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Numerical development: current issues and future perspectives

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Abstract

The goal of this article is to give an overview about current issues and future perspectives in numerical development. First, we shortly discuss the evidence for an innate number sense. As one core representation of a supposed innate number sense is pre(non)verbal quantity representation, we discuss contrasting models of such a pre(non)verbal quantity representation. We then introduce calculation and transcoding models and discuss – particularly for the first – whether and when we need developmental calculation models that are distinct from adult calculation models. After discussing the influence of other cognitive abilities on number processing, we review the evidence for the postulated functional-anatomical link between parietal regions and numerical development. In the final part about future perspective, we elaborate how dyscalculia patient studies and intervention studies might enhance our understanding of numerical development. Based on this review of quite heterogeneous and parallel literatures about different topics, we conclude that a stronger integration of these different approaches and models is needed for the future.

Key words: numeric development, quantity representation, number processing, neural correlates, dyscalculia, intervention

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Introduction

Children master numerical core concepts such as magnitude, counting and number conservation already before they are exposed to explicit school mathematics (e.g., Fuson, 1988; Gallistel & Gelman, 1992; Ginsburg, 1977; Resnick, 1982; Siegler & Shrager, 1984). Likewise, even illiterate children demonstrate rather advanced levels of arithmetic computation skills without ever being explicitly trained in mathematics (Nunes, 1993). Hence, during the past two decades, there has been increasing interest in studying preverbal children (some examples of infant habituation studies: Bijeljac-Babic, Bertoncini, & Mehler, 1993; Carey & Xu, 2001; Köchlin, Dehaene, & Mehler, 1998; Wynn, 1996; Xu & Spelke, 2000) and even neonates (Antell & Keating, 1983). Overall, there is converging evidence that even a few months old infants are capable of discriminating small sets of objects, even after controlling for confounding variables such as physical stimulus properties like contour length, density, luminosity etc. (however, for contrasting views, see Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, 2002; Mix, Huttenlocher, & Levine, 2002; Tan & Bryant, 2000). Moreover, maybe even more surprisingly, infants as young as 5 months old do have additive and subtractive expectations (Simon, Hespos, & Rochat, 1995; Wynn, 1992) and according to Brannon (2002), 11-months-old infants do demonstrate ordinal numerical knowledge (whereas 9-months-olds do not master these greater than/less than numerical relationships). While earlier respective studies focused on children's manipulation abilities within the so-called subitizing range (that is small item sets up to 3 or 4 objects), Xu & Spelke (2000) were able to show that infants are able to discriminate sets far beyond the subitizing range, provided the ratio is large enough (i.e., infants in their study were able to discriminate 8 from 16, but not 8 from 12 items).

Consequently, some researchers believe that numerical abilities are genetically determined, and terms such as 'number sense' (Dehaene, 1992) or 'number module' (Butterworth, 1999) came into play (see also Geary, 1993; Kosc, 1974; Spelke, 1996). The assumption of an inherited, language independent basis of core numerical knowledge has been further corroborated by non-human primate studies (Boysen, & Berntson, 1989; Brannon & Terrace, 1998, 2000; Hauser, Carey, & Hauser, 2000; Matsuzawa, 1985; Nieder, Freedman, & Miller, 2002; Nieder & Miller, 2003). Recently, Nieder and colleagues found number-specific neurons in primates ('numerons') which were selectively activated by specific numerosities in the range from 1 to 5. While some researchers interpreted this finding as strong evidence for a biological predisposition of numerosity (Dehaene, Piazza, Pinel, & Cohen, 2003), others claim that numerosity-specific neuronal firing patterns can also be learnt via simple learning mechanisms in very few trials (Verguts, Fias, & Stevens, 2003; Verguts & Fias, 2004). Thus, although most researchers tend to favour the idea of strong biological determination of the numerical competence found, the issue is not fully resolved yet.

