A Model for the Thickness and Salinity of the Upper Layer in the Arctic Ocean and the Relationship between the Ice Thickness and Some External Parameters

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ABSTRACT

This paper presents a dynamical model for the salinity and thickness of the upper layer in the Arctic. The parameters are the river runoff to the Arctic, the buoyancy supply through the Bering Strait, the export of ice from the Arctic and a parameter characterizing the vertical mixing. An ice model is formulated, having the following two important properties: 1) the horizontal surface area of the exported ice is essentially determined by external parameters (the wind field over the Arctic); and 2) there is a relationship between the ice thickness and the fraction of open water in the Arctic. The model for the upper layer and the ice model are used together with a heat budget for the Arctic, also including the effect of different albedo for ice and open water. A relationship between the freshwater supply and the ice thickness is derived. Also investigated are the effects on the ice thickness of a changed export of ice area and changed properties of the flow through the Bering Strait. It is found that a decrease of the fresh-water supply by as much as 50% would have only a small effect upon the ice thickness and the fraction of open water in the present Arctic Ocean. However, if such a decrease of the freshwater supply is combined with a moderate decrease of the flow through the Bering Strait and with a likewise moderate increase of the area of the exported ice, the pack ice might disappear.

1. Introduction

The Arctic Ocean receives large amounts of buoyancy in the form of freshwater from rivers and low salinity water from the Pacific. These buoyancy supplies are in the following termed freshwater buoyancy. Relative to typical Atlantic water (of salinity 35‰) these two sources of freshwater buoyancy are of approximately equal strength. Some of the freshwater buoyancy is transformed from liquid to solid state within the Arctic by the growth of sea ice with low salinity. Most of the rest of the freshwater buoyancy is diluted by mixing processes with saltier and denser water of Atlantic origin and forms a relatively fresh top layer of polar water resting on the underlying Atlantic water.

Freshwater buoyancy is removed from the Arctic by ocean currents and outflowing ice. The polar surfacewater flows out through the Fram Strait (into the East Greenland current) but also through the Canadian Archipelago, mainly through Lancaster Sound (see the map in Fig. 1). The ice exits mainly through the Fram Strait.

The ice moves across the polar basin in the so called Transpolar Drift. A reflection of this motion is found in the distribution of the thickness of the pack ice which is largest in the downstream, convergent area north of the Canadian Archipelago and Greenland. The thinnest ice is naturally found in the upstream, divergent area along the coasts of Alaska and Siberia. For a review of the ice conditions in the Arctic Ocean (see Vinje, 1981; Hibler, 1979).

On the annual basis the Arctic Ocean loses large quantities of heat to the atmosphere. By these heat losses the surface layer is cooled and during a large portion of the year ice forms as the water releases its heat of fusion. During the freezing process salt is rejected from the ice. Especially in shallow shelf areas overlain with thin ice (along the coasts of Alaska and Siberia), this may lead to the generation of very dense water which occasionally may be even denser than the Atlantic water below the top layer. This cold, dense water may penetrate deep into the lower layer [see SCOR Working Group 58 (1979), hereafter referred to as SWG 58].

In an ordinary "positive" estuary, for instance a fjord, the large buoyancy (freshwater) supply leads to the generation of a surface layer that is lighter than the seawater outside the estuary. There also are "negative" estuaries, e.g., the European Mediterranean, where buoyancy loss due to evaporation creates water that is denser than the water outside the basin. The Arctic Ocean seems to possess both these properties, thus creating both lighter and denser water than the seawater outside the basin. In the mean, the entire Arctic basin receives a net buoyancy supply but locally atmospheric cooling may lead to the generation of deep water.

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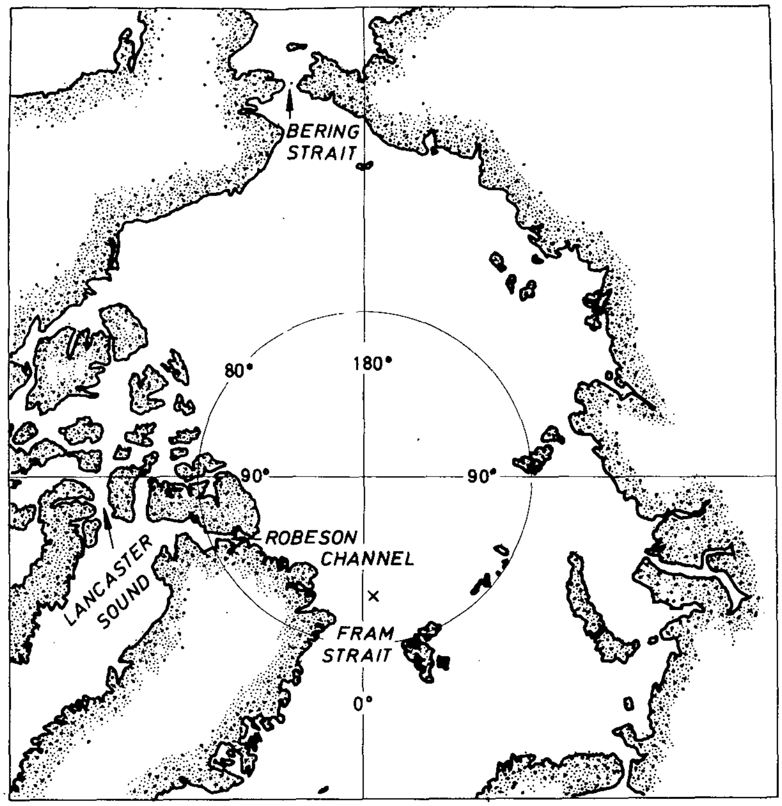

FIG. 1. Map of the Arctic Ocean.

However, we will later see that this local generation of deep water probably removes only small amounts of freshwater buoyancy from the surface layer of the present day Arctic Ocean.

Experience from deep fjords with freshwater runoff has shown that it is possible to model the properties of the brackish layer (salinity and thickness) as functions of wind mixing, topographic parameters and the freshwater supply (see Stigebrandt, 1975, 1980a, 1981). In the models described in these papers the width of the mouth of the fjord is supposed to be small compared to the internal Rossby radius (calculated from the thickness and buoyancy of the brackish layer). In this way effects of the rotation of the earth may be excluded from the models. The outflow of brackish water from the fjords was assumed to be topographically controlled and critical flow was assumed to occur in the mouth (the densimetric Froude number then attains a critical value). The vertical mixing within the fjord was assumed to be mainly of the entrainment type: the wind-generated turbulence in the brackish layer lifts dense seawater through the pycnocline. The rate of entrainment was modeled by the Kato-Phillips formula. The hydrography of the upper part of the Arctic Ocean is strongly affected by the supply of freshwater buoyancy and in the next section we will develop a two-layer model of the Arctic Ocean. The model is in the spirit of the above-mentioned fjord models, the main difference being that the assumption of critical flow at the mouth has been replaced by a condition of geostrophically balanced transports in the straits.

One may look on the model for the upper layer presented in this paper as a further development of the volume and salt budgets presented by Aagaard and Greisman (1975). The model thus concerns a steady-state (mean) Arctic Ocean. The effects of the annual variation in the river runoff, for example, are accordingly not considered. The new elements in the present model consist of the use of some dynamical constraints, namely, the assumption of a geostrophically balanced transport of polar surface water and a dynamical law for the vertical mixing. This makes the model prognostic and changes of long period (compared to the residence time for the water in the basin) in the hydrographic state, caused by changes in for instance the freshwater runoff from rivers, may be predicted.

Some basic properties for the pack ice are needed for the analysis. First, is is assumed that the horizontal area of the exported ice is dependent on external parameters (the wind field over the Arctic). This seems to be a good assumption (see Vinje, 1981). Second, it is assumed that there is a relationship between the ice thickness and the fraction of open water in the Arctic. As there are no other ice-covered parts of the ocean similar to the Arctic and there is only one observed state of the Arctic (the present state), it is hard to justify rigorously this assumption.

A heat balance for the upper layer of the Arctic Ocean is formulated. It contains both the oceanic advection of heat and the heat exchange through the air-sea interface. The exchanges through open and ice-covered surfaces are treated separately. In this way the albedo effect is accounted for. Finally, the models for the upper layer and the ice are combined with the heat balance and the relationship between the ice thickness and some external parameters (the freshwater supply, the area of the exported ice, the Bering Strait flow parameters) are derived.

2. A model of the Arctic surface layer

In the following we will assume that the vertical stratification in the Arctic Ocean may be described by two superposed layers: a relatively fresh and cold top layer of polar water (density ρ_1, salinity S_1, temperature T_1 and thickness H_1) overlying a saltier and warmer water of Atlantic origin (density ρ_2, salinity S_2 and temperature T_2) (see Fig. 2).

The density of sea water depends on its salinity, temperature and pressure. For the present purpose the pressure effect may be neglected and the following linear equation of state, valid around some reference state (ρ_0, S_0, T_0), is sufficiently accurate for small ranges in temperature and salinity

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)], \qquad (1)$$

where $\alpha = -1/\rho_0(\partial\rho/\partial T)_{S=\text{const.}}$ and $\beta = 1/\rho_0(\partial\rho/\partial S)_{T=\text{const.}}$, α varies both with salinity and temperature while β is fairly constant (e.g., Defant, 1961). In the temperature range (−2°C to +3°C) and the salinity range (32–35‰) the following values of the coefficients may be used: $\alpha = 5.5 \times 10^{-5}$ and $\beta = 8 \times 10^{-4}$ (see Appendix for list of symbols).

