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# High power circular waveguide $TE_{0n}$-$TE_{11}$ mode conversion*

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**Abstract**: Based on the theory of mode coupling, this paper discusses the circular waveguide mode conversion with waveguide axis curved and radius tapered in detail. It also carries out the optimized analysis about the geometry structure of $TE_{0n}$-$TE_{11}$ mode converter. Adopting different phase rematch techniques, the optimal geometry parameter is obtained. The mode converter designed in this way owns high conversion efficiency exceeding 98 %.

**Key words**: circular waveguide; mode converter; coupling equation; phase rematch

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Mode conversion has important application value in the transmission of high power millimeter wave, transmit, measure. Gyrotron have the output mode $TE_{01}$ and mix up $TE_{0n}$, which is very inconvenient for direct use, so a mode conversion must be taken. Based on the virtual need of gyroklystron, high efficient wideband $TE_{01}$-$TE_{11}$ circular waveguide mode converter is designed in this paper. In overmoded circular waveguide a selective transformation of one specific mode into another can be achieved by means of periodic structure of the inner waveguide wall under the condition that the geometric period $\lambda_w$ of the wall perturbations and the unperturbed wave number $\beta_1$ and $\beta_2$ of the interacting modes satisfy the resonance relationship[1]: $\Delta\beta = |\beta_1 - \beta_2| = 1 \times 2\pi/\lambda_B = l \times 2\pi/\lambda_W$, ($l = \pm 1, \pm 2, \cdots\cdots$), where, $\lambda_B$ is the beat wavelength of the two modes. This condition guarantees that the conversion to the desired mode is continuously increased, while conversion to other unwanted modes that are also coupled by the waveguide perturbations suffers destructive interference. If the completely power conversion from one mode to another mode can be realized, the length of waveguide should satisfy the relation: $L = N\lambda_w$, where $N$ is the number of inner waveguide wall geometry wave period, and the best value of $N$ is decided by the three needs: limiting the conversion to unwanted modes, enhancing conversion efficiency of the desired mode and satisfying the bandwidth requirements.

## 1 Fundamental equation and theory of mode converters

The unevenness in a waveguide (the axis bent in a circular waveguide, gradual radius changes of the waveguide) will cause the energy coupling among different propagation modes and thus create mode conversion. The coupling wave equations for studying axis curved circular waveguide mode converter are[2]

$$\frac{dA^{+}_{m'n'}}{dz} = j\gamma_{m'n'}A^{+}_{m'n'} - j\sum_{mn}[C^{+}_{(m'n')(mn)}A^{+}_{mn} + C^{-}_{(m'n')(mn)}A^{-}_{mn}] \tag{1}$$

$$\frac{dA^{-}_{m'n'}}{dz} = j\gamma_{m'n'}A^{-}_{m'n'} + j\sum_{mn}[C^{+}_{(m'n')(mn)}A^{-}_{mn} + C^{-}_{(m'n')(mn)}A^{+}_{mn}] \tag{2}$$

The equations of radius taper circular waveguide mode conversion are[3]

$$\frac{dA^{+}_{mn'}}{dz} = -\frac{1}{2}\frac{d(\ln\gamma_{mn'})}{dz}A^{-}_{mn'} - \gamma_{mn}A^{+}_{mn'} + \sum_{+mn}A^{+}_{mn}C^{+}_{(mn')(mn)} + \sum_{-mn}A^{-}_{mn}C^{-}_{(mn')(mn)} \tag{3}$$

$$\frac{dA^{-}_{mn'}}{dz} = -\frac{1}{2}\frac{d(\ln\gamma_{mn'})}{dz}A^{+}_{mn'} + \gamma_{mn}A^{-}_{mn'} + \sum_{+mn}A^{+}_{mn}C^{-}_{(mn')(mn)} + \sum_{-mn}A^{-}_{mn}C^{+}_{(mn')(mn)} \tag{4}$$