In line with the biological determination account, some researchers claim that arithmetic ability/aptitude constitutes a specific cognitive domain (Rossor, Warrington, & Cipolotti, 1995) that dissociates – among others - from linguistic ability (e.g., Dehaene, 1992; Dehaene, Dehaene-Lambertz, & Cohen, 1998). Likewise, it has been claimed that arithmetic knowledge does not necessarily correlate with reading skills (e.g., Geary & Hoard, 2001; Hanich, Jordan, Kaplan, & Dick, 2001; Rourke & Strang, 1983; Shalev, Manor, Amir, & Gross-Tsur, 1993). Nevertheless, there is converging evidence for the fact that some aspects of arithmetic are associated with specific language skills. For instance, solving of simple mental calculation is initially facilitated by verbal counting strategies (Barouillet & Fayol, 1998; Geary, 1994) and furthermore, it is supposed to be acquired in a language-specific format (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Geary, 1993). Additionally, Butterworth (1999) stresses the importance of finger counting strategies for understanding numbers and simple calculation. Likewise, the decoding and understanding of text problems crucially depend on intact receptive language abilities (e.g., Hanich et al., 2001). In sum, the available evidence suggests that numerical cognition is a cognitive domain of its own right that is neither strongly determined by other cognitive capabilities nor fully independent of them.

Another crucial finding reported in the literature is that arithmetic abilities are composed of different, quite dissociable knowledge domains, both at a functional (e.g., Baroody & Ginsburg, 1986; Geary, 1993; Shalev et al., 1993; Temple, 1989, 1991) and neuro-anatomical level (adult neuropsychological studies: Cipolotti & Butterworth, 1995; Dehaene & Cohen, 1997; Delazer & Benke, 1997; Hittmair-Delazer, Sailer, & Benke, 1995; McCloskey, Caramazza, & Basili, 1985; Pesenti, Seron, & Van der Linden, 1994; Warrington, 1982; see also Domahs & Delazer, this issue; brain imaging studies of adults: Dehaene et al., 1999; Gruber, Indefrey, Steinmetz, & Kleinschmidt, 2001; Pesenti, Thioux, Seron, & De Volder, 2000; Rickard, Romero, Basso, Wharton, Flitman, & Grafman, 2000; Zago, Pesenti, Mellet, Crivello, Mazoyer, & Tzourio-Mazoyer, 2001). To further complicate the matter, one has to keep in mind that average arithmetic development does not pursue a straight, fully predictable course of acquisition, but rather can be characterised by quite impressive individual differences (e.g., Dowker, 1998; Geary & Widaman, 1987; Lemaire & Siegler, 1995; Siegler, 1987; Siegler & Jenkins, 1989).

In the following sections, we will outline a) contrasting models of pre(non)verbal quantity representations; b) developmental calculation models and discuss reasons why they should be distinct from adult calculation models; c) transcoding models. Finally, we discuss how evidence from various fields (such as numeracy intervention studies, dyscalculia research, math anxiety research, brain imaging studies) might enhance our understanding of numerical development in the future.

Models of pre(non)verbal quantity representation

Strictly hierarchical accounts

Up to date, there is still a controversy about the order of acquisition of numerical/arithmetic abilities. Traditionally, according to Piaget (1952) and post-Piagetian researchers, arithmetic development follows a strict hierarchical sequence of acquisition (such as proposed by the stage model of Ginsburg, 1977; see also Siegel, 1982; Siegler & Robinson, 1982). In other words, it has been argued that specific numerical skills need to be mastered before other – maybe more difficult – skills can be acquired. This view is the basis of almost all traditional developmental models. Consequently, a developmental disability or learning difficulty would imply a failure to acquaint a further step in the learning sequence, whereas the previously acquired skills should still be mastered. However, this strict view has been largely discarded because it does not provide a sound explanatory framework of the developmental behavioural data reported so far. For instance, Temple's research (1989,

1991) has shown that – similar to acquired calculation disorders - also in developmental dyscalculias double dissociations can be found, a finding which renders the classical developmental (stage) models implausible. Another nice piece of evidence against traditional hierarchical accounts is provided by Censabella and Noël (this issue). They tested the hypothesis that acquisition of multiplication facts interferes with previously learned mastery of addition problems. Indeed, the addition performance deteriorated after multiplication knowledge was established. Hence, the authors concluded that the acquisition of a new skill is not based on the mastery of an earlier stage, but rather it interferes with capabilities learned earlier in life. Thus, clearly, a strictly hierarchical sequence account is not compatible with the data obtained recently.