The freshwater buoyancy supply dominates over the buoyancy removal by cooling if the salinity difference, $S_2 - S_1$, between the mixed water (S_1; T_1) and the Atlantic water (S_2; T_2) fulfills the following condition:

$$S_2 - S_1 > \frac{\alpha}{\beta}(T_2 - T_1). \qquad (2)$$

In principle, a lighter less saline top layer than is present in the basin and a positive estuarine circulation would be expected. [If the condition (2) is not met, dense water is created in the basin maintaining a negative estuarine circulation).] The actual magnitude of $S_2 - S_1$ and $T_2 - T_1$ will depend on the magnitude of the freshwater buoyancy supply, the amount of cooling, the topography and the general dynamics of the basin (e.g., the mixing processes and the outflow dynamics).

The freezing of ice, and the accompanying release of salt, seems to be a process that has sensible effects in the surface layer over the whole Arctic Ocean. In Fig. 3 are shown typical vertical wintertime profiles of salt and temperature. The salinity is homogeneous down to ~50 m: below this depth is a halocline down to 200–250 m. The temperature, however, is almost homogeneous (and near the freezing temperature) down to ~100 m. The fact that the water in the upper part of the strong halocline is near freezing temperature most probably means that it has recently been in contact with ice. Thus, from the area of origin, the water sinks until the surrounding water is of the same density. Then the water flows into the basin along its own density surface. When sinking the water may mix with the surrounding water. However, this will not change the temperature very much as the surrounding water

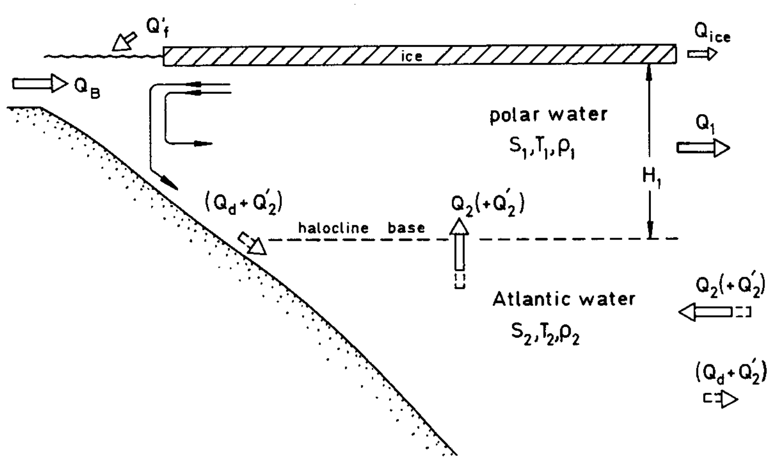

FIG. 2. Definition sketch showing some of the parameters in the two-layer model for the upper layer.

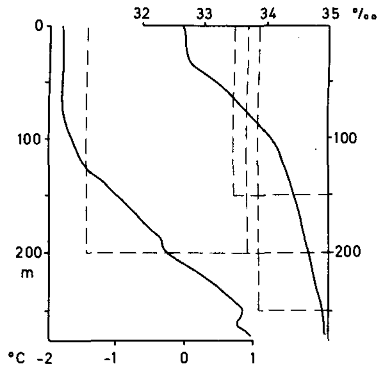

FIG. 3. The vertical distribution of salinity and temperature upstream of the Fram Strait. The position of this hydrographic station is marked by a cross in Fig. 1 (from McPhee, 1980a).

down to ~100 m has a near freezing temperature. Superposed on this wintertime convection there is an upward mixing of salt and heat from the underlying Atlantic water. This mixing is probably mainly driven by mechanically generated turbulence, maintained by the action of winds and tides. The vertical mixing in combination with the outflow of the polar surface water from the Arctic Ocean introduces a vertical velocity (directed upward) at the base of the pycnocline. In a steady state this advective vertical velocity must be balanced by the entrainment velocity of the turbulence.

We will now develop a model for the Arctic Ocean in which the freshwater buoyancy dominates and the condition (2) is fulfilled. The Arctic Ocean will be looked upon as an estuary and experience from modeling smaller scale estuaries will be used. Concerning the dynamics of the outflow a first-order description of the baroclinic effects is generally obtained using a two-layer model. A better representation may possibly be achieved by also invoking higher baroclinic modes. However, this is done at the cost of simplicity. At the present level of modeling the Arctic Ocean this is not considered worthwhile.

The volume flow of surface polar water out of the Arctic is denoted by Q_1. This outflowing water is composed of entrained Atlantic water Q_2 water flowing into the Arctic from the Pacific (through the Bering Strait) Q_B and freshwater from rivers and excess precipitation Q_f'. Ice is formed from the surface water in the Arctic Ocean and the flow of ice out of the Arctic is denoted by Q_{ice} which is the volume flow of ice multiplied by the density ratio between ice and freshwater (ρ_{ice}/ρ_f). Conservation of volume in the upper layer then gives

$$Q_1 = Q_2 + Q_B + Q_f,$$

where

$$Q_f = Q_f' - Q_{ice} - Q_d. \quad (3)$$

There are some indications (see SWG 58) that, connected to the generation of very dense water on the shelf, there is a transport of fresh water down into the lower layer. This may occur locally where condition (2) is not met. From the generation area the very dense water may flow along the bottom down the shelf. Like other density currents it should mix with the lighter surrounding water. If the water is dense enough to penetrate the base of the halocline and if its salinity is less than S_2 then, obviously, fresh water is transported, at the rate Q_d, down into the lower layer. This deep-reaching convection may in this context be described in the following way: there is an extra flow Q_2' of Atlantic water into the upper layer. Before leaving the upper layer this water is cooled and mixed with fresh water (see the sketch in Fig. 2). The net effect of this convection on the present model for the upper layer, which takes account only of salinity effects on the density of the water, is the loss Q_d of fresh water.

It is known that polar surface water leaves the Arctic only where guided by a land mass to the right (west) of the current. If the current is assumed to be in geostrophic balance, and if the width of the current is less than the width of the strait, and if the maximum thickness of the current is assumed to be the thickness of the upper layer in the Arctic, the transport should be equal to $g'H_1^2/2f$ (cf. e.g., Stommel, 1965, p. 111). Thus the magnitude of the transport is determined by the stratification (thickness and buoyancy) of the layer. Here $g' = g(\rho_2 - \rho_1)/\rho_2$ is the buoyancy parameter and g is the acceleration of gravity, f is the Coriolis parameter. Two-layer flows in rotating channels and straits are treated by, for example, Whitehead *et al.* (1974), and a review of the subject is given by Whitehead (1980).

There should be two main currents of polar surface water out of the Arctic, one through Fram Strait and one through Lancaster Sound in the Canadian Archipelago where there also are some minor channels (the most important of these are Robeson-Kennedy Channel and Jones Sound) (see, also, the map in Fig. 1). We write the outflow from the upper layer of the Arctic Ocean in the following way:

$$Q_1 = \gamma g'H_1^2/2f, \quad \text{where} \quad \gamma = \gamma_F + \gamma_L + \gamma_r. \quad (4)$$

Here γ_F (γ_L) are the coefficients for the Fram Strait (Lancaster Sound). These should be nearly equal to 1 because of the large widths (compared to the internal Rossby radius) of the two straits. γ_r is the coefficient for all the small contributions taken together and it is probably in the range 0.1–0.5 [cf. the suggestive Fig. 315 of Defant, (1961), which shows the dynamic topography of the sea surface in the northern part of Baffin Bay].

The upper layer of the Arctic Ocean has an open border in the Barents Sea. Johannessen and Foster (1978) have found that here the polar front approximately follows the 100 m isobath. (The front was identified with the strongest horizontal temperature gradient.) This front seems to be rather permanent. It is tempting to compare it with the so-called shelf fronts, occurring during the spring and summer heating period in shelf areas with strong tides e.g., Simpson, *et al.* (1978) or Stigebrandt (1980b). The shelf fronts are maintained by the nonuniformity in vertical mixing, caused by variations in water depth. The persistence and sharpness of shelf fronts indicates that cross-frontal mixing is relatively small. In the following we assume that mixing across the polar front is negligible.

Conservation of salt in the upper layer gives the equation

$$Q_1S_1 = Q_2S_2 + Q_BS_B, \tag{5}$$

where S_1 is the salinity of the outflowing polar water (assumed to be the same in all the outflows), S_2 is the salinity of the underlying Atlantic water being entrained into the upper layer, and S_B is the salinity of the inflowing Pacific water. In Eq. (5) we have neglected the salt transport performed by the ice flowing out of the polar basin. According to Aagaard and Greisman (1975) $S_{\rm ice}$ = 3‰. Since $Q_{\rm ice}$ is an order of magnitude smaller than Q_1, $Q_{\rm ice}S_{\rm ice}$ is two orders of magnitude smaller than Q_1S_1.

In laboratory experiments on entrainment flows in two-layer systems, like those described by Turner (1973), the pycnoclines are usually thin compared to the thickness of the mixed layers. In the Arctic, however, the upper mixed layer is thin compared to the thickness of the pycnocline. The horizontal variations of the surface salinity (and the internal advective transports caused by these) in combination with relatively weak vertical mixing are responsible for this.

