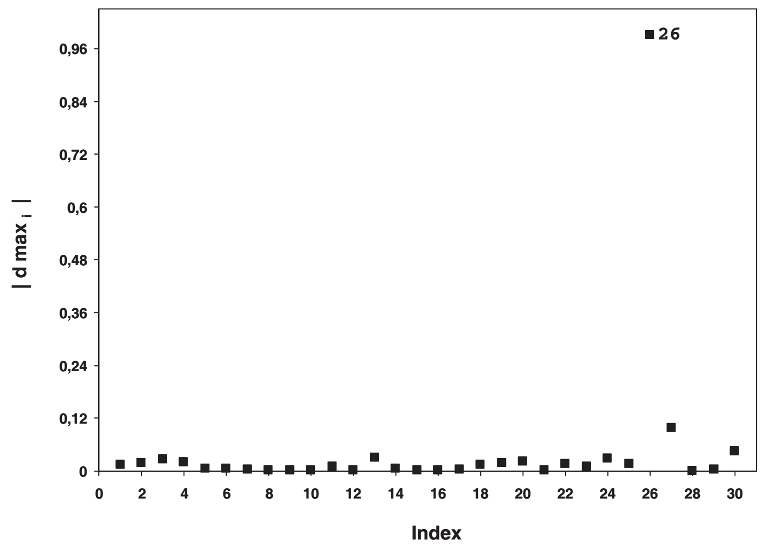

Figure 1: *Index plot of* $\mathbf{d}_{max}$ *for* γ *the simulated data (case-weights perturbation).*

Considering local influence with case weights, we obtain the maximum curvature $C_{\mathbf{d}_{\max}} = 14.677$. Vector $\mathbf{d}_{\max}$ corresponding to the direction of maximum curvature is plotted against the observation index in Figure 1, where it is clearly noted that observation 26 stands out as a possible influential observation. Similarly, Figure 2 total local influence for all observations. Observation 26 again stands out.

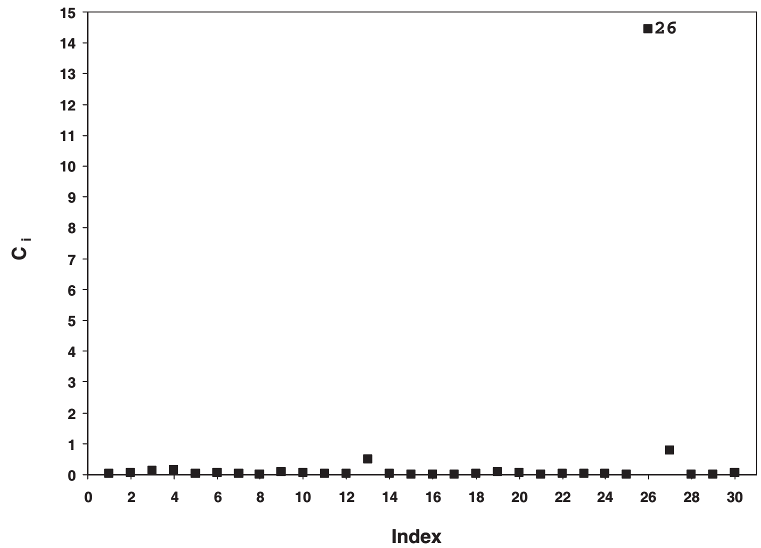

Figure 2: *Total local influence on the estimates* γ *the simulated data (case-weights perturbation).*

6.2 Golden shiner data

Survival times for the golden shiner, Notemigonus crysoleucas, were obtained from field experiments conducted in Lake Saint Pierre, Quebec, in 2005 (Laplante *et al.*, unpublished data). Individual fish were attached by means of a monofilament cord to a chronographic tethering device that allowed the fish to swim in midwater. A timer in the device was set off when the tethered fish was captured by a predator. The device was retrieved approximately 24 h after the onset of an experiment, and survival time was then obtained from the difference: time elapsed between onset of experiment and retrieval-time elapsed in device timer since predation event. The variables involved in the study were:

- y_i: survival time observed (in hours);
- $cens_i$: censoring indicator (0 = censoring, 1 = lifetime observed);
- x_{i1}: north or south bank of the lake (0 = north, 1 = south);
- x_{i2}: distance over the longitudinal axis of the lake (in km);
- x_{i3}: size of the fish (in cm);
- x_{i4}: depth of the place (in cm);
- x_{i5}: abundance index of macro-thin plants (in percentage);
- x_{i6}: transparency of the water (in cm);
- x_{i7}: initial time.