Weakly hierarchical accounts and preverbal counting models

A weaker version of the hierarchical account has been put forward by those researchers concerned with the development of counting skills. Specifically, it has been claimed that core numerical skills, namely pre(non)verbal quantity representations, need to be established before conventional, language dependent verbal counting skills can be assimilated (e.g., Fuson, 1988; Gallistel & Gelman, 1992; Resnick, 1983)[2]. With respect to arithmetic fact (like 4+3, 4x3) and procedural knowledge (correct execution of complex arithmetic operations such as multi-digit written problems), some authors argue that the two components are partly independent of each other and thus their order of acquaintance does not need to follow a strict hierarchical sequence, but rather should be regarded as a continuous interplay (e.g., Baroody, 2003; Rittle-Johnson, Siegler, & Alibali, 2001). Regarding counting skills, Fuson's work (1988) implies a rather hierarchical fashion of learning: the most immature counting strategy being mastered first (such as counting all), that is followed by more mature counting strategies that reflect some knowledge about cardinality (such as counting on or counting on from the larger). Similarly, it has been proposed that preverbal counting/computational knowledge precedes verbal counting/computational skills (preverbal/implicit skills: Gallistel & Gelman, 1992; protoquantitative knowledge: Resnick, 1992). In the following, we will briefly describe various models of pre(non)verbal models of quantity representation.

In their influential work, Gallistel & Gelman (1992) explicate a preverbal counting model by adopting the so-called *accumulator model* that was initially proposed by Meck and Church (1983) to account for the timing and counting behaviour of rats. Meck, Church, and Gibbon (1985) were able to train rats to discriminate two sets of numerosities (2 versus 8 events). Moreover, they demonstrated that rats trained on duration discrimination only spontaneously generalised their discrimination behaviour to non-trained numerical differences. They concluded that in animals, both aspects of magnitude (durational and numerical) are likely to be represented by the same mental magnitude representations. According to the accumulator model (Meck & Church, 1983), a given set of objects - a specific numerosity - is transformed (mapped)- to their respective mental magnitude representation. This is supposed to be achieved by a stage-wise processing, in which a burst of impulses needs to pass a

[2] However, it has to be noted that there is disagreement with respect to the direction of causality: some believe that principles come first (e.g., Gallistel & Gelman, 1992), while others propose that knowledge about counting principles develops after the mastery of rote verbal counting routines (e.g., Fuson, 1988).

gate in order to enter and subsequently to increment the current state of the accumulator. Moreover, the pairing between the states of the accumulator and the objects (impulses) to be counted presumably occurs on a one-to-one basis. Hence, it reflects the isomorphism between the ordering of magnitudes and numbers. Finally, Gallistel and Gelman (1992; Gelman & Gallistel, 1978) claimed that 'the operation of this mechanism conforms to the principles that define counting processes' (p. 51) and proposed a *bidirectional mapping hypothesis*: arithmetic tasks requiring the manipulation of Arabic digits - i.e., upon attempting to count and/or compute sums - need to be mapped to their corresponding preverbal representations of numerosity (magnitudes) and vice versa (for a similar view, see Wynn, 1995).[3]

However, the accumulator model is not the only candidate for a plausible explanation of pre(non)verbal quantity representations and it has been criticised that it can not account for the successful discrimination/enumeration behaviour of simultaneously presented visual object sets as found in nonverbal animals and/or preverbal infants (Mix et al., 2002). Alternative views are the subitizing account (e.g., Starkey & Cooper, 1980; Trick & Pylyshyn, 1994), the object representation account (Simon, 1997; Uller, Carey, Huntley-Fenner, & Klatt, 1999; for a review, see Carey & Xu, 2001) and mental models (Huttenlocher, Jordan, & Levine, 1994). Nevertheless, we will come back to the accumulator model later, discussing it's potential usefulness in comparative studies of number-related research areas.