The polar surface water is the water that flows out of the Arctic Ocean. It is essentially the water that is lighter than the Atlantic water forming the underlying layer. Thus, the polar surface water is the water above the base of the pycnocline. The entrainment of Atlantic water through the base of the pycnocline is for simplicity described by a uniform entrainment velocity. Thus,

$$Q_2 = Aw_e, \tag{6}$$

where A is the surface area of the pycnocline and w_e is the entrainment velocity. Such an entrainment flow may be driven by the turbulence generated by the motions of the ice. However, it is quite probable that entrainment and other vertical mixing processes occur to a significant degree in some active regions such as along the margin of the basin where the pycnocline hits the bottom. Also, the process of generation of dense water by the formation of ice on the shelf and the subsequent sinking and mixing of this water may be of some importance. At the present time we are not able to parameterize these kinds of vertical mixing and we choose, for simplicity, to use Eq. (6).

In the mean state of the Arctic Ocean there are two contributions of different kinds to the freshwater buoyancy flux into the polar surface water: one of dense Atlantic water from below and one of fresh water (essentially from rivers) from above. For a similar situation where the buoyancy flux from above was caused by heating the present author (Stigebrandt, 1980b) derived an expression for the entrainment velocity. In the present application it should be written

$$w_e = \frac{2m_0u_*^3}{g\beta(S_2 - S_1)H_1} - \epsilon\frac{Q_fS_1}{A(S_2 - S_1)}. \tag{7}$$

Here β is defined by Eq. (1) and u_* is the friction velocity in the upper layer. m_0 is a constant (=1.25) (Niiler, 1977). The meaning of the parameter ϵ is explained below. As long as we do not know exactly where and by which processes the mixing takes place it does not seem to be meaningful to include also the effect of the rotation of the earth in Eq. (7). The first term on the right-hand side of Eq. (7) is equivalent to the Kato-Phillips expression for the entrainment velocity (Turner, 1973). The second term on the same side gives the modification of the entrainment velocity caused by the buoyancy flux from above. When $Q_f > 0$ there is a net buoyancy flux from above and ϵ should attain the value 1 in this case. Eq. (7) shows that the entrainment of dense water from below is then suppressed. When $Q_f < 0$ the ice growth is larger than the supply of freshwater (from rivers and excess precipitation) minus the transport of fresh water down into the lower layer. Then there is a negative buoyancy flux from above as salt is rejected by the freezing process and a haline convection below the ice follows. This alone could lead to some entrainment of water from below (penetrative convection), a feature also shown by Eq. (7). However, it is known that the efficiency of this process is quite low, on the order of a few percent (e.g. Farmer, 1975). Therefore, ϵ is small in this case (we use $\epsilon = 0.05$ in the following computations).

In Eq. (7) we have approximated the buoyancy parameter $g(\rho_2 - \rho_1)/\rho_2$ by $g\beta(S_2 - S_1)$ as $\beta(S_2 - S_1) \gg \alpha(T_2 - T_1)$ in the Arctic. However, when there is ice on the surface the effect of temperature on the buoyancy is easily included because we can then expect that T_1 is near the freezing point for the upper layer. $T_2 - T_1$ can then with a good approximation be considered constant. Thus, $g\Delta\rho/\rho_2 = g[\beta(S_2 - S_1) - \alpha(T_2 - T_1)] = g\beta(S_2' - S_1)$, where $S_2' = S_2 - \alpha/\beta(T_2 - T_1)$. Note that S_2' may only be used in the buoyancy term. For later convenience we introduce the parameter $P = S_2/$

$(S_2 - S_1)$. Eq. (7) may then be written

$$w_e = \frac{2m_0 u_*^3 P}{g\beta S_2 H_1} - \epsilon \frac{Q_f}{A}(P - 1). \qquad (7')$$

We will solve for H_1 and S_1 (or P). Eq. (5) may be written

$$Q_1 = Q_2 P/(P - 1) + Q_B P(P_B - 1)/P_B(P - 1). \qquad (8)$$

Eqs. (8) and (3) together give

$$Q_2 = Q_f(P - 1) + Q_B(P/P_B - 1) \qquad (9)$$

or

$$Q_1 = P(Q_f + Q_B/P_B), \qquad (10)$$

where the parameter P_B is defined by $P_B = S_2/(S_2 - S_B)$. Eq. (4) may be written

$$Q_1 = \frac{\gamma g \beta S_2}{2f}\frac{H_1^2}{P}. \qquad (11)$$

Eqs. (10) and (11) give

$$H_1 = P\left[\frac{2f}{\gamma g\beta S_2}(Q_f + Q_B/P_B)\right]^{1/2}. \qquad (12)$$

Eqs. (6) and (7′) give

$$Q_2 = \frac{2m_0 u_*^3 AP}{g\beta S_2 H_1} - \epsilon Q_f(P - 1). \qquad (13)$$

Eqs. (9) and (13) give, with the help of Eq. (12),

$$P = \frac{Q_f(1+\epsilon) + Q_B}{Q_f(1+\epsilon) + Q_B/P_B} + \frac{\dfrac{2m_0 u_*^3 A}{g\beta S_2}}{[Q_f(1+\epsilon) + Q_B/P_B]\left[\dfrac{2f}{\gamma g\beta S_2}(Q_f + Q_B/P_B)\right]^{1/2}}. \qquad (14)$$

For the thickness of the upper layer, we then obtain from Eqs. (12) and (14)

$$H_1 = \frac{Q_f(1+\epsilon) + Q_B}{Q_f(1+\epsilon) + Q_B/P_B}\left[\frac{2f}{\gamma g\beta S_2}(Q_f + Q_B/P_B)\right]^{1/2} + \frac{2m_0 u_*^3 A}{g\beta S_2[Q_f(1+\epsilon) + Q_B/P_B]}. \qquad (15)$$

Eqs. (14) and (15) should give the essential properties of the upper layer in the Arctic Ocean (H_1, S_1) provided the relevant external parameters (Q_f', Q_B, S_B, S_2, A) and the parameters Q_{ice}, Q_d and u_* are known.

The parameters Q_{ice}, Q_d and u_* require specific models to express these quantities in terms of H_1, S_1 and external parameters (those determining the heat and momentum transfer through the air-sea interface). In the fjord models referred to in Section 1 a relation between u_* and the wind speed was used. In the present case the coupling between u_* and the wind is more complicated because of the existence of pack ice. Also vertical mixing caused by other energy sources may be important and u_* may get contributions from these as well. For a determination of Q_{ice} we actually need a model for the generation and export of ice. Later we will assume that the area of the exported ice is controlled by the wind field over the Arctic Ocean and thus independent of the ice thickness. The ice thickness will be determined from the heat budget of the Arctic which also depends on the dynamics of the upper layer. In this way a coupling between the dynamic model, presented in this section, and the thermodynamics for the Arctic Ocean is obtained. The removal of freshwater buoyancy from the upper layer by the generation of dense water on the shelves Q_d is today hard to parameterize. Fortunately, Q_d is a small quantity in the present Arctic Ocean (see Section 3 in this paper). However, this term may possibly increase in importance if, for instance, the freshwater supply decreases.

The residence time τ_r, of fresh water may be defined as the stored volume of fresh water divided by the input. Thus,

$$\tau_r = \frac{\left(\dfrac{H_1}{P} + \bar{H}_i\right)A}{Q_f + Q_B/P_B} = A\left\{\left[\frac{2f}{\gamma g\beta S_2(Q_f + Q_B/P_B)}\right]^{1/2} + \frac{\bar{H}_i}{Q_f + Q_B/P_B}\right\}, \qquad (16)$$

where $\bar{H}_i$ is the mean thickness of the ice which is considered to be fresh. The vertical stratification in the Arctic Ocean is not an ideal two-layer stratification. Thus there is some ambiguity in the choice of the "observed" salinity and thickness of the upper layer. In the next section we will therefore, using all available information, establish the best two-layer approximation to the existing stratification. By this we tune the model. Thereafter, we may use the model to investigate effects induced by changed magnitude of the external parameters.

3. Tuning the model

We will now use independent estimates of Q_{ice}, Q_1 and Q_2 in order to tune the model. The vertical stratification varies horizontally in the Arctic. However, the dynamically most important areas are those just upstream of the outflows, i.e., around northern Greenland. The stratification here is

TABLE 1. Values of the external parameters used in the two-layer model of the Arctic Ocean. The values of the parameters in the left column are adopted from Aagaard and Greisman (1975).

$Q_B = 1.5$ ($\times 10^6$ m^3 s^{-1})	$f = 1.4 \times 10^{-4}$	(s^{-1})
$S_B = 32.4$ (‰)	$m_0 = 1.25$	
$S_2 = 35.0$ (‰)	$\beta = 8 \times 10^{-4}$	(‰$^{-1}$)
$A = 10^7$ (km^2)	$g = 10$	(m s^{-2})
$Q_f' = 0.10$ ($\times 10^6$ m^3 s^{-1})	$\gamma = 2.3$	(see comment in text)
	$\rho_f/\rho_{ice} = 1.1$	

supposed to determine the rate of outflow [see Eq. (4)]. A typical example of the vertical stratification of the upper layer northeast of Greenland is shown in Fig. 3.