We present now results from fitting the model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \beta_6 x_{i6} + \beta_7 x_{i7} + \delta z_i \qquad (16)$$

where variable Z_i follows the log-exponentiated-Weibull distribution given in (6), $i = 1, 2, \ldots, 106$. To obtain the maximum likelihood estimates for the parameters in the model, we used the subroutine MAXBFGS in Ox, whose results are given in the following table.

Table 2: *Maximum likelihood estimates for the complete data set.*

Parameter	Estimate	SE	p-value
θ	9.4958	136.570	—
δ	4.5059	3.759000	—
β_0	−0.07456	30.482000	0.5053
β_1	2.1253	0.261380	<0.0001
β_2	0.0093338	0.0001398	0.2150
β_3	−0.12357	0.000951	<0.0001
β_4	0.033788	0.000083	<0.0001
β_5	0.022252	0.000276	0.0900
β_6	0.22427	0.040549	0.1320
β_7	−0.049872	0.025275	0.3771

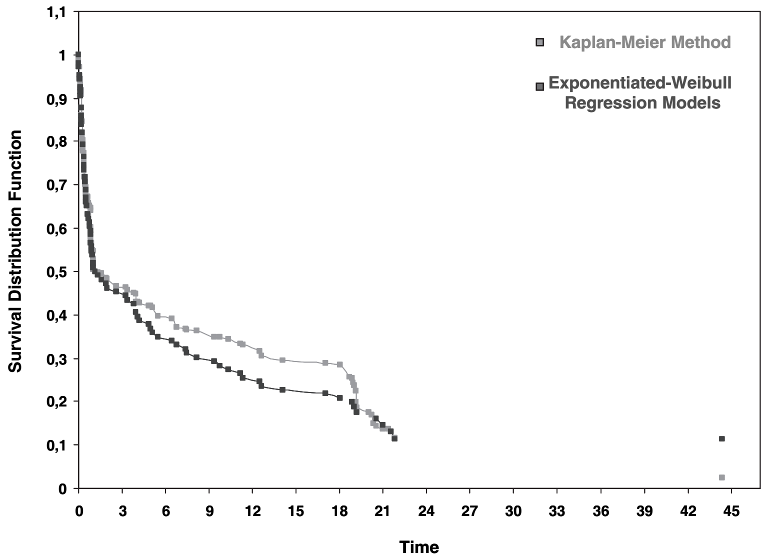

Figure 3: *Plot of the Survivor Function.*

We can see that variables x_1, x_2 and x_3 are significant for the model. We can also observe, in Figure 3, the empirical distribution function for the survival function as well as the survival function estimated by the exponentiated-Weibull regression model, where it is possible to notice a distant point in time.

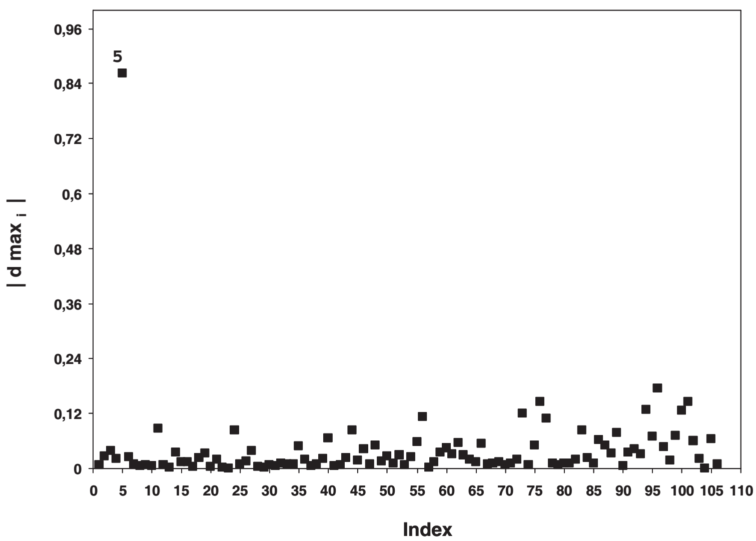

Figure 4: *Index plot of* $\mathbf{d}_{max}$ *for* γ *(case-weights perturbation).*