The mental model

According to Mix et al. (2002), the models that fit best to a wide range of empirical data seem to be the *mental model* (Huttenlocher, Jordan, & Levine, 1994) and the object representation account (object files: Kahneman, Treisman, & Gibbs, 1992; FINST [Fingers of Instantiation, see below for an explanation]: Trick & Pylyshyn, 1994). Although Mix et al. (2002) admit that their view is quite speculative, they excel in providing an in-depth and critical review of the existing respective literature by questioning the conclusiveness of most of the popular findings. Based on the findings of their own as well as other laboratories (Feigenson et al., 2002; Simon, 1997; Tan & Bryant, 2000; for a review, see Mix et al., 2002), the authors stress the importance to distinguish between perception of continuous amount on the one hand and discrete number on the other hand (i.e., mass versus number). A key assumption of Mix et al. (2002) is that many studies claiming that infants can discriminate small numerosities (and maybe larger ones, too, if the ratio is large enough; Xu & Spelke, 2000), did confound numerosity (i.e., discrete amount) with physical properties of the relevant stimuli such as length, density, luminosity etc. (i.e., continuous amount). Hence, Mix et al. (2002) argue that infants should be less or not at all sensitive to discrete number if continuous stimulus variables have been controlled for. And indeed, that was what they

[3] In contrast to Dehaene's assumption of the logarithmic compressive mapping of number words (digits) onto the mental number line (Dehaene, 1992), the preverbal magnitude representations purported by Gallistel & Gelman (1992) are assumed to be characterised by a linear mapping from number words to the mental number line (so-called 'scalar variability' account; Dehaene, 2001; Seron & Pesenti, 2001; for that controversy in the special issue of Mind and Language). The latter distinction becomes important with respect to the subjective difference between two equally well discriminable numbers: it is a constant in the logarithmic mapping model, while it increases linearly with a fixed scalar parameter with the mean numerical value in the scalar variability mapping model.

found (e.g., Clearfield & Mix, 1999; Gao, Levine, & Huttenlocher, 2000; Simon, 1997; Tan & Bryant, 2000). As Tan and Bryant (2000) point out, continuous amount frequently is associated with number in real life, too (consider the overall amount of two pieces of cake versus one piece or three pieces) and thus, estimates of overall amount would generally suffice to make a 'quasi-numerical' judgement. Moreover, it can be assumed that a learning process (that presumably requires higher level cognitive skills such as feature decomposition of visual displays, abstracting the relevant stimulus dimension) must be mastered that enables the child to separate these two dimensions of number processing.

The *mental model account* originally has been developed to provide an explanatory framework for nonverbal quantity transformation knowledge exhibited already by three year-old children that are otherwise not able to solve the same tasks presented verbally, either in word or story format (Levine, Jordan, & Huttenlocher, 1992). In classical transformation studies, children are required to mentally perform additive and/or subtractive transformations on previously hidden object sets. The assumptions of the mental model are the following: upon trying to solve such transformation tasks, children seem to "construct" a mental image of the hidden display. Likewise, they seem to perform mentally - and hence without drawing on linguistic competencies such as verbal counting strategies - all crucial and salient arithmetic transformations which lead them to the problem's numerical solution. Importantly, it has been suggested that the mental images formed to represent the (hidden) objects - including featural object characteristics - can be assigned sequentially. Moreover, they can be manipulated according to the transformations performed on the (hidden) set (like adding/subtracting objects). According to Huttenlocher and colleagues (1994), the latter ability to symbolically construct and/or mentally manipulate/update a visual display of discrete items, seems to occur simultaneously with mental imagery abilities in other functional domains such as language, symbolic play etc.

