Electronics and Communications

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Up to now we have covered:

- Basic structure of telecommunication systems
- Signal representation in time and frequency domains
- Amplitude Modulation (AM)
  - » DSB-FC
  - » DSB-SC
  - » SSB-SC

Contents of Lecture 12

- Another form of modulation called Frequency Modulation (FM).

Frequency Modulation (FM)

- In Amplitude Modulation (AM) the amplitude of the carrier signal is varied in proportion to the message signal.
- But there are other characteristics of the carrier signal which can also be varied e.g. frequency and phase.
- In Frequency Modulation (FM) we vary the instantaneous frequency of the carrier signal in proportion to the message signal.

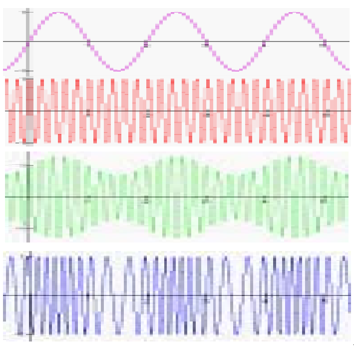
Frequency Modulation (FM)

message signal:

carrier signal:

DSB-FC
AM signal:

FM signal:



Frequency Modulation ctd

- Generally, we can express the sinusoidal carrier signal c(t) as:
  $$c(t) = A(t)\cos(\theta(t))$$
- When amplitude A(t) is varied according to message m(t) ($A(t) \propto m(t)$) - we get Amplitude Modulation.
- When angle θ(t) is varied according to message *m(t)*, we get Angle Modulation (Phase Modultion or Frequency Modulation).

Instantaneous Frequency

- Instantaneous frequency f(t) of a sinusoid is related to the angle θ(t) through:

$$f(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$$

- Hence

$$2\pi f(t)dt = d\theta$$

and

$$\theta(t) = 2\pi\int_0^t f(t)dt$$

Instantaneous Frequency: Example

- Determine the instantaneous frequency in Hz of the following signals
- a) x(t) = 2cos(2π10t)
- b) x(t) = 2cos(2π10t + πt^2)

Instantaneous Frequency: Example - Solution

- For the case (a) we have:

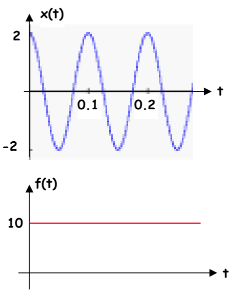
$$x(t) = 2\cos(2\pi \times 10t)$$

- Hence, the instantaneous frequency is:

$$f(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$$

$$= \frac{1}{2\pi}(2\pi \times 10) = 10 \text{ Hz}$$

which is a constant of time.



Instantaneous Frequency: Example - Solution

- For the case (b) we have:

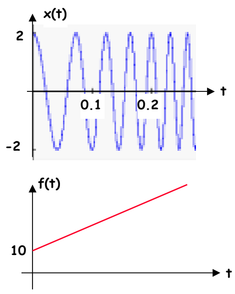
$$x(t) = 2\cos(2\pi \times 10t + 10\pi t^2)$$

- Hence, the instantaneous frequency is:

$$f(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$$

$$= \frac{1}{2\pi}(2\pi \times 10 + 20\pi t)$$

$$= 10 + 10t \text{ Hz}$$



VCO and Instantaneous Frequency

- We want to vary instantaneous frequency f(t) in proportion to the message signal m(t)
- Recall that we can use a Voltage Controlled Oscillator (VCO) to produce a signal whose frequency is proportional to the input voltage.