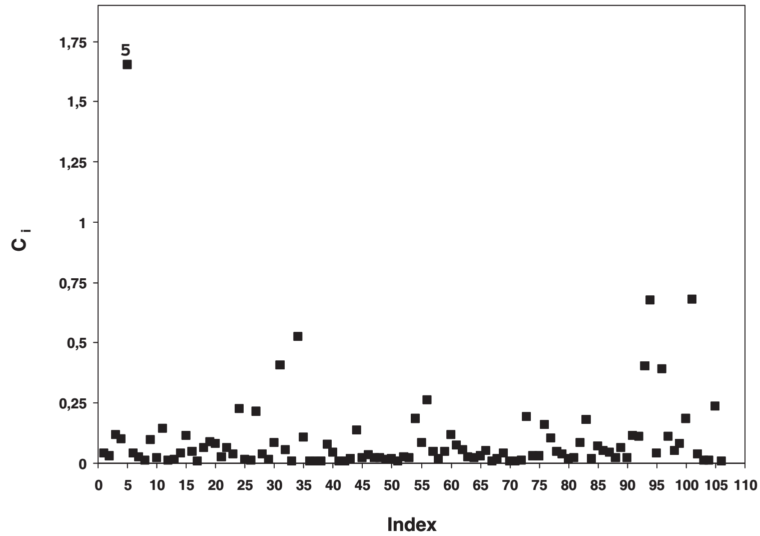

Figure 5: *Total local influence on the estimates* γ *(case-weights perturbation).*

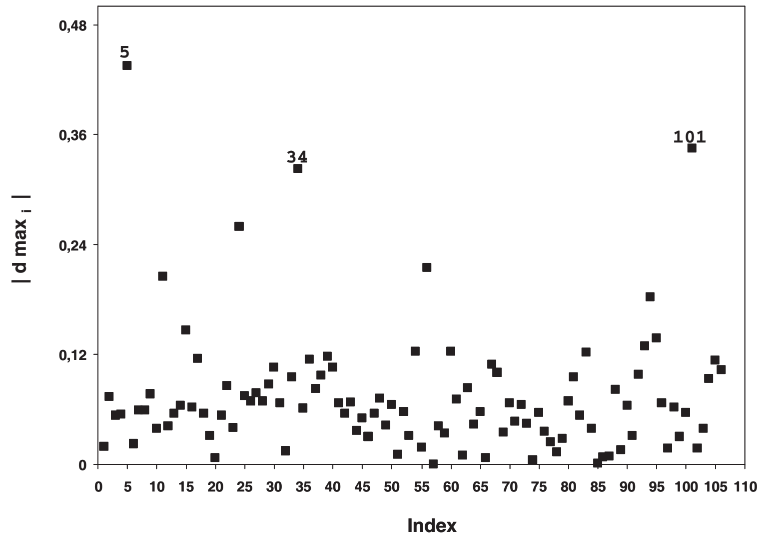

Figure 6: *Index plot of* $\mathbf{d}_{max}$ *for* γ *(response perturbation).*

6.2.1 Cases-weights perturbation

Using the exponentiated-Weibull regression model in (16), it follows that $C_{\mathbf{d}max} = 2.0943$ with eigenvectors corresponding to $C_{\mathbf{d}max}$plotted in Figure 4 presents the plot of the eigenvector corresponding to the whole vector γ. Clearly, the most influential is

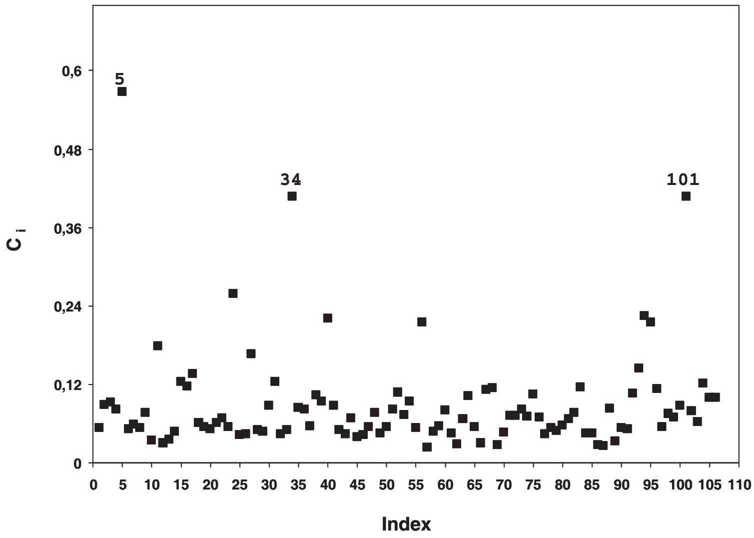

Figure 7: *Total local influence on the estimates γ (response perturbation).*

observation 5. We also use the total local influence index given in (12), whose response be found in Figure 5. We also found observation 5 as a possible influent point.