The stratification is not an ideal two-layer stratification. Thus, there is some difficulty in establishing the "true" stratification in an equivalent two-layer model. In Fig. 3 some alternatives are drawn. Is the layer 150 m thick with salinity 33.5‰ or is it as much as 250 m thick with salinity 33.9‰?

Using the parameter values given in Table 1, H_1 and S_1 are calculated from Eqs. (14) and (15) for various values of Q_f and u_*. The results of these computations are shown in Fig. 4 where S_1 vs H_1 is drawn. For a given value of Q_f (solid line) the actual value of the pair (H_1, S_1) depends on the mixing rate u_* (dashed line). As can be seen an increase in the mixing rate results in a saltier and thicker layer provided Q_f is constant.

The number of "geostrophic outlets" γ is certainly not only an external parameter. Of course, it depends on the topography but also on the hydrographic state. For instance, if there was no mixing in the Baffin Bay, $\gamma_L + \gamma_r$ would be approximately equal to 1 even if there were several sufficiently deep and wide connections between the Arctic Ocean and the Baffin Bay. The reason for this should be the following: the stratification (g' and H_1 in a two-layer system) determines the baroclinic transport capacity of the coastal current acting as a sink for the outlets from the basin (Arctic). If there is no friction or mixing in the outer system (Baffin Bay) already, the first outlet, if sufficiently wide and deep, will feed the coastal current with all the water it may transport with the given stratification. Outside all subsequent (downstream) outlets the stratification will be the same as inside the outlets and thus there will be no transport through these.

In order to show the effects on the Arctic of different numbers of outlets, the relation between H_1 and S_1 for $Q_f = 0$ is shown in Fig. 5. The case $\gamma = 1$ should describe an Arctic Ocean with only one outlet (e.g., the Fram Strait). As can be seen the surface layer would be fresher and thicker than with the present number of outlets for the same values of the external parameters.

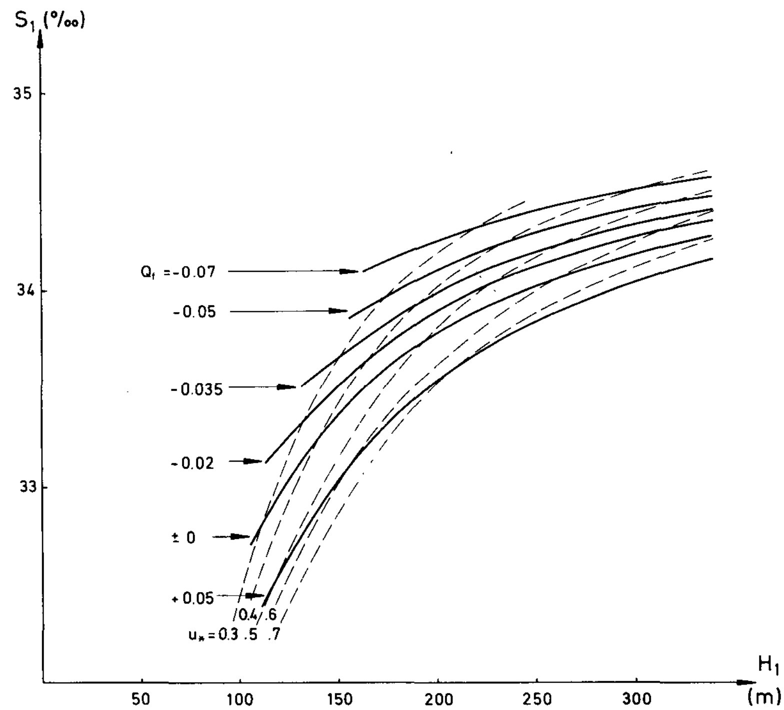

FIG. 4. Salinity S_1 versus thickness H_1 of the upper layer for different values of Q_f (solid line) and u_* (dashed line).

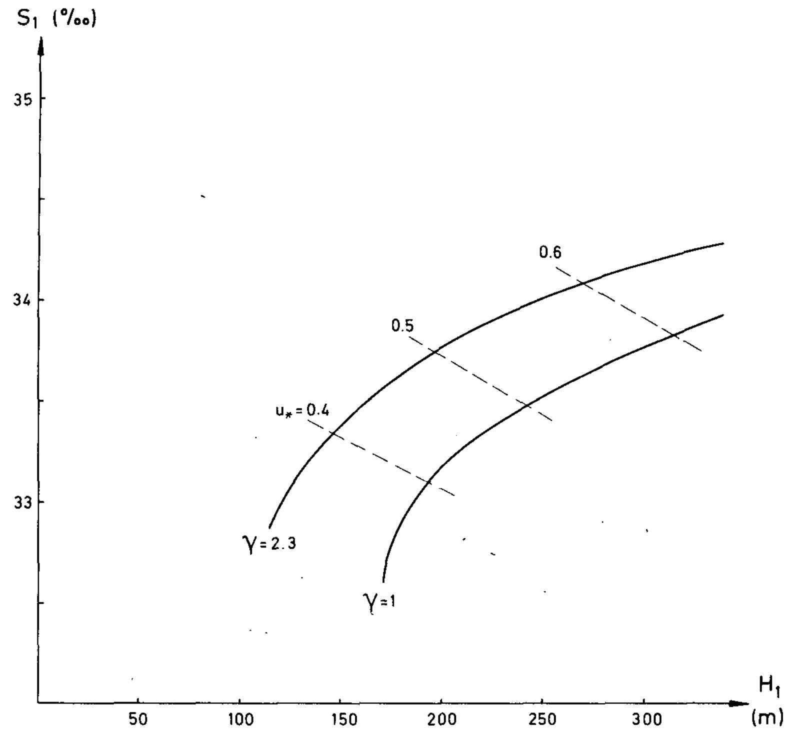

FIG. 5. A plot of the effect of varying the number of "geostrophic outlets" γ on the salinity and thickness of the upper layer ($Q_f = 0$).

We will now select the best estimate of H_1 and S_1. For this reason we have calculated the predictions of the model (Table 2) for four different combinations of H_1 and S_1 which are in harmony with the observed vertical salinity profile in Fig. 3. There exist independent estimates of Q_{ice} and Q_1, Q_2 and we discuss these below.

a. Estimates of the ice export Q_{ice}

According to Vinje (1981) the horizontal area of the annually exported ice floes through the Fram Strait is ~1 × 10⁶ km². The average thickness of the floes may be estimated to be ~3 m (Vinje, personal communication). This gives an annual ice export of 3000 km³. The uncertainty of this figure is perhaps ±20%. Thus the ice volume export ($Q_{ice} \times \rho_f/\rho_{ice}$) through the Fram Strait should be 0.095 × 10^6 m³ s⁻¹ ± 20%.

TABLE 2. The predictions of the model for certain combinations of (H_1, S_1) with $Q_d = 0$.

H_1 (m)	150	175	200	250
S_1 (‰)	33.5	33.6	33.7	33.8
Q_f ($\times 10^6$ m³ s⁻¹)	−0.02	0.00	0.02	0.04
u_* (cm s⁻¹)	0.35	0.45	0.55	0.65
Q_1 ($\times 10^6$ m³ s⁻¹)	2.1	2.75	3.5	4.8
Fram Strait	0.9	1.2	1.5	2.1
Can. Archipel.	1.2	1.55	2.0	2.7
Q_2 ($\times 10^6$ m³ s⁻¹)	0.6	1.25	2.0	3.3
(Q_{ice} ($\times 10^6$ m³ s⁻¹)	0.12	0.10	0.08	0.065)
$Q_{ice} \times \rho_f/\rho_{ice}$	0.13	0.11	0.09	0.07

The annual ice export through the Canadian Archipelago has been determined by Sadler (cited in SWG 58) to be ~220 km³. Thus, it seems to be a reasonable estimate that the ice volume export, $Q_{ice} \times \rho_f/\rho_{ice}$, is in the range 0.08–0.12 × 10^6 m³ s⁻¹. The pairs of (H_1, S_1) in Table 2 that fulfill this requirement are (175, 33.6) and (200, 33.7).

b. Estimates of the transports Q_1 *and* Q_2

According to SWG 58 several transport estimates of the West Spitsbergen current, based on dynamic calculations, all indicate transports in the range 2–4 × 10^6 m³ s⁻¹. However, according to Aagaard and Greisman (1975), current measurements in a few positions in the Fram Strait suggest a larger transport, ~7 × 10^6 m³ s⁻¹. Volume transport estimates of the East Greenland current from dynamic sections give transports in the same 2–4 × 10^6 m³ s⁻¹ range (see SWG 58). There also may be inflows between Spitsbergen and the European mainland. To the present author's knowledge there is no reliable estimate of this transport.

The inflow of Atlantic water thus seems to be in the range 2–8 × 10^6 m³ s⁻¹. However, not all of this water is permanently lifted up into the polar surface layer by mixing processes. A portion of the inflowing

Atlantic water is possibly circulated below the surface layer. This circulation is supposed to be driven by sinking heavy water formed on the shelf (see SWG 58).