- Frequency f(t) of e(t) is given by:

$$f(t) = f_c + k_o m(t)$$

VCO and Instantaneous Frequency (ctd)

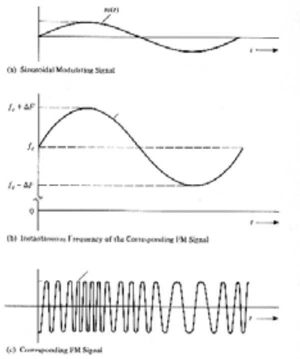
- k_o is proportionality (or sensitivity or gain) constant of the VCO (in Hz/volt)
- *Frequency deviation* from the free running frequency f_c is given by:

$$\Delta f(t) = f(t) - f_c = k_o m(t)$$

- *Peak frequency deviation* from the free running frequency f_c is given by:

$$\Delta F = \max(\Delta f(t)) = k_o \max(m(t))$$

VCO and Instantaneous Frequency (ctd)



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FM Analysis

- A VCO can be used to generate an FM signal.
- The resulting FM signal e(t) is then given as:

$$e(t) = A\cos[\theta(t)]$$
$$= A\cos[2\pi\int_0^t f(t)dt]$$
$$= A\cos[2\pi f_c t + 2\pi k_o \int_0^t m(t)dt]$$

FM Analysis

- To simplify our analysis we use a sinusoidal message signal (tone signal) $m(t) = -A_m\sin(2\pi f_m t)$
- Hence our FM signal becomes:

$$e(t) = A\cos[2\pi f_c t + \frac{k_o A_m}{f_m}\cos(2\pi f_m t)]$$
$$= A\cos[2\pi f_c t + \frac{\Delta F}{f_m}\cos(2\pi f_m t)]$$
$$= A\cos[2\pi f_c t + m_f\cos(2\pi f_m t)]$$

- m_f is called the modulation index for FM.

Modulation Index in FM

- Recall that modulation index measures the extent to which the carrier signal is varied by the message signal.
- Modulation index of an FM signal modulated by a *tone* signal is given by:

$$m_f = \frac{\Delta F}{f_m} = \frac{\text{peak frequency deviation}}{\text{frequency of tone signal}}$$

- In general, modulation index of an FM signal modulated by an arbitrary signal is given by:

$$m_f = \frac{\Delta F}{B} = \frac{\text{peak frequency deviation}}{\text{bandwidth of modulating signal}}$$

Example

- A 1MHz VCO with sensitivity of 3kHz/V is modulated with a tone signal of peak amplitude equal to 2V and frequency equal to 4kHz. Calculate the index of modulation of the output FM signal.
- Solution:
  - » f_c=1MHz
  - » k_0=3kHz/V
  - » A_m=2V
  - » f_m=4kHz

$$m_f = \frac{\Delta F}{f_m} = \frac{k_0 A_m}{f_m} = \frac{2\times 3}{4} = 1.5$$

Narrowband and Wideband FM

- Recall that for a sinusoidal message signal our FM signal is given mathematically as:

$$e(t) = A\cos[2\pi f_c t + m_f\cos(2\pi f_m t)]$$

- If $m_f < 0.25$ then we have Narrowband FM.
- If $m_f \geq 0.25$ then we have Wideband FM.

- Next we look at how to obtain magnitude spectra for Narrowband FM and Wideband FM.

Magnitude Spectrum of Narrowband FM

- Our FM signal in the time domain is given by:

$$e(t) = A\cos[2\pi f_c t + m_f \cos(2\pi f_m t)]$$

- Using trigonometric identity:

$$\cos(a+b) = \cos(b)\cos(a) - \sin(b)\sin(a)$$

we get:

$$e(t) = A\cos[m_f \cos(2\pi f_m t)]\cos(2\pi f_c t) - A\sin[m_f \cos(2\pi f_m t)]\sin(2\pi f_c t)$$

Magnitude Spectrum of Narrowband FM (ctd)

- Let

$$\Delta\Phi = m_f \cos(2\pi f_m t)$$

- Then for $m_f < 0.25$ we have the following approximations:

$$\cos(\Delta\Phi) \approx 1$$
$$\sin(\Delta\Phi) \approx \Delta\Phi$$

and our FM signal becomes:

$$e(t) = A\cos(2\pi f_c t) - Am_f \cos(2\pi f_m t)\sin(2\pi f_c t)$$

- Hence the narrowband FM is equivalent to DSB-SC with a sin (not cos) carrier plus a cos carrier.