6.2.2 Prediction influence using response variable perturbation

We consider now the influence on predictions by using model (16) and the objective function proposed by Thomas and Cook(1990) as discussed in Section (5). Figure 6 and 7 present influence on the predictions by using additive perturbation in the observed response **y** ($C_{\mathbf{d}max} = 2.6454$).

6.3 Residual analysis

In order to study departures from the error assumption as well as the presence of outliers, we will first consider the martingale residual proposed by Barlow and Prentice (1988) (see also Therneau *et al.*, 1990). This residual was introduced in counting processes and can be adapted for the exponentiated-Weibull regression models as

$$r_{M_i} = \delta_i + log[S(y_i, \hat{\gamma})]$$

where $\delta_i = 0$ denotes censored observation, $\delta_i = 1$ uncensored and $S(y_i, \hat{\gamma})$ is as defined in Section 2. Due to the skewness distributional form of r_{M_i}, it has maximum value +1 and minimum value $-\infty$, transformations to achieve a more normal shaped form would be more appropriate for residual analysis. Another possibility is to use the deviance

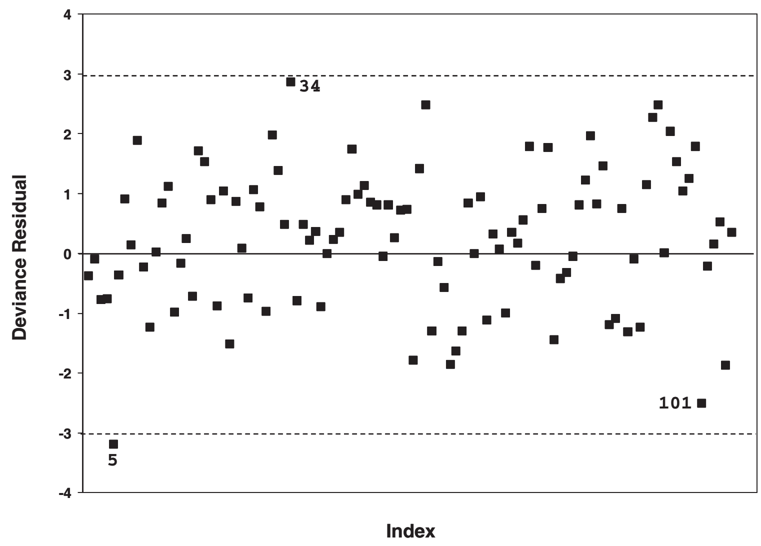

Figure 8: *Index plot of the deviance residual* r_{D_i}.

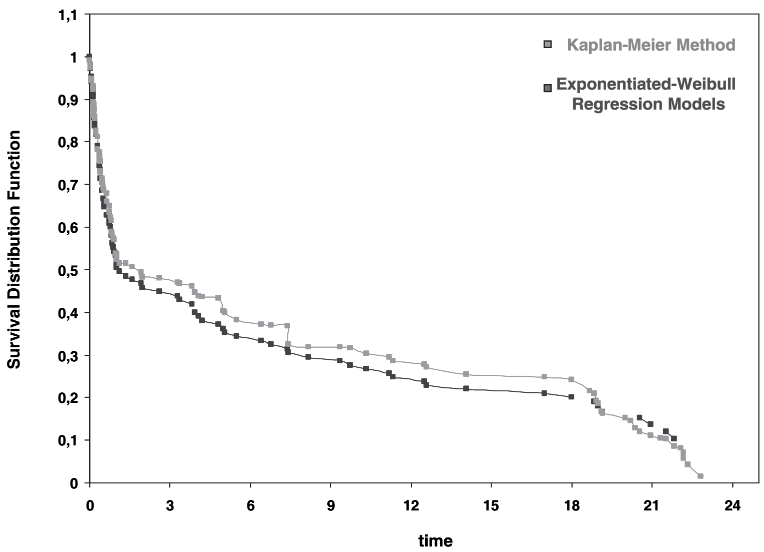

Figure 9: *Plot of the Survivor Function.*

residual (see, for instance, definition in McCullagh and Nelder, 1989, section 2.4) that has been largely applied in generalized linear models (GLMs). Various authors have investigated the use of deviance residuals in GLMs (see, for instance, Williams, 1987; Hinkley *et al.*, 1991; Paula 1995) as well as in other regression models (see, for example, Farhrmeir and Tutz, 1994). In the exponentiated-Weibull regression model, the residual deviance can be expressed here as