According to Aagaard and Greisman (1975) the inflowing Atlantic water in the west Spitsbergen current has a salinity of 35‰, while there is a subsurface outflow of modified Atlantic water with salinity 34.9‰ through the western Fram Strait. Effectively Atlantic water has thus been mixed by freshwater approximately in the ratio 350:1 within the Arctic basin. This gives the circulation in the lower layer to be about $350 \times Q_d$. For example, if we take our (200, 33.7)-case, we find that $Q_2 = 2 \times 10^6\ \mathrm{m^3\ s^{-1}}$ and, if the inflow estimates by Aagaard and Greisman ($\sim 7.7 \times 10^6\ \mathrm{m^3\ s^{-1}}$ of Atlantic water) are correct, this gives $Q_d = 5.7/350 = 0.016 \times 10^6\ \mathrm{m^3\ s^{-1}}$. There, however, are some reasons to doubt the figure $7 \times 10^6\ \mathrm{m^3\ s^{-1}}$ for the West Spitsbergen inflow into the Arctic Ocean. According to Aagaard and Greisman only about half of this inflow is baroclinically balanced (in the geostrophic sense). One possibility is that there is a stationary barotropic cyclonic gyre in the Fram Strait. A fraction of the northward transport in the West Spitsbergen current is by this turned southward in the strait, thus joining the East-Greenland current. According to Greisman (1976) already early investigators of the circulation in the Fram Strait reported indications of a high-latitude cyclonic eddy. Greisman supported by his own measurements, considered the existence of a high-latitude cyclonic eddy most probable. The cyclonic gyre may be bottom-steered as there is a circular depression, centered at 79°15′N, 3°E, with a radius of 60 km, which is 2000 m deeper than the surrounding ocean bottom (Vinje, 1981). The fraction of the transport in the West Spitsbergen current that belongs to the barotropic gyre may not play any role within the Arctic Ocean. However, the gyre may have some conspicuous local effects and it has been suggested that it may be responsible for the often observed surface eddies centered near 79°30′N, 3°E (Vinje, 1981). Another possibility may be that there is some sort of low-frequency "tidal pumping" through the strait (cf., e.g., Stigebrandt, 1977). If we take the figure of Aagaard and Greisman (1975) as an upper limit for the circulation in the lower layer of the Arctic Ocean we find that $Q_d \leq 0.016 \times 10^6\ \mathrm{m^3\ s^{-1}}$ which is of the second order for the dynamics of the upper layer. However, as will be seen later, it may be of importance for the advective heat budget for the Arctic Ocean.

The volume flow out of the Canadian Archipelago has, according to SWG 58 been determined by Sadler to be $2.1 \times 10^6\ \mathrm{m^3\ s^{-1}}$ (this figure was also used by Aagaard and Greisman). This transport figure is very close to that given by our (200, 33.7) prediction (see Table 2).

c. *The two-layer stratification of the Arctic Ocean*

To the present author's knowledge there is no further significant information available concerning the circulation and dynamics of the Arctic surface layer. Thus we find that the pair of (H_1, S_1) equal to (200, 33.7) makes the best fit to the presently available pieces of information about the Arctic Ocean.

Thus, by using accepted values of the external parameters, it is possible to tune the model to fit reasonably the observed quantities, *viz.*, the vertical stratification, the rate of ice export and the flow of polar surface water through the Canadian Archipelago.

The rate of vertical mixing as measured by u_* is also determined by the model. For the (200, 33.7) case, we find from Table 2 that $u_* = 0.55\ \mathrm{cm\ s^{-1}}$. The mean work conducted by the turbulence in the buoyancy field is then $m_0 u_*^3 \approx 0.2\ \mathrm{erg\ cm^{-2}}$. Actually this figure should be a bit higher as the surface area of the pycnocline we have used ($A = 10^7\ \mathrm{km^2}$) rather is the area of the polar sea surface. (The ratio of these two surfaces is ~ 0.5).

The observed mean drift speed in the Transpolar Drift is about $2.8\ \mathrm{cm\ s^{-1}}$ (see Vinje, 1981). In that paper there is a reference to a paper by Dunbar and Wittman who calculated a meandering coefficient which is the actual distance a drift station travels divided by the net displacement during the same period. From the drift of station SP-6 they found meandering coefficients up to 7.2 in the Transpolar Drift. From this information we conclude that a typical instantaneous speed of a pack ice floe may be $15\ \mathrm{cm\ s^{-1}}$.

The bottom of the pack ice is rough and the meandering motion of the ice should generate turbulence in the water below the ice. In order to obtain a rough estimate of the stress τ generated by the ice motion we assume that a typical velocity difference between the ice and the underlying water is u. Then $\tau/\rho = u_*^2 = c_d u^2$ or $u_* = u(c_d)^{1/2}$. The drag coefficient c_d is fairly well known in other situations. In an oscillating tidal stream over a rough sea bed the bottom stress may be calculated using $c_d = 3 \times 10^{-3}$ (e.g., Proudman, 1953). We adopt this value for the present case and with $u = 10\ \mathrm{cm\ s^{-1}}$, we get $u_* \approx 0.6\ \mathrm{cm\ s^{-1}}$, the same figure the model demands. Thus, on the basis of this preliminary estimate it appears that the oscillating ice motions constitute an important source for vertical mixing in the polar surface layer. However, it should be stressed that the short calculation above is of a rather preliminary character as it includes neither the effect

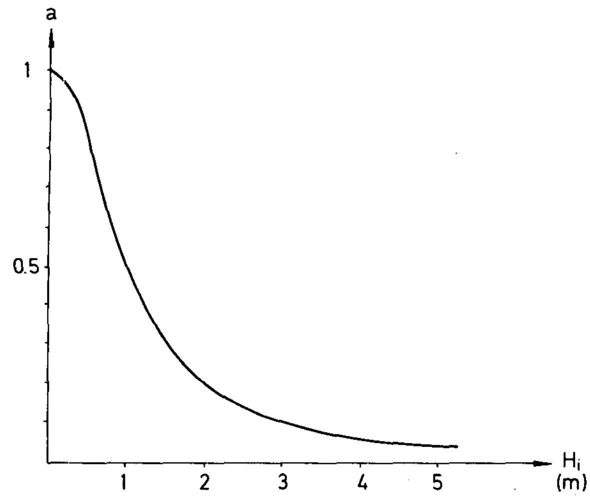

FIG. 6. The suggested relation Eq. (17) between the ice-free fraction a of the Arctic Ocean and the ice thickness H_i with $H_{i0} = 1$ m.

of the rotation of the earth nor the effect of deeply penetrating ice keels (ridges).

The polar surface water is composed of a relatively thin mixed layer and a thick pycnocline. For our model of the polar surface water the magnitude of the transport of Atlantic water into this layer (through the base of the pycnocline) is of the utmost importance. The distribution of the Atlantic water within the layer, however, is of secondary importance as here we work with a horizontally integrated Arctic Ocean.

It is believed that the model used here for the entrainment velocity through the base of the pycnocline is an acceptable first-order approximation. More detailed work on different aspects of local mixed-layer dynamics in the Arctic has been published. For instance Solomon (1973) studied the wintertime surface layer convection and, later, McPhee (1980b) studied the local boundary layer below the ice. However, neither of these papers are directly applicable to the problem of the transport of Atlantic water through the base of the pycnocline.

4. Some properties of the pack ice

From the drift of manned and unmanned ice stations and from observations from aircraft, satellites and submarines a great deal of information about the Arctic pack ice has been gathered (for a review see Vinje, 1981). Most of the pack ice leaves the Arctic through Fram Strait although some ice also exits through the Canadian Archipelago. The ice moves across the polar basin in the so called Transpolar Drift. There is also a clockwise gyre in the Beaufort Sea. The ice thickness is largest north of the Canadian Archipelago and Greenland and smallest along the Alaskan and Siberian coasts. The latter area also is the area with the largest fraction of open water during the summer. There are large local variations in ice thickness as well (see Thorndike *et al.*, 1975).

For our modeling of the mean state of the Arctic Ocean we need to know a few mean properties of the Arctic pack ice. Thus, a lot of less important local properties, although interesting in themselves, may be overlooked in this connection. The first important mean property is the mean rate of export of ice area. From Vinje (1981) one may conclude that the magnitude of the area of the exported ice A_{ice} is essentially determined by the wind field over the Arctic. Thus, we can decouple A_{ice} from the hydrodynamics of the upper layer in the Arctic. It may be the case that A_{ice} also is dependent on the ice-free fraction of the Arctic. However, an increased ice-free fraction will have largest effects far upstream of the ice exit. For the present crude modeling we do not consider this possible dependence to be important.

The thickness H_i of the exported ice is of course dependent on both A_{ice} and the dynamics and thermodynamics of the system. We will later assume that the mean thickness $\bar{H}_i$ of the ice in the Arctic is proportional to the thickness H_i of the exported ice. Thus, a given change in the mean thickness will give rise to a proportional change in the thickness of the exported ice.