Finally, the *subitizing account* draws upon well established behavioural data: both adults and young children employ differential counting/enumeration mechanisms that depend on the numerosity of the set (adults: e.g., Jensen, Reese, & Reese, 1950; Mandler & Shebo, 1982; 5-year old children: Chi & Klahr, 1975; however, for a contrasting view, see Balakrishnan & Ashby, 1992). Specifically, small sets (up to three or four items) seem to be recognised quickly (as displayed by rather flat response latency curves), whereas sets larger than four generally require more time to be counted (as reflected by a linear increase in response latencies). Hence, it has been proposed that the quick enumeration of small sets (coined as 'subitizing') operates at a preattentive processing stage (Trick & Pylyshyn, 1994). Subsequently, subitizing has been considered as a plausible explanation for the quantity discrimination performance of pre(non)verbal infants and animals (see also Starkey & Cooper, 1980). Moreover, by stressing that subitizing reflects a preattentive - and hence obligatory - activation ('individuation') of a visually perceived scene Trick and Pylyshyn (1994) proposed a somewhat different conceptualisation of subitizing: the so-called FINST (Finger of Instantiation) is a metaphor for the assignment process that maps a reference token to each visually perceived feature of the scene. The FINST is thought to mediate object individuation and spatial tracking, the latter being limited capacity operations.

Another influential proposal is the so-called pattern recognition view purported by Mandler and Shebo (1982) claiming that enumerating small sets is quicker relative to large sets because the former can be simultaneously processed as they are easily recognised as patterns. However, as response latencies are not completely flat - as should be the case for simultaneous processing of set sizes within the subitizing range (but generally exhibit a slope of 20 ms per item; for a review, see Mix et al., 2002) - the pattern recognition view has been criticized as well. Another important distinction has been drawn between item and event subitizing. While item subitizing implies that a visual object set is presented simultaneously, the latter should be able to account for sequentially presented stimulus/object sets (such as sounds; Davis & Perusse, 1988).

To conclude, the models outlined above might be more or less suited to exemplify pre(non)verbal quantity knowledge. However, they neither attempt to provide an explanatory framework for the many remaining building blocks that constitute arithmetic knowledge (and even less so for the cognitive components that are involved in higher mathematical reasoning such as geometry, algebra etc.) nor do they attempt to link the diverse components of numerical and arithmetic thinking. Instead, most of these models do only account for few or even only one elementary numerical task which - in our view- is not entirely satisfying.

In the next sections, we will try to sketch the most influential adult calculation models and reason why we believe that developmental models should be designed 'out from the scratch'.

Calculation models: Can adult models be recruited to test developmental data?

The most influential adult calculation models are those of McCloskey and collaborateurs (1985; McCloskey, 1992; see also Cipolotti & Butterworth, 1995, for a modification) and Dehaene and Cohen (1995, 1997; Dehaene, 1992; see also Dehaene et al., 2003, for a recent specification). Both models depict the modular architecture of number processing and calculation by claiming that specific arithmetic skills can be dissociated from each other. The McCloskey model proposes systems for number comprehension and number production as well as a calculation system that mediates fact and procedural knowledge. The comprehension and production systems are thought to be format specific for Arabic digits and spoken/written number words. Likewise, Dehaene's 'triple-code' model proposes the interplay of three distinct modules: visual Arabic representations (parity judgements, multi-digit calculations), phonological/verbal word frame (counting, number fact retrieval) and analogue magnitude representations (approximate numerical abilities, magnitude comparisons). However, one disagreement among the models is the one regarding the nature of the mental magnitude representations: whereas the McCloskey model posits that every numerical input has to be converted into an internal semantic representation before it can be further processed, Dehaene's triple-code model suggests that the three kinds of representations are directly interlinked, and thus number processing and calculation do not need to pass through a semantic route. In McCloskey's model, it is assumed that magnitude is organized according to the base-10 power system of Arabic numerals. So 307 would be 3EXP{2} + 7EXP{0}. In contrast, in Dehaene's model, magnitude is assumed to be represented holistically on an analog mental number line. While earlier data were more compatible with Dehaene's model (Dehaene, 1992, 1997; Dehaene & Changeux, 1993; see also Brysbaert, 1995), more recent evidence suggests that the base-10 structure of Arabic digits also affects their magnitude representation (adult data: Nuerk, Weger, & Willmes, 2001; Nuerk, Geppert, van Herten, & Willmes, 2002; Ratinckx, Brysbaert, & Fias, submitted; developmental literature: Nuerk, Kaufmann, Zoppoth, & Willmes, 2004). This led Nuerk and colleagues to suggest a hybrid view that magnitude of multi-digits numbers is represented analog as well as decomposed for their constituent digits (see Nuerk & Willmes, this issue for details).