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where $A^{+}_{mn}$ and $A^{-}_{mn}$ are the forward and backward wave complex amplitudes of the ($mn$) mode. $C^{+}_{(m'n')(mn)}$ and $C^{-}_{(m'n')(mn)}$ stand for the coupling coefficients between ($mn$) mode and ($m'n'$) mode whose direction of propagation are the same and opposite respectively. $\gamma_{mn} = \alpha_{mn} + j\beta_{mn}$ is the propagation constant of the ($mn$) mode, with $\beta_{mn}$ the wave number, $\alpha_{mn}$ the attenuation constant for circular waveguides, the study results about coupling coefficients at Ref. [4] in detail. It is supposed that the length of mode converter is $L$, and there exists an incident wave at its input terminal and a zero-valued backward wave at its output terminal. The boundary condition[5] together with equations (1), (2) and (3), (4) reveal the problem of boundary value of a coupling wave differential equation groups. The solution solves the distribution of $A^{+}_{mn}$ and $A^{-}_{mn}$ along the $z$ axis, and $z$ is the arc length of waveguide axis.

The couple principles of curved axis and periodic radius perturbations circular waveguides are $\Delta m = \pm 1$ and $\Delta m = 0$. In order to restrain other unwanted mode amplitude and to rise the desired mode, the coupling structure is often adopted as[6~8]

(a) Waveguide mode converter of axisymmetric, periodic radius perturbations

$$a(z) = a_0 \frac{[1 - \sum_{m=1} \varepsilon_m \sin(mk_p t)]}{1 - \sum_{m=1} \varepsilon_m} \qquad (1 \leq m \leq 4) \tag{5}$$

and $\alpha(z)$ must be the function of $k_p = 2\pi/\lambda_B$.

(b) Waveguide mode converter of axis curved in plane

$$y(x) = \varepsilon_1 \cos \frac{2\pi z}{\lambda_{W[mp, m'q]}} - \varepsilon_2 \sin \frac{2\pi z}{\lambda_{W[mp, m_1 n_1]}} - \varepsilon_3 \sin \frac{2\pi z}{\lambda_{W[m'q, m_2 n_2]}} \tag{6}$$

(c) Slightly changed perturbation period

$$\lambda_W = (1 + \delta) \lambda_{B[mp, mq]} \tag{7}$$

(d) Set a proper placement of phase delay sections of straight waveguide to adjust the phase, so that coupling to unwanted mode is minimized.

And adopting corresponding structure, mode complete conversion can be realized.

## 2 Result of numerical calculation

Adopting the periodic perturbation of radius, the $TE_{03}$-$TE_{02}$, $TE_{02}$-$TE_{01}$ circular waveguide mode converter is optimized with input mode $TE_{03}$ and frequency of 28GHz, radius of 20mm. And its structure is shown in Fig. 1 (a), (b), and Table 1, Fig. 2 (a) and (b) show the result of the optimization. For the structure is symmetry, the transportable mode is only $TE_{03}$, $TE_{02}$ and $TE_{01}$ with input frequency and original radius, furthermore, the beat wavelength of $TE_{03}$ and $TE_{02}$, $TE_{02}$ and $TE_{01}$ is very short, thus high efficient conversion in fewer periods can be realized, even with arriving at a complete conversion.