For our own heat balance for the Arctic, which follows in the next section, we need a relation between the ice-free fraction a of the Arctic and the mean thickness $\bar{H}_i$ of the ice. The ice-free fraction of the Arctic must depend on a number of parameters. The ice thickness certainly is one of them. We expect that the thinner the ice the larger is the area of the polynyas and open leads. Furthermore, the wind field is surely important. The diameter of the basin and also the geometrical properties of the outlet region may at least indirectly be important (by its effects on the ice thickness). Of course the mechanical properties of the ice are of importance. Unfortunately, we are not able to parameterize all possible influences. However, physical intuition tells us that the following statements are likely to be true for a surface with pack ice: 1) a should approach 1 (ice free) when the ice thickness approaches zero and 2) a approaches zero when the ice becomes very thick. These suggest the following simple relationship

$$a = 1/[1 + (H_i/H_{i0})^2], \tag{17}$$

where we have used the assumed proportionality between $\bar{H}_i$ and H_i. Here H_{i0} should not be considered as a length having a universal value. It is rather a length specific for the Arctic. In principle, it should be possible to express this length in the parameters mentioned above. This cannot be done at the present time, however, and H_{i0} thus has to be determined from empirical data. As there is only one Arctic Ocean and we know only one mean state of it (the present) there is just one data point (H_t = 3 m, a = 0, 1 which gives H_{i0} = 1 m). The suggested relation between a and H_i is drawn in Fig. 6.

5. The heat budget of the Arctic Ocean

In connection with the problem of the possible existence of an ice-free state of the Arctic Ocean there seem to be two main questions to answer:

1) Under what conditions will the ice disappear from the Arctic?

2) Under what conditions will an open Arctic remain ice-free?

The physical processes that can transfer significant amounts of energy through the air-sea interface are (i) radiation—short- and long-wave; (ii) evaporation—condensation; and (iii) conduction. The magnitude of the energy exchange performed by these processes depends on a number of atmospheric and oceanic parameters. The most important of these are the sea surface temperature, the temperature and humidity of the air, the wind speed and the cloudiness. Of course, the latitude of the area under consideration is of great importance. An ice cover on the sea surface may have large effects on the albedo of the incoming shortwave radiation as well as on other energy transfer processes.

We will try to answer the first of the above questions later in this paper. Fletcher (1965) and Donn and Shaw (1968), among others, have suggested that an ice-free Arctic will remain open on account of the albedo effect. However, this conclusion was drawn without taking into account the fact that the Arctic receives large amounts of freshwater buoyancy, most of it in phase with the heating. This will limit the summer heating to a shallow layer. Whether ice will reform or not in winter will depend on how dense and deep this layer becomes in winter. We hope to return to this problem in another paper.

a. The advective heat budget for the Arctic Ocean

The contemporary Arctic Ocean receives heat from the Atlantic, mainly through the West Spitsbergen current, and from the Pacific through the Bering Strait. The Arctic also exports a heat deficit in the form of ice. Besides, of course, the Arctic gains and looses heat by exchange through the air-sea interface. However, before we look at the complete heat balance we will concentrate our attention on the advective part of it.

A number of estimates of the advective heat budget for the Arctic Ocean have been presented in the past (see SWG 58). The estimates differ from ~5 × 10^9 kcal s^{-1} (see Maykut and Untersteiner,

TABLE 3. Heat budget for the upper layer of the Arctic Ocean (annual mean) essentially based on the two-layer model, see the (200, 33.7) case in Table 2 and Fig. 5. Additional figures needed are adopted from Aagaard and Greisman (1975).

Source	Mass transport (10^6 ton s^{-1})	Mean temperature (°C)	Heat transport* (10^9 kcal s^{-1})
Bering Strait	1.5	0.5	2.9
Arctic Archipelago	−2.0	−1.4	0
Fram Strait polar water	−1.5	−1.4	0
ice	−0.08		6.4
Atlantic water	2.0	2.2	7.2
Runoff	0.1	5.0	0.5
	Net heat exchange: 17.0		

* Heat transport is relative to −1.4°C.

1971), through ~16 × 10^9 kcal s^{-1} (Mosby *et al.*, see SWG 58) to 25.8 × 10^9 kcal s^{-1} (Aagaard and Greisman, 1975). The advective heat budget based on the two-layer model presented in Sections 2 and 3 gives a heat supply to the Arctic surface layer which is 17 × 10^9 kcal s^{-1}, see Table 3. Thus the present estimate gives nearly identically the same result as the estimates by Mosby and Vowinkel & Orwig although the magnitudes of the different terms differ between these estimates.

The heat transports are calculated relative to the temperature of the outflowing polar surface water. From Table 3 it follows that the advective heat balance contains three large terms. The inflowing Atlantic water supplies slightly more than the heat deficit exported by the ice. The heat supply from the Pacific is ~17% of the total advective heat supply.

Connected to the deep-water formation process, however, there may be further advective heat gains. We found in Section 3 of this paper that the upper layer requires 2 × 10^6 m^3 s^{-1} of Atlantic water (the 200, 33.7 case) in order to fulfill the salt and volume balances. The rest of the inflowing Atlantic water takes part in the deep-water circulation (see Fig. 2). According to Aagaard and Greisman (1975) the Atlantic water circulating below the surface layer suffers a temperature reduction from 2.2 to 0.5°C. From the inflow estimates of Aagaard and Greisman, about 5.5 × 10^6 m^3 s^{-1} should circulate in the lower layer. Thus, there might be an extra advective heat flux of 9.5 × 10^9 kcal s^{-1}. If this heat flux is added to the estimate from the present model (Table 3) one obtains approximately the estimate of Aagaard and Greisman (1975).

Thus, there is no contradiction between the results from the present model for the upper layer of the Arctic Ocean and the estimates by Aagaard and Greisman. The estimate of the magnitude of the circulation in the lower layer seems, however, to be very uncertain (see Section 3 in this paper).

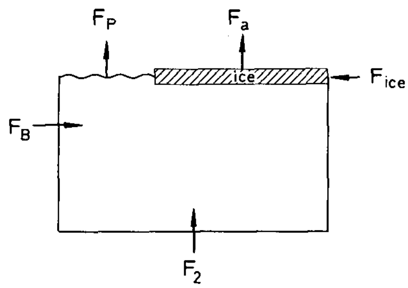

FIG. 7. Definition sketch showing the components of the heat balance for the upper layer in the Arctic Ocean.

The advective heat exchange between the Arctic and the surrounding seas is such that the Arctic gains 16–25 × 10^9 kcal s^{-1}, mainly through the exchange with the Atlantic Ocean. In a steady state the advectively gained heat must be lost through the air-sea interface.

b. The heat budget for the Arctic Ocean

In order to study the effects on the ice cover of a changed advective heat flux into the upper layer of the Arctic, caused by, say, a change in the supply of freshwater buoyancy, we will construct a heat budget for the upper layer of the Arctic. Like the dynamic model developed in Section 2 this model is for a steady-state (mean) Arctic.

The heat exchange through the air-sea interface from the ice-covered part of the Arctic is denoted by F_a, the exchange through polynyas and open leads by F_p. There is advective flow of sensible heat into the Arctic from the Atlantic Ocean, denoted by F_2, and from the Pacific, F_B. The latent heat deficit exported from the Arctic by the ice drifting out of the Arctic is F_{ice}. The following heat balance should then apply:

$$F_2 + F_{ice} + F_B + F_a + F_p = 0. \qquad (18)$$

The heat balance is illustrated in Fig. 7. We will now parameterize the terms in Eq. (18).

The horizontal surface area A_{ice} of the ice floes exported per unit time is supposed to be constant and thus independent of the thickness H_i of the ice. The heat flux by the exported ice may then be written

$$F_{ice} = \nu H_i A, \qquad (19)$$

where $\nu = (A_{ice}\rho_{ice}L)/A$ and L is the heat of fusion for ice.

The net mean heat transfer through the ice F_a should be inversely proportional to the thickness of the ice and proportional to the area of the ice-covered surface. According to Fletcher (1965), F_a is essentially determined by the radiation imbalance. The ice-free fraction of the Arctic is denoted by a; thus

$$F_a = \mu A(1 - a)/H_i, \qquad (20)$$

where μ is the heat exchange per unit time and unit length.

The sensible and latent heat flow/unit surface area from the polynyas and open leads is certainly dependent on the size of the ice-free area. A small polynya has a larger loss of sensible and latent heat per unit surface area than a large one. The reason for this is that the properties of air of continental origin are modified when flowing over the open sea surface. A simple way to include this variation is to assume that the flow of sensible and latent heat is proportional to the area of the polynyas and open leads raised to some power <1. We will frankly assume that the appropriate power is equal to ½. The mean radiation balance for an open sea surface is very different from that of a surface covered by ice or snow. The largest difference is caused by the difference in the albedo for the shortwave radiation from the sun. A simple relation describing these features is the following:

$$F_p = baA + cA(a)^{1/2}, \qquad (21)$$

where b is the radiative heat exchange per unit time and unit surface area and $c(a)^{-1/2}$ is the heat exchange by conduction and evaporation per unit time and unit surface area.