$$r_{D_i} = \text{sign}(r_{M_i})\Big[-2\Big\{r_{M_i} + \delta_i \log(\delta_i - r_{M_i})\Big\}\Big]^{\frac{1}{2}}$$

where r_{M_i} is the residual martingale corresponding to the exponentiated-Weibull regression model.

Analyzing the residual deviances, obtained after computing the residual martingales, it follows that individual 5 presented residual deviances greater than 3 (Figure 8).

6.4 Impact of the detected influential observations

To reveal the impact of the detected influential observations, we estimate the parameters again without the influential observations. Let $\hat{\gamma}$ and $\hat{\gamma}^0$ be the maximum likelihood estimates of the models that are obtained from the data sets with and without the influential observations, respectively. Lee, Lu and Song (2006) define the following two quantities to measure the difference between $\hat{\gamma}$ and $\hat{\gamma}^0$:

$$\text{TRC} = \sum_{\text{i}=1}^{\text{n}_\text{p}} \frac{|\hat{\gamma}_\text{i} - \hat{\gamma}_\text{i}^0|}{\hat{\gamma}_\text{i}} \quad \text{and} \quad \text{MRC} = \frac{\max_\text{i}|\hat{\gamma}_\text{i} - \hat{\gamma}_\text{i}^0|}{\hat{\gamma}_\text{i}}$$

where TRC is total relative changes, MRC maximum relative changes and n_p is the number of parameters.

We find that $TRC = 5.490$ and $MRC = 0.415$. In order to compare the impact of the non-influential observations, we repeat the analysis after removing the same number randomly selected from non-influential observations. We find that $TRC = 1.786$ and $MRC = 0.123$. Hence, the ML results are more sensitive to the influential observations.

Table 3: *Maximum likelihood estimates for the complete data set.*

Parameter	Estimate	SE	p-value
θ	5.556	24.761	—
δ	3.561	1.6481	—
β_0	0.28453	14.625	0.4705
β_1	2.216	0.24192	<0.0001
β_2	0.10296	0.0012952	0.0021
β_3	−0.12659	0.00083604	<0.0001
β_4	0.038063	0.0000811	<0.0001
β_5	0.0021795	0.00026622	0.4468
β_6	0.2631	0.038086	0.0888
β_7	0.028371	0.022559	0.4251

6.5 A reanalysis of golden shiner data

The model was estimated one more time, but without observation 5. Next, we present the results of the model fitting

We can observe from Table 3 that the variable x_2 became significant. The survival function was also fitted again for the exponentiated-Weibull regression model (see Figure 8) in which we can observe a good model fitting.

7 Concluding remarks

In this work, we have discussed applications of influence diagnostics in exponentiated-Weibull regression models with censored data. Appropriate matrices for assessing local influence as well as predictions on the fitted models under different perturbation schemes are obtained. Model fitting is also considered by using deviance residuals and graphs of the survival function. The approach was applied to simulated and real data sets, which clearly indicates the usefulness of the approach.

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Appendix A: Matrix of second derivatives $\ddot{\mathbf{L}}(\gamma)$

Here, we derive the necessary formulas to obtain the second order partial derivatives of the log-likelihood function. After some algebraic manipulations, we obtain

$$\mathbf{L}_{\theta\theta} = -\frac{r}{\theta^2} + \sum_{i\in C}\left\{\frac{g_i^{\theta}[\log(g_i)]^2}{(1-g_i^{\theta})^2}\right\};$$

$$\mathbf{L}_{\theta\delta} = -\frac{1}{\delta}\left\{\sum_{i\in F}\frac{z_i h_i}{g_i} + \theta\sum_{i\in C}\left[\frac{z_i h_i g_i^{\theta-1}(\log(g_i) - g_i^{\theta} + 1)}{(1-g_i^{\theta})^2}\right]\right\};$$