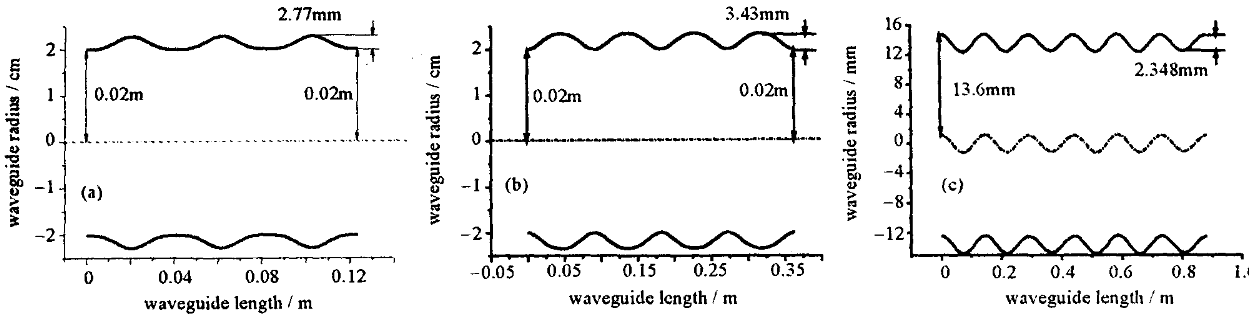


Fig. 1 Geometry structure of $TE_{03}$-$TE_{02}$, $TE_{02}$-$TE_{01}$ mode converter with radius taper in (a), (b), and $TE_{01}$-$TE_{11}$ with axis curved in (c)

Adopting the perturbation of axis curved, the $TE_{01}$-$TE_{11}$ circular waveguide mode converter is optimized with frequency of 35GHz, radius of 13.6mm. Because of the beat wavelength of $TE_{01}$ and $TE_{11}$ is longer, and its beat wavelength is very close to the beat wavelength of $TE_{01}$ and $TE_{12}$, thus the high efficiency conversion can hardly be realized in a fewer period number. But the conversion efficiency can be increased from added period number of wave and changed the waveguide inner radius, which the unwanted mode amplitude became minimized. At the same time, the length of mode converter became longer. For the length of mode converter is usually decided by four factors: wavelength of beat wave, coupling coefficient, transport constant and coupling to other mode, so the converter length is longer, Fig. 1 (c), compared to the former, Fig. 1 (a, b). The optimal results in Table 1 and Fig. 2 (c, d), show that because of adopting phase rematch technology, the undesired mode became smaller and smaller in output end. Six coupled modes were included in the theoretical analysis: $TE_{01}$, $TE_{11}$, $TE_{12}$, $TE_{21}$, $TM_{11}$ and $TM_{21}$. Ohmic attenuation is included in the coupling matrices. The influence of $TM_{11}$ and $TM_{21}$ turned out to be negligible because there is a continuous and coherent conversion (with no change in phasing) between $TE_{01}$ and the degenerate $TM_{11}$ mode for the $TM_{21}$ mode is only weakly coupled. There exist strong coupling between $TE_{01}$ and $TE_{12}$, $TE_{11}$ and $TE_{21}$, so if adopting fold perturbation items to rematch the phase of $TE_{12}$ and $TE_{21}$, the conversion efficiency can exceed 98 %.

**Table 1 Geometry parameter and conversion efficiency of $TE_{0n}$-$TE_{11}$ mode converter**

| case | $TE_{03}$-$TE_{02}$ | $TE_{02}$-$TE_{01}$ | $TE_{01}$-$TE_{11}$ |
|---|---|---|---|
| beat wavelength $\lambda_B$/mm | 35.33 | 74.60 | 143.90 |
| geometric period $\lambda_W$/mm | 41.14 | 90.81 | 146.94 |
| number of periods | 3 | 4 | 6 |
| converter length /mm | 123.4 | 363.2 | 881.6 |
| perturbation amplitudes $\varepsilon_1$ | 0.065 67 | 0.078 30 | 0.084 01 |
| $\varepsilon_2$ | - 0.012 86 | 0.008 64 | 0.005 41 |
| geometric period factor $\delta$ | 0.164 47 | 0.217 25 | 0.021 09 |
| outer power level: | 0.000 27 ($TE_{03}$) | 0.009 25 ($TE_{03}$) | 0.000 17 ($TE_{01}$) |
| | 0.984 93 ($TE_{02}$) | 0.000 42 ($TE_{02}$) | 0.980 38 ($TE_{11}$) |
| | 0.013 42 ($TE_{01}$) | 0.989 08 ($TE_{01}$) | 0.002 22 ($TE_{12}$) |
| | | | 0.011 69 ($TE_{21}$) |
| | | | 0.000 18 ($TM_{11}$) |
| power transmission | | | 0.000 03 ($TM_{21}$) |
| efficiency: $P_{sum}$ | 0.998 62 | 0.998 75 | 0.994 68 |