Insertion of Eqs. (19)–(21) into Eq. (18), using Eq. (17), then gives

$$F_2/A + F_B/A + \nu H_i + (\mu H_i/H_{i0}^2 + b)/(1 + (H_i/H_{i0})^2) + c/(1 + (H_i/H_{i0})^2)^{1/2} = 0. \qquad (22)$$

According to Table 3, $F_{ice} = 6.4 \times 10^9$ kcal s^{-1}. If $A = 10^{17}$ cm^2 this gives $F_{ice}/A = 2.02$ kcal cm^{-2} $year^{-1}$. As $H_i = 300$ cm, Eq. (18) then gives $\nu = 7 \times 10^{-3}$ kcal cm^{-2} $year^{-1}$. The ice-free fraction a of the present Arctic varies a lot over the year (e.g., Hibler, 1979). The model developed in this paper, however, does not take care of such variations. It is believed that $a = 0.1$ is a reasonable estimate of the mean ice-free fraction of the Arctic. Eq. (17) then gives $H_{i0} = 100$ cm. According to Fletcher (1965), the radiation imbalance for the present ice-covered Arctic is about −2.5 kcal cm^{-2} $year^{-1}$ ($=F_a/A(1 - a)$). Thus, we get from Eq. (20) $\mu = -750$ kcal cm^{-1} $year^{-1}$. The radiation imbalance for an ice-free Arctic has been estimated by Fletcher to be about +20 kcal cm^{-2} $year^{-1}$ which directly gives $b = 20$. In order to determine the value of c, we use the advective heat balance proposed in Table 3. We denote the resulting value of c by c_1. We also will compare with the advective budget proposed by Aagaard and Greisman (1975) and the corresponding value of c is denoted by c_2. For our own balance we find $F_2/A = 2.3$, $F_B/A = 0.9$, $F_{ice}/A = 2.0$, $F_a/A = -2.3$ and $ba = 2$. Thus, $c_1 = -15.3$.

Let us then consider Aagaard and Greisman's transport numbers. As the reference temperature we take the temperature of the outflowing polar

TABLE 4. Parameter values for the heat balance estimates. The numbers for the Aagaard–Greisman case are found inside parentheses if unequal.

$\mu = -750$ kcal cm^{-1} year^{-1}	$b = 20$ kcal cm^{-2} year^{-1}
$F_2/A = 2.3$ (2.2) kcal cm^{-2} year^{-1}	$F_a/A = -2.3$ kcal cm^{-2} year^{-1}
$F_B/A = 0.9$ (0.8) kcal cm^{-2} year^{-1}	$F_{2'}/A = 0$ (2.7) kcal cm^{-2} year^{-1}
$F_{ice}/A = 2.0$ (2.5) kcal cm^{-2} year^{-1}	$c_{1(2)} = -15.3$ (−25) kcal cm^{-2} year^{-1}
$H_{i0} = 100$ cm	

surface water (−1.2°C). Their case then becomes (in units of kcal cm^{-2} year^{-1}) $F_B/A = 0.8$, $F_{ice}/A = 2.5$, $F_2/A = 2.2$ and $F_{2'}/A = 2.7$, where $F_{2'}$ is the heat transmitted to the upper layer in connection with the deep-water formation (~5 × 10^6 m^3 s^{-1} of Atlantic water whose temperature drops from 2.2 to 0.5°C). We are not able to parameterize $F_{2'}$, so we consider this term not to vary with H_i. We then obtain $c_2 = -25$. When calculations later are made, the term $F_{2'}/A$ is added to Eq. (22). The parameter values used in the heat balances are summarized in Table 4. The relation between the ice thickness H_i and the amount of heat supplied to the upper layer by vertical mixing F_2 has been calculated from Eq. (22) with the parameter values given in Table 4. The result is shown in Fig. 8. The two curves (cases 1 and 2) represent the present model (case 1) and the Aagaard–Greisman case. The curves have several common features. There is a maximum heat flow F_2/A possible with an ice cover. The maximum heat flow gives an ice thickness of 1.5 and 1 m, respectively. If the advective heat flow is larger than the maximum value, an extensive ice cover cannot exist on the surface of the Arctic Ocean provided the heat exchange coefficient c remains unchanged. The ice-free fraction a of the Arctic at maximum heat flow for ice, according to Fig. 6, is 30 and 50%, respectively. Furthermore, two stationary states are possible for a range of values of F_2/A, one with thick ice and one with thin ice. There are also some differences between the curves. The maximum value of F_2/A with an ice cover is much larger in the Aagaard–Greisman case. This case does not permit an ice-free Arctic without advective heat flow from the Atlantic. The present model, however, gives a negative value of F_2/A for $H_i = 0$ which indicates that an ice-free Arctic would have a surface temperature above the present (whereby c increases in magnitude). This feature of an ice-free Arctic has earlier been deduced by Budyko (see Fletcher) and Fletcher (1965).

In the following we will use the heat budget based on the present dynamic model only (case 1). Our advective heat supply to the Arctic, as noted earlier, is larger than that used by Maykut and Untersteiner (1971) but smaller than that estimated by Aagaard and Greisman (1975). Thorndike *et al.* (1975) and Hibler (1979) essentially used the balance by Maykut and Untersteiner.

One striking feature of the curves in Fig. 8 is that for a certain range of advective heat flows, less than maximum for ice, there are two steady-state solutions possible. This might stimulate thoughts about the possibility for the Arctic to alternate between these states if disturbed in some way. However, the left-hand state in Fig. 8 is probably not a stable condition as perturbations in this region appear to lead to an ice-free Arctic. Thus, if the ice-thickness decreases to <1.5 m (case 1), one would expect the ice to disappear as the Arctic is then in the, apparently, unstable range. The physical reason for this is the following: in the *stable range* a small increase of the open-water surface area leads to an increased net loss of heat. The losses by increased conduction and evaporation override the radiative gains due to the albedo effect. This tends to generate more ice and, thus, the perturbation is counteracted. In the *unstable range*, however, a perturbation giving a small increase in the open-water surface area gives rise to radiative gains that are larger than the increased conductive and evaporative losses (remember that the heat exchange by conduction and evaporation per unit time and unit surface area is assumed to be proportional to $a^{-1/2}$). Thus, in this range, the perturbation seems to be amplified and, hence, it should grow. Questions concerning possible oscillations between different states

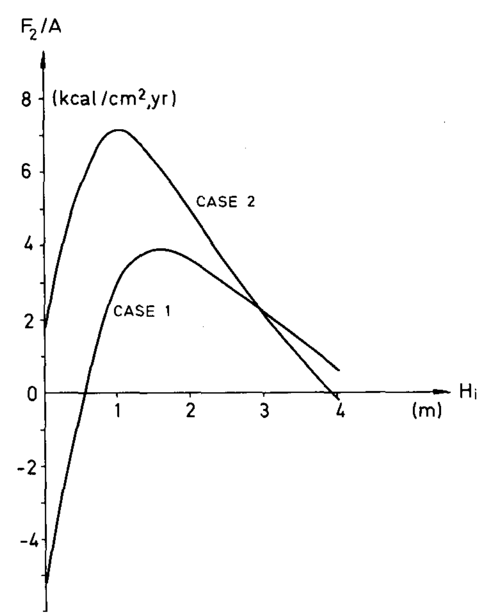

FIG. 8. Solutions of Eq. (22) showing the entrained heat flow F_2/A versus the ice thickness H_i. Case 1 is with the advective heat transports from Table 4. Case 2 is based on the advective heat transports given by Aagaard and Greisman (1975).

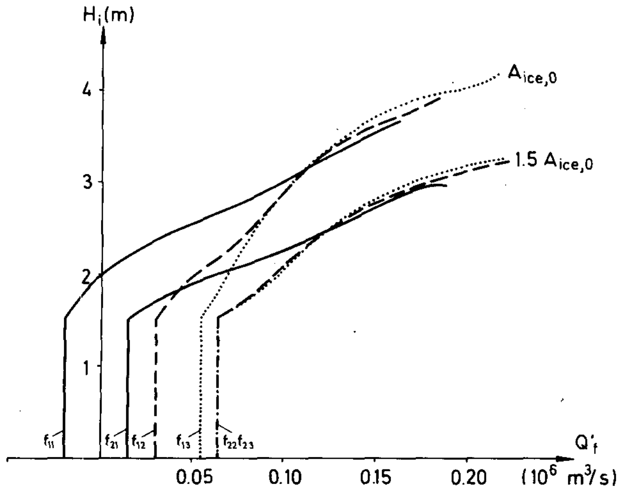

FIG. 9. A plot of the ice thickness versus the freshwater inflow to the Arctic ocean. Three different flows in the Bering Strait are used. A solid line shows the relation with the present Bering Strait flow, large dashed line is for half the present buoyancy flow and small dashed line for the closed Bering Strait. The area of the exported ice has two values. $A_{ice,0}$ is with the present ice-export and $1.5 \times A_{ice,0}$ is for a case where the ice-export (area) is 50% larger. For further information see the text.

and their stability must be investigated by the means of a time-dependent model. We leave that particular subject here.

6. The dependence of the ice thickness on some external parameters

The important freshwater parameter in the dynamic model developed in Section 2 is Q_f. It is equal to the freshwater supply Q_f' minus the freshwater contained in the exported ice Q_{ice} minus the flow of freshwater down into the lower layer in connection with deep-water formation Q_d. As Q_d has been shown to be a small quantity, at least in the present Arctic, see Section 3, the following relation should be approximately valid

$$Q_f' = Q_f + Q_{ice}. \tag{23}$$

Thus, the dynamic model does not give any information about how much of the freshwater buoyancy that flows out of the Arctic Ocean as ice. However, the model gives the relation between the rate of entrainment Q_2 and Q_f. The heat budget, on the other side, gives the relation between the ice thickness (and thereby Q_{ice}) and the rate of heat entrainment, F_2 (and thereby Q_2). Thus it is possible to establish the relation between the freshwater supply Q_f' and the ice thickness H_i by a combination of the model in Section 2 and Eqs. (22) and (23).