$$\mathbf{L}_{\theta\beta} = -\frac{1}{\delta}\left\{\sum_{i\in F}\frac{x_{ij} h_i}{g_i} + \sum_{i\in C}\left[\frac{x_{ij} h_i g_i^{\theta-1}(-g_i^{\theta} + \theta\log(g_i) + 1)}{(1-g_i^{\theta})^2}\right]\right\};$$

$$\mathbf{L}_{\delta\delta} = \frac{r}{\delta^2} + \frac{(\theta-1)}{\delta^2}\sum_{i\in F}\left[\frac{g_i z_i h_i(2+h_i-h_i\exp\{z_i\})-(z_i h_i)^2}{g_i^2}\right]+$$
$$+\frac{1}{\delta^2}\sum_{i\in F}\left[2z_i(1-\exp\{z_i\})-z_i^2\exp\{z_i\}\right]+$$
$$+\frac{\theta}{\delta^2}\sum_{i\in C}\left\{\frac{z_i h_i g_i^{\theta-1}}{(1-g_i^{\theta})^2}[z_i g_i^{-1}h_i(\theta-1)+z_i(1-\exp\{z_i\})-z_i g_i^{\theta}+\right.$$
$$\left. z_i g_i^{\theta}(g_i^{-1}h_i+\exp\{z_i\})]\right\};$$

$$\mathbf{L}_{\delta\beta} = -\frac{1}{\delta^2}\sum_{i\in F}\frac{(\theta-1)x_{ij}h_i}{g_i^2}\left\{\left[-g_i(1+z_i-z_i\exp\{z_i\})+z_i h_i\right]-\right.$$
$$\left. x_{ij}g_i^2\left[1-\exp\{z_i\}-z_i\exp\{z_i\}\right]\right\}$$
$$-\frac{\theta}{\delta^2}\sum_{i\in C}\frac{x_{ij}g_i^{\theta-1}h_i}{(1-g_i^{\theta})^2}\left\{(1-g_i^{\theta-1})\left[1-z_i g_i^{-1}h_i+z_i(1-\exp\{z_i\})\right]+\right.$$
$$\left.\theta z_i g_i^{-1}h_i\right\};$$

$$\mathbf{L}_{\beta\beta} = -\frac{(\theta-1)}{\delta^2}\sum_{i\in F}\frac{x_{ij}x_{ik}h_i\left[g_i(-1+\exp\{z_i\})+h_i\right]}{g_i^2}-\frac{1}{\delta^2}\sum_{i\in F}x_{ij}x_{ik}\exp\{z_i\}+$$
$$+\frac{\theta}{\delta^2}\sum_{i\in C}\frac{x_{ij}x_{ik}h_i g_i^{\theta-1}\left\{(1-g_i^{\theta-1})\left[-1+\exp\{z_i\}-(\theta-1)h_i\right]-\theta h_i g_i^{\theta-1}\right\}}{(1-g_i^{\theta})^2},$$

where $h_i = \exp\left[z_i-\exp\{z_i\}\right]$, $g_i = 1-\exp\left[-\exp\{z_i\}\right]$ and $z_i = \frac{y_i - x_i^T\beta}{\delta}$.

Appendix B: Local influence on predictions: Response perturbation

Here, we provide the derivatives of elements $\mathbf{\Delta}_{ij}$ of matrix $\mathbf{\Delta}$ considering the response variables perturbation scheme. The elements of vector $\mathbf{\Delta}_1$ take the form

$$\Delta_{1i} = \begin{cases} \dfrac{\widehat{h_i^*}\, s}{\widehat{g_i^*}\,\widehat{\delta}} & \text{if} \quad i\in F \\ -\dfrac{(\widehat{g_i^*})^{\widehat{\theta}-1}\widehat{h_i^*}\, s}{\widehat{\delta}\,[1-(\widehat{g_i^*})^{\widehat{\theta}}]}\left\{\widehat{\theta}\log(\widehat{g_i^*})\left[\dfrac{(\widehat{g_i^*})^{\widehat{\theta}}}{1-(\widehat{g_i^*})^{\widehat{\theta}}}+1\right]+1\right\} & \text{if} \quad i\in C \end{cases}$$