Importantly, both adult models do *not* incorporate components such as arithmetic reasoning, although the latter ability has been long emphasised in the respective developmental literature (also termed conceptual or teleological knowledge, Resnick, 1982; van Lehn, 1990). In the adult literature, Delazer was the first to empirically demonstrate that conceptual knowledge can be selectively disturbed/spared, and hence conceptual knowledge seems to be separable from other calculation skills such as fact retrieval in acquired acalculia, too (Delazer & Benke, 1997; Hittmair-Delazer et al., 1995; for a review, see Delazer, 2003). By demonstrating a double dissociation between approximate and exact calculation abilities in patients with acquired calculation disorders, Dehaene & Cohen (1997) suggest that the analogue magnitude representation mediates approximation skills (for a review, see Dehaene et al., 2003).[4]

Moreover, *estimation skills* have not been considered explicitly in adult models. However, it has to be noted, that (non-arithmetic) estimation abilities might be quite distinct from approximation skills. The former term generally denotes the enumeration of very large object sets and/or continuous quantities, and might be referred to when task difficulty is enhanced by limiting exposure time. Thus, estimation could be thought of as a specific enumeration process that is distinct from both subitizing and counting (Trick & Pylyshyn, 1994; for a review, see Mix et al., 2002).

Approximation, as used by Dehaene and Cohen (1991) probably refers to arithmetic/computational estimation (that is quite distinct from non-arithmetic forms of estimation such as numerosity judgement, magnitude comparison). The authors contrasted the patient's verification performance on exact and approximate calculation by using identical items (i.e., one-digit problems: which in itself is an elegant way to ensure that performance differences can surely be attributed to the relevant research question – in this case the different task requirements – rather than being due to stimulus differences across tasks). However, one has to note that Dehaene's approximate calculations (requiring the individual to judge which of two presented – however incorrect – answers is numerically closer to the problem at hand) easily can be solved by exact calculation, because many individuals automatically will retrieve the respective number fact from long-term-memory (supporting the latter notion, LeFevre and Kulak [1994] provide evidence for the obligatory and task-irrelevant activation of fact knowledge). If they indeed do, then the so-called approximation task turns into a fact production/retrieval task that needs to be accompanied by a comparison process (with the suggested answers) and a subsequent response decision. In contrast, the exact task turns in a delayed match-to-sample task in which the retrieved fact has to be matched with one or two response numbers.

Regarding the developmental literature on arithmetic estimation, Dowker (1997) points out that previous research on this matter is hardly comparable as the definitions of estimation have been quite heterogeneous, ranging from '...certain specific approximate calculation

[4] It is important to note that approximate calculation skills are not identical and/or interchangeable with pre(non)verbal quantity representations as outlined above. Rather, the term approximate calculation has been used in various contexts and one needs to specify what is meant when referring to that term. Hence, there has been no attempt to link and/or integrate pre(non)verbal numerical abilities to those calculation components already being specified in adult models.

strategies to any guess as to a numerical quantity.' (p. 142). Therefore it is not surprising that with respect to the age range when estimation generally can be applied and it's relation to conventional calculation skills, the findings are quite ambiguous, too. It has been proposed that estimation proficiency in children can not be expected until around 10 years of age (Case & Sowder, 1990; for a contradicting view, see Dehaene & Cohen, 1991). Interestingly, while mathematics educators have put forward a hierarchical (stage model) view by claiming that the ability to perform computational estimations depends on mental calculation skills (e.g., Reys, 1984), the findings of double dissociations between exact and approximate calculation in the adult literature of acquired calculation disorders do not corroborate the stage model view but rather imply that the two components are separable from each other (e.g., Dehaene & Cohen, 1991; Warrington, 1982).