Magnitude Spectrum of Narrowband FM (ctd)

- The first term is just the carrier, and the second one we can write as:

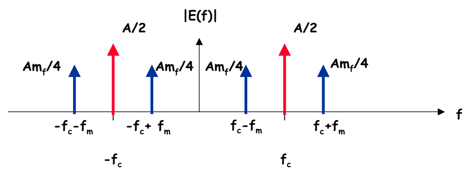
$$Am_f \cos(2\pi f_m t)\sin(2\pi f_c t) = = \frac{Am_f}{2}\sin[2\pi(f_c - f_m)t] + \frac{Am_f}{2}\sin[2\pi(f_c + f_m)t]$$

using the trigonometric identity:

$$\cos(a)\sin(b) = \frac{1}{2}\sin(a-b) + \frac{1}{2}\sin(a+b)$$

Magnitude Spectrum of Narrowband FM (ctd)

- Resultant magnitude spectrum for narrowband FM is:



Magnitude Spectrum of Wideband FM

- In genaral, our FM signal is given by:

$$e(t) = A\cos[2\pi f_c t + m_f \cos(2\pi f_m t)]$$

- If m_f is not small (i.e $m_f > 0.25$) we cannot use the approximations which we used for narrowband FM.
- Analysis becomes much more complex.
- However, frequency components of the FM signal can be written in terms of Bessel functions.

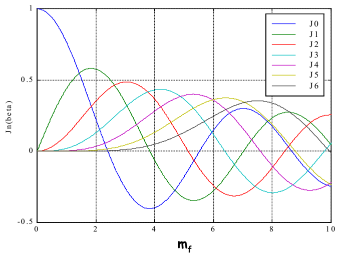
Magnitude Spectrum of Wideband FM ctd

- It turns out (without proving it in this subject) that FM signal e(t) can be written as the following sum:

$$\begin{aligned} e(t) &= AJ_0(m_f)\cos(2\pi f_c t) \\ &+ AJ_1(m_f)\{\cos[2\pi(f_c+f_m)t] - \cos[2\pi(f_c-f_m)t]\} \\ &+ AJ_2(m_f)\{\cos[2\pi(f_c+2f_m)t] + \cos[2\pi(f_c-2f_m)t]\} \\ &+ \ldots \\ &= A\sum_{n=0} J_n(m_f)\{\cos[2\pi(f_c+nf_m)t] + \cos[2\pi(f_c-nf_m)t]\} \end{aligned}$$

- The functions $J_n(m_f)$ are referred to as the Bessel functions of the first kind of the nth order and tables are usually used to facilitate calculations.

Magnitude Spectrum of Wideband FM ctd



- Bessel functions of the first kind of the orders n=0 to 6.

Magnitude Spectrum of Wideband FM Summary

- The magnitude spectrum |E(f)| of e(t) resulting from frequency modulating of a sinusoidal carrier with even a single tone

  $$m(t) = A_m \cos(2\pi f_m t)$$

  contains a carrier component and an infinite number of sidebands located on either side of the carrier frequency, spaced at integer multiples of f_m.

- Since FM is a nonlinear modulation, the spectrum of an FM signal must be evaluated on a case-by-case basis for any modulating wave shape.

Bandwidth of FM Signals

- The bandwidth of an FM signal depends on both f_m and m.
- It can be shown that 98% of the total power is contained in the bandwidth:

  $B_T = 2(m + 1) f_m$ (Upper bound)

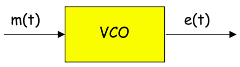
  $B_T = 2\Delta f$ (Lower bound)

- The above approximation is called Carson's rule.
- For small values of modulation index (m <<1), the spectrum of an FM signal is effectively limited to the carrier frequency f_c and one pair of side bands.