On the other hand, the elements of the vector $\boldsymbol{\Delta}_2$ are expressed as

$$\Delta_{2i} = \begin{cases} -\dfrac{(\widehat{\theta}-1)\,\mathrm{s}\,\widehat{h}_i^*}{\widehat{\delta}^2\,\widehat{g}_i^*}\Big[-\dfrac{\widehat{z}_i^*\,\widehat{h}_i^*}{\widehat{g}_i^*}+\widehat{z}_i^*(1-\exp\{\widehat{z}_i^*\})\Big]+\dfrac{\mathrm{s}}{\widehat{\delta}^2}\Big[\exp\{\widehat{z}_i^*\}(\widehat{z}_i^*+1)-1\Big] & \text{if} \quad i \in F \\ \dfrac{\mathrm{s}\,\widehat{\theta}\,\widehat{h}_i^*\,(\widehat{g}_i^*)^{\widehat{\theta}-1}}{\widehat{\delta}^2\,[1-(\widehat{g}_i^*)^{\widehat{\theta}}]}\Big\{\widehat{z}_i^*\Big[\dfrac{\widehat{h}_i^*}{\widehat{g}_i^*}\Big(\dfrac{\widehat{\theta}\,(\widehat{g}_i^*)^{\widehat{\theta}}}{1-(\widehat{g}_i^*)^{\widehat{\theta}}}+\widehat{\theta}-1\Big)-\exp\{\widehat{z}_i^*\}+1\Big]+1\Big\} & \text{if} \quad i \in C, \end{cases}$$

while the elements of the vector $\boldsymbol{\Delta}_j$, $j = 3, \ldots, p+2$ are expressed as

$$\Delta_{ji} = \begin{cases} \dfrac{\mathrm{s}\,x_{ij}}{\widehat{\delta}^2}\Big[-\dfrac{(\widehat{\theta}-1)\,\widehat{h}_i^*}{\widehat{g}_i^*}\Big(1-\dfrac{\widehat{h}_i^*}{\widehat{g}_i^*}-\exp\{\widehat{z}_i^*\}\Big)+\exp\{\widehat{z}_i^*\}\Big] & \text{if} \quad i \in F \\ \dfrac{\mathrm{s}\,\widehat{\theta}\,x_{ij}\,\widehat{h}_i^*(\widehat{g}_i^*)^{\widehat{\theta}-1}}{1-(\widehat{g}_i^*)^{\widehat{\theta}}}\Big\{\dfrac{\widehat{h}_i^*}{\widehat{g}_i^*}\Big[\dfrac{\widehat{\theta}\,(\widehat{g}_i^*)^{\widehat{\theta}}}{1-(\widehat{g}_i^*)^{\widehat{\theta}}}+\widehat{\theta}-1\Big]+1-\exp\{\widehat{z}_i^*\}\Big\} & \text{if} \quad i \in C, \end{cases}$$

where $\widehat{h}_i^* = \exp\big[\widehat{z}_i^* - \exp\{\widehat{z}_i^*\}\big]$, $\widehat{g}_i^* = 1 - \exp\big[-\exp\{\widehat{z}_i^*\}\big]$ and $\widehat{z}_i^* = \frac{y_i^* - \mathbf{x}_i^{\mathrm{T}}\widehat{\beta}}{\widehat{\delta}}$.

Appendix C: Local influence on predictions: Explanatory variable perturbation

In this appendix we provide the derivatives of elements $\boldsymbol{\Delta}_{ij}$ of matrix $\boldsymbol{\Delta}$, considering the explanatory variables perturbation scheme. The elements of vector $\boldsymbol{\Delta}_1$ are expressed as

$$\Delta_{1i} = \begin{cases} -\dfrac{\mathrm{s}\,\widehat{h}_i^*\,\widehat{\beta}_t}{\widehat{\delta}\,\widehat{g}_i^*} & \text{if} \quad i \in F \\ \dfrac{\mathrm{s}\,\widehat{h}_i^*\,\widehat{\beta}_t\,(\widehat{g}_i^*)^{\widehat{\theta}-1}}{\widehat{\delta}[1-(\widehat{g}_i^*)^{\widehat{\theta}}]}\Big\{1+\widehat{\theta}\log(\widehat{g}_i^*)\Big[1+\dfrac{(\widehat{g}_i^*)^{\widehat{\theta}}}{1-(\widehat{g}_i^*)^{\widehat{\theta}}}\Big]\Big\} & \text{if} \quad i \in C, \end{cases}$$