We have calculated the relation between the ice thickness and the freshwater supply and the details of this calculation are given here. For the calculation of Q_2 [from Eqs. (14) and (9)] for a given value of Q_f we use the values of f, m_0, β, g, γ, S_2 and A given in Table 1. For the Bering Strait parameters we use three different sets: $(Q_B, S_B) = (1.5, 32.4)$, $(1.0, 33.25)$ and $(0, 0)$, where Q_B is in 10^6 m^3 s^{-1} and S_B in ‰. Again, we use $T_B = 0.5°$C. We make the calculations with two different sets in areal export, A_{ice}. In one the area of the exported ice is as today $A_{ice,0}$ and in the other it is 50% greater. This gives us the following six combinations:

$f_{11} = (1.5, 32.4, A_{ice,0})$ $f_{21} = (1.5, 32.4, 1.5 \times A_{ice,0})$

$f_{12} = (1.0, 33.25, A_{ice,0})$ $f_{22} = (1.0, 33.25, 1.5 \times A_{ice,0})$

$f_{13} = (0, 0, A_{ice,0})$ $f_{23} = (0, 0, 1.5 \times A_{ice,0})$.

In Section 3 we found that $u_* = 0.55$ cm s^{-1} which value we use. From Q_2 we get the value of F_2 by the relation $F_2/A = Q_2[\rho_2 c_p(T_2 - T_1)]$. We then use the solution of Eq. (22), case 1, to determine the corresponding H_i. We choose the larger of the two possible values. Q_{ice} is then immediately obtained as $Q_{ice} = A_{ice} H_i \rho_{ice}/\rho_f$ and A_{ice} is given in Section 3. For the given Q_f we thus have the corresponding Q_{ice} and Eq. (23) then gives Q_f'.

The results of the computations are shown in Fig. 9. The general trend for all these curves is that a lower runoff from rivers and excess precipitation, Q_f', gives a thinner ice than a higher runoff. Already from Fig. 8 we could see that the ice disappears when the heat entrained from below, F_2/A, is larger than a certain critical value. For case 1 in that figure we see that the ice thickness at maximum heat entrainment with ice is ~1.5 m. This can already be seen in Fig. 9 where the curves fall to zero ice thickness when approaching this value from the right. The great importance of the Pacific water can be seen from Fig. 9. For the present Arctic (curve f_{11}) to become ice-free all the freshwater supply from rivers has to be removed and, besides, about 0.02×10^6 m^3 s^{-1} of fresh water must be taken away from the Arctic by, say, excess evaporation. The Pacific water thus has a large buffering effect. If the Bering Strait flow is stopped partly (case f_{12}) or completely (case f_{13}) a reduction of the freshwater supply by 55 and 45%, respectively, is sufficient to make the Arctic ice-free. Such reductions might have occurred in the past when glaciations led to a lower sea level and thereby to a lower transport rate through the shallow Bering Strait. [In a recent paper Coachman and Aagaard (1981) estimated, from current measurements in a section north of the Bering Strait, the flow through the Strait to be slightly $<1 \times 10^6$ m^3 s^{-1}. If this is correct the present Arctic Ocean is rather described by case f_{12}.]

The influence of the magnitude of the area of the exported ice is also clearly seen in Fig. 9. A larger rate of export of ice area gives a significantly thinner ice in the Arctic. It also leads to ice-free states for higher values of the freshwater runoff. The combined effect of a reduced flow in the Bering Strait and an increased rate of the exported ice area (case f_{22}) gives an ice-free state for a 35% reduction of the freshwater supply compared with the present freshwater supply (0.1×10^6 m^3 s^{-1}).

The effects on the pack ice of a suggested diversion of Siberian rivers have been discussed for many years (e.g. Aagaard and Coachman, 1975). As a complete model for the Arctic has not existed earlier the answers to that question have been more or less guesswork. The present model, however, may be used for an estimate of the effects of a diversion and Fig. 9 gives, in fact, the large-scale effect. In addition, as pointed out by Aagaard and Coachman, there may be local and regional effects in the neighborhood of the mouth of a diverted river. With the present runoff from rivers the Arctic Ocean is strongly buffered and minor concomitant disturbances in climate, ice export and flow through the Bering Strait do not lead to an ice-free Arctic. However, if a large portion of the present runoff were diverted the Arctic will be much less buffered and rather small concomitant disturbances might lead to the disappearance of the ice.

In Section 5 we raised the question: Under what conditions will the ice disappear from the Arctic? We claim that an approximate answer to that question is found in Fig. 9. However, this figure is not complete as it does not include effects of climate changes on the ice thickness. Climate changes will lead to changes in the coefficients μ, b and c. The effects of changed climate are, however, not treated in this paper.

The rate of entrainment may change when the ice-free fraction a changes. The reason for this should be that the wind stress is transmitted to the water in a different way when the pack ice is absent (c.f. Solomon, 1973). However, we do not attempt to parameterize this effect in the present work. Finally, we point to the fact that in the computations undertaken in this paper we have neglected the effect of the temperature on the buoyancy of the upper layer. Qualitatively, this means that the buffering effect of the freshwater buoyancy supply is somewhat overestimated in the present calculations. However, the temperature effect is of second order. It may be included in the way suggested in the text following Eq. (7) in this paper.

7. Concluding remarks

This work attempts to model the dependence of the ice thickness (alternatively, the ice-free fraction) on some external parameters by the simultaneous use of models for the dynamics of the upper layer, some ice properties and the heat balance. One of the guiding principles followed throughout this work is the one that the physics of the problem has to be modeled as correctly as possible and only to the first order. Thus many effects of less importance, although interesting in themselves, are deleted. The salinity and thickness of the upper layer are not constant over the Arctic Ocean. There are horizontal gradients mirroring the internal dynamics, the distribution of freshwater sources and the inflow of Pacific water. Also the horizontal distribution of vertical mixing and freezing-melting of ice are factors in determining the mass field. At the present time we do not possess all the knowledge necessary for the construction of detailed baroclinic models for the Arctic Ocean.

In constructing the models presented in this paper it was necessary to make a number of assumptions (e.g., the two-layer nature of the Arctic Ocean, geostrophically balanced flows in the straits, an entrainment law, the ice export dependency on the wind field, the ice thickness dependency on the fraction of open water, etc.) Such a long list of uncertainties might be considered a weakness. However, it might perhaps rather be looked upon as a logically constructed list of priority items for future field work.

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APPENDIX

List of Symbols

α	coefficient of thermal expansion
β	the fractional increase in density per unit increase in salinity
γ	number of "geostrophic outlets"
ϵ	entrainment parameter
μ	coefficient of heat exchange
ν	parameter $[=(A_{ice}\cdot\rho_{ice}\cdot L)/A]$
$\rho_1(\rho_2)$	density of upper (lower) layer
$\rho_f(\rho_0)$	density of fresh (reference) water
ρ_{ice}	density of ice
τ	stress caused by the ice motion
τ_r	residence time of fresh water in the Arctic Ocean
A	horizontal area of the pycnocline in the Arctic Ocean
A_{ice}	mean rate of export of horizontal ice area
F_2	advective heat flow from the lower to the upper layer
$F_{2'}$	advective heat flow from the lower to the upper layer (connected to the deep water formation)
F_a	vertical heat flow through the ice
F_B	advective heat flow through the Bering Strait

$F_{\rm ice}$	flow of latent heat deficit by the exported ice
F_p	vertical heat flow through polynyas
H_1	thickness of the upper layer
H_i	thickness of the exported ice
$\bar{H}_i$	mean thickness of the ice of the Arctic Ocean
H_{i0}	length, specific for the ice in the Arctic Ocean
L	heat of fusion for ice
$P(P_B)$	mixing parameter for the upper (Bering Strait) water
Q_1	volume flow of upper water out of the Arctic Ocean
Q_2	volume flow from the lower to the upper layer
$Q_{2'}$	volume exchange between the lower and upper layers connected to the deep-water formation
Q_B	volume flow through the Bering Strait
Q_d	volume flow of fresh water down into the lower layer
Q_f'	volume flow in rivers emptying into the Arctic Ocean
Q_f	"effective" volume flow of fresh water to the upper layer ($=Q_f' - Q_{\rm ice} - Q_d$)
$Q_{\rm ice}$	volume flow of fresh water as ice out of the Arctic Ocean (volume flow of ice times $\rho_f/\rho_{\rm ice}$)
$S_1(S_2)$	salinity of upper (lower) layer
$S_0(S_B)$	salinity of reference (Bering Strait) water
$S_{\rm ice}$	salinity of the exported ice
$T_1(T_2)$	temperature of upper (lower) layer
$T_0(T_B)$	temperature of reference (Bering Strait) water
a	ice-free fraction of the mean Arctic Ocean
b	coefficient of radiative heat exchange
$c(a)^{-1/2}$	coefficient of heat exchange by conduction and evaporation
c_d	drag coefficient for the ice
f	Coriolis parameter
g	acceleration of gravity
g'	reduced acceleration of gravity
m_0	entrainment constant
u	scale of velocity difference between ice and underlying water
u_*	friction velocity in the upper layer
w_e	entrainment velocity

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