Break

FM Generation

- As we saw earlier we can generate an FM signal using a VCO whose free running frequency is set to equal to the carrier frequency:



free running frequency = carrier frequency f_c

Indirect FM Modulator

- The narrow-band, (m_f<<1), FM signal can be approximated as:

$$s(t) = A\cos(2\pi f_c t) - A\Delta\Phi(t)\sin(2\pi f_c t)$$

- This indicates that a narrowband angle modulated signal consists of two terms:
  - a discrete carrier component
  - a sideband term.
- This is similar to AM-type signalling, however, the sideband term is 90° out of phase with the carrier term.

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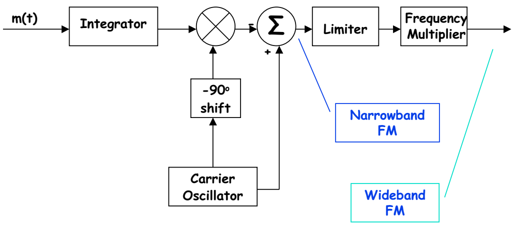
Indirect FM Modulator ctd



Block diagram of an indirect (Armstrong) wideband FM modulator.

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FM Demodulation

- The objective of all FM demodulators is to produce a transfer characteristic that is the inverse of that of the frequency modulator.
- There are many types of FM demodulators:
  - slope detectors (based on differentiation),
  - zero-crossing detectors,
  - phase locked discriminators,
  - quadrature detectors.

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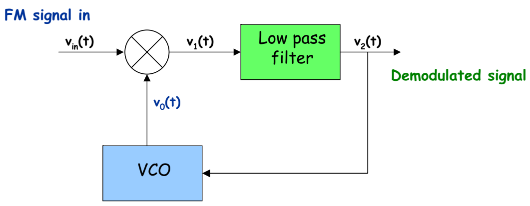
FM Demodulation ctd

- A common method to demodulate an FM signal (i.e. to recover the message from an FM signal) is to use a phase-locked loop (PLL)



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FM Demodulation ctd

- The PLL VCO's free running frequency is set to the FM carrier frequency f_c.
- Hence, the frequency of the VCO output signal $v_o(t)$ is equal to f_c when its input signal $v_2(t)$ = 0.
- However, signal $v_2(t)$ changes as the FM signal e(t) changes.
- This signal $v_2(t)$ forces the VCO to track the frequency of the input signal.
- As a result, $v_2(t)$ is proportional to the frequency variations of the FM signal e(t).
- So $v_2(t)$ is also proportional to the message signal contained in the FM signal e(t) and hence it is our demodulated signal as required.

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FM Applications

- **Radio broadcasting (wideband FM).**
- **TV broadcasting (of audio signal) - video signal is AM.**
- **Mobile communications (narrowband FM).**

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FM Summary

- Time domain representation:

  $e(t) = A\cos[2\pi f_c t + 2\pi k_o]$ for a general signal m(t)

  $e(t) = A\cos[2\pi f_c t + m_f \cos(2\pi f_m t)]$ for a tone signal
- Frequency domain representation:
  - narrowband FM ($m_f \le 0.25$) - spectrum similar to DSB-FC AM.
  - wideband FM ($m_f > 0.25$) - spectrum consists of potentially many sidebands.
  - Spectrum for both narrowband and wideband FM can be obtained using *Bessel function tables.*

FM Summary ctd

- FM generation - use a voltage controlled oscillator (VCO).
- FM demodulation - use a phase-locked loop (PLL).

Phase Modulation (PM)

- Phase Modulation is a form of angle modulation in which the information carrying phase component θ(*t*) varies linearly with the modulating signal, i.e.

$$\theta(t) = k_\theta m(t)$$

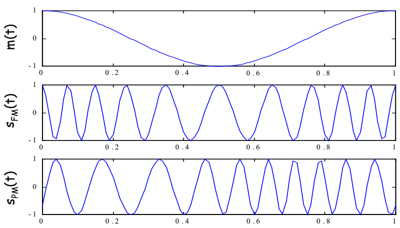
- This result in the PM signal being of the form:

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_\theta m(t)]$$

- The constant k_θ is the phase deviation constant, measured in rad/V.