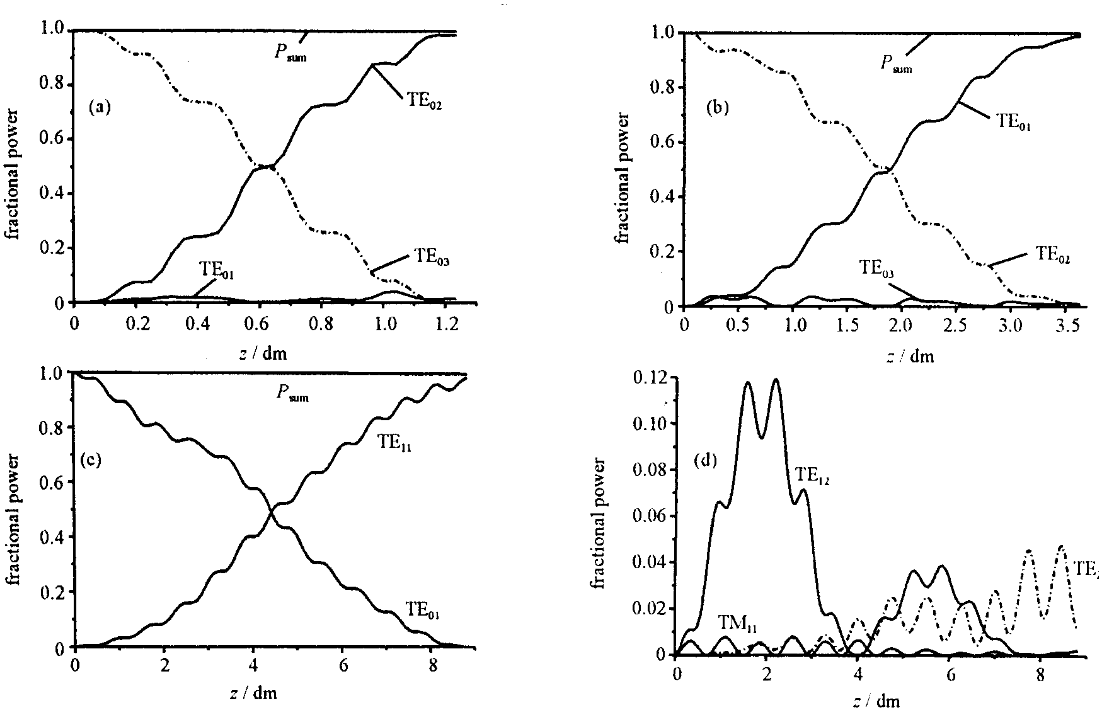


Fig. 2 Fractional power distributions along mode converter

## 3 Conclusion

This paper optimizes the $TE_{03}$-$TE_{02}$, $TE_{02}$-$TE_{01}$ and $TE_{01}$-$TE_{11}$ with frequency 28GHz and 35GHz. Adopting corresponding structure, the reliable optimal geometry parameter can be obtained. This results contribute to designing 8mm gyroklystron $TE_{01}$-$TE_{11}$ circular waveguide mode converter with tighten, high efficiency and broad bandwidth.

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# 高功率圆波导 $TE_{0n}$-$TE_{11}$模式变换研究


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**摘 要**: 在模式耦合理论的基础上,详细讨论了波导轴线弯曲与波导半径渐变的圆波导模式变换,并对 $TE_{0n}$-$TE_{11}$模式变换器的几何结构进行了优化分析,采用不同的相位重匹配技术,得到了可靠的最优几何参量。以此数据设计的 8mm 回旋速调管 $TE_{01}$-$TE_{11}$模式变换器的转换效率可达 98%。



**关键词**: 圆波导; 模式变换器; 耦合波方程; 相位重匹配