the elements of vector $\boldsymbol{\Delta}_2$ are expressed as

$$\Delta_{2i} = \begin{cases} \dfrac{\mathrm{s}\widehat{\beta}_t}{\widehat{\delta}^2}\Big\{\dfrac{(\widehat{\theta}-1)\,\widehat{h}_i^*}{\widehat{g}_i^*}\Big[\widehat{z}_i^*\Big(-\dfrac{\widehat{h}_i^*}{\widehat{g}_i^*}-\exp\{\widehat{z}_i^*\}+1\Big)+1\Big]-\exp\{\widehat{z}_i^*\}(1+\widehat{z}_i^*)+1\Big\} & \text{if} \quad i \in F \\ -\dfrac{\mathrm{s}\widehat{\beta}_t\,\widehat{\theta}\,\widehat{h}_i^*\,(\widehat{g}_i^*)^{\widehat{\theta}-1}}{\widehat{\delta}^2\,[1-(\widehat{g}_i^*)^{\widehat{\theta}}]}\Big\{\widehat{z}_i^*\Big[\dfrac{\widehat{h}_i^*}{\widehat{g}_i^*}\Big(\dfrac{\widehat{\theta}\,(\widehat{g}_i^*)^{\widehat{\theta}}}{1-(\widehat{g}_i^*)^{\widehat{\theta}}}+\widehat{\theta}-1\Big)-\exp\{\widehat{z}_i^*\}+1\Big]+1\Big\} & \text{if} \quad i \in C \end{cases}$$

the elements of vector $\boldsymbol{\Delta}_j$, for $j = 1, \ldots, p$ and $j \neq t$, take the forms

$$\Delta_{ji} = \begin{cases} \dfrac{x_{ij}\,\mathrm{s}\beta_t}{\delta^2}\left\{\dfrac{(\theta-1)h_i^*}{g_i^*}\left[\dfrac{h_i^*}{g_i^*}+\exp\{\mathrm{z}_\mathrm{i}^*\}-1\right]+\exp\{\mathrm{z}_\mathrm{i}^*\}\right\} & \text{if} \quad i \in F \\ \dfrac{\mathrm{s}\beta_t\,\theta\,h_i^*\,x_{ij}(g_i^*)^{\theta-1}}{\delta^2\,[1-(g_i^*)^{\theta}]}\left\{\dfrac{h_i^*}{g_i^*}\left[-\dfrac{\theta\,(g_i^*)^{\theta}}{1-(g_i^*)^{\theta}}\right]-\theta+1\right\} & \text{if} \quad i \in C \end{cases}$$

the elements of vector $\boldsymbol{\Delta}_t$ are given by

$$\Delta_{ti} = \begin{cases} \dfrac{\mathrm{s}}{\delta}\left\{-\dfrac{(\theta-1)\,h_i^*}{g_i^*}\left[\dfrac{x_{it}\,\beta_t}{\delta}\Big(\dfrac{h_i^*}{g_i^*}+\exp\{\mathrm{z}_\mathrm{i}^*\}\Big)-1\right]+\exp\{\mathrm{z}_\mathrm{i}^*\}+\left[1-\dfrac{\beta_\mathrm{t}\,\mathrm{x}_\mathrm{it}}{\delta}\right]-1\right\} & \text{se} \quad i \in F \\ \dfrac{\mathrm{s}\,\theta\,h_i^*\,(g_i^*)^{\theta-1}}{\delta\,[1-(g_i^*)^{\theta}]}\left\{-\dfrac{\beta_t x_{it}}{\delta}\left[\dfrac{h_i^*}{g_i^*}\Big(\dfrac{\theta(g_i^*)^{\theta}}{1-(g_i^*)^{\theta}}+\theta-1\Big)-\exp\{\mathrm{z}_\mathrm{i}^*\}+1\right]+1\right\} & \text{se} \quad i \in C \end{cases}$$

where $\widehat{h_i^*} = \exp\left[\widehat{\mathrm{z}_\mathrm{i}^*}-\exp\{\widehat{\mathrm{z}_\mathrm{i}^*}\}\right]$, $\quad \widehat{\mathrm{g}_\mathrm{i}^*} = 1-\exp\left[-\exp\{\widehat{\mathrm{z}_\mathrm{i}^*}\}\right]$ and $\widehat{\mathrm{z}_\mathrm{i}^*} = \frac{\mathrm{y_i}-\mathbf{x}_\mathrm{i}^{*\mathrm{T}}}{\widehat{\delta}}$

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