FM and PM Example

- Example plots for a harmonic modulating signal



Phase Modulation Index

- The phase modulation index is given by:

$$\beta_P = k_\theta A_m = \Delta\theta$$

where Δθ is the peak phase deviation of the transmitter.

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Revision

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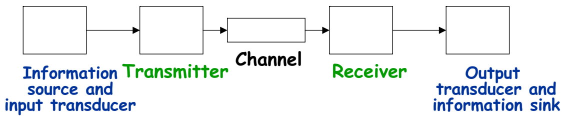
Main Areas Covered

- Basic structure of telecommunication systems.
- Signal representation in time and frequency domains.
- Amplitude Modulation (AM).
- Frequency Modulation (FM).

Telecommunication Systems

Aim: to transmit information from one point to another (or point to multi-point)



Explain the purpose of each element

Signal Representation in Time and Frequency Domains

- Represent signals in time domain:
  - amplitude (voltage, current, etc) as a function of time.
- Represent signals in frequency domain:
  - amplitude as a function of frequency
  - double-sided and single-sided magnitude spectra
  - for non-periodic signals - use Fourier Transform table directly
  - for periodic signals - use theorem:

$$W(f) = f_0 \sum_{n=-\infty}^{n=\infty} H(nf_0)\delta(f - nf_0)$$

Amplitude Modulation (AM)

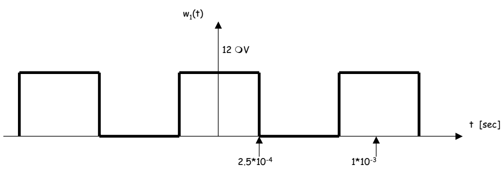
- Explain the concepts of:
  - » DSB-FC
  - » DSB-SC
  - » SSB-SC
- Represent various AM types in time domain (except for SSB-SC) and frequency domain.
- Draw basic structures and explain operation of AM modulators and demodulators (mixers, envelope detector, product detectors).
- Explain methods of obtaining coherent local carrier signal.

Frequency Modulation (FM)

- Represent FM signals in time domain and frequency domain (for narrowband and wideband FM with sinusoidal message only).
- Draw structures and explain operation of FM modulators and demodulators (VCO, PLL).

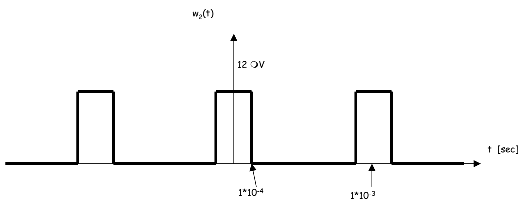
Sample Problems

- *(Frequency domain representation (double sided spectrum) for a periodic signal - problem 1)*
- For the square wave shown below obtain its double-sided spectrum in the range -5000 to 5000 Hz.



Sample Problems (ctd)

- *(Frequency domain representation (double sided spectrum) for a periodic signal - problem 2)*
- For the square wave shown below obtain its double-sided spectrum in the range -5000 to 5000 Hz.



Sample Problems (ctd)

- *(Representation of AM signals in time and frequency domains)*
- **Given the message signal:**

$$m(t) = 4\cos(2\pi 5\times 10^{3} t) \text{ volts}$$

and a sinusoidal carrier signal with frequency f_c = 25 kHz

a) sketch to scale in the range 0 to 0.4 ms the resultant Amplitude Modulated (AM) signal for 70% modulation,

b) sketch the magnitude spectrum of the AM signal.

Sample Problems (ctd)

- *(Representation of FM signals in time and frequency domains)*
- **For the FM signal:**

$$e(t) = 1000\cos[2\pi 10^{7} t + 5\cos(2\pi 10^{4} t)]$$

determine the amplitudes of all its spectral components. Draw the resulting magnitude spectrum (show only positive frequency components) and determine the bandwidth requirement of the FM signal. Using Carson's rule evaluate the bandwidth of the FM signal and compare it to the value obtained from your spectral plot.

Questions & Comments