Finally, it has to be mentioned, that approximation is not the only method to roughly estimate computational results. Alternatively, the correct answer retrieval can be facilitated by (mostly implicit) knowledge about it's parity status, too (e.g., Berch, Foley, Hill, & McDonough, 1999; Krueger, 1986; Krueger & Hallford, 1984; Lemaire & Fayol, 1995; Lochy, Seron, Delazer, & Butterworth, 2000).

Problems with translating adult models to the children data

In the last decade, there have been efforts to demonstrate that behavioural data of children diagnosed with dyscalculia fit the adult models. Temple (1991) has been the first to show a double dissociation between fact and procedural knowledge in developmental dyscalculia (see also Kaufmann, 2002), and moreover, has described a child exhibiting specific diffficulties in processing Arabic digits (Temple, 1989).

However, in our view, there are several reasons why one should defer from utilising adult (calculation) models in order to obtain an explanatory framework for the observed data set.

No reliable premorbid functional levels. Firstly, perhaps the most obvious reason is that in adults, one could get rather reliable estimates of premorbid functional levels to which the acquired deficit profile can be compared. To the contrary, by interpreting developmental data, one has to deal with many unknown facts (both regarding numerical/arithmetic skill development as well as with those cognitive domains that might subserve the understanding and manipulation of numbers; such as general symbolic abilities, linguistic abilities, visual-spatial abilities etc.).

Intra- and interindividual differences. Secondly, an important reason to stress the need for newly emerging, specifically developmental calculation models, is the fact that inter- and intraindividual differences regarding numerical/arithmetic abilities (e.g., Dowker, 1998) are likely to be accompanied by inter- and intraindividual differences regarding other cognitive domains as well. Thus, developmental models need to incorporate many links to cognitive abilities that are likely to modulate and/or exert an influence upon both or either the acquaintance of numerical/arithmetic skills as well as their availability/ease of access and manipulation. For example, it has been repeatedly shown that working memory plays a crucial role not only in complex mental calculation (e.g., Fuerst & Hitch, 2000), but also in number fact retrieval (Kaufmann, 2002; for a review, see Ashcraft, 1995). Furthermore, it has been repeatedly emphasised that visual-spatial skills are correlated to specific aspects of arithmetic (and even more so to geometry; Casey, Pezaris, & Nuttall, 1992; Geary, 1993, 1996; Rourke & Strang, 1983). Likewise, a kind of spatial attention ('attentional orientation') has been thought to mediate/facilitate the utilisation of the mental number line (Dehaene et al., 2003; Zorzi, Priftis, & Umiltà, 2002. With respect to the neuro-anatomical correlates of the latter ability, Dehaene et al. claim bilateral posterior superior parietal systems as crucial cerebral areas. Another – in adult calculation models crucially neglected function – is linguistic ability (but see MARC-effect, Nuerk, Iversen, & Willmes, 2004, for linguistic influences on elementary number processing): despite having been generally acknowledged as being important for specific arithmetic abilities like counting, fact retrieval, solving of word problems, there has been no attempt to explicitly define and characterise the interplay between linguistic and numerical abilities. We believe that developmental models should be designed that incorporate the differential effects of clearly defined linguistic abilities on various components of number processing and calculation (going beyond the well-established literature on preverbal/verbal counting).

Here, we also wish to draw the readers attention to strategy use: despite the well-known findings of LeFevre's group (e.g., LeFevre & Kulak, 1994; LeFevre, Bisanz, Daley, Buffone, Greenham, & Sadesky, 1996; LeFevre, Sadesky, & Bisanz, 1996) demonstrating that even adults seem to apply a wide range of strategies upon solving simple calculations (that are not restricted to retrieval strategies but also include various procedural strategies), it seems logical to conclude that this might be even more so true for children (see Siegler, 1987, 1988; Siegler & Jenkins, 1989). It is important to note, that individual variability regarding strategy use is likely to be even more pronounced in children with developmental (calculation) disorders. In the latter, a specific disability or cognitive dysfunction might result in the application of compensatory strategies. Further below, we will discuss in more detail the potential usefulness of explicitly incorporating non-numerical cognitive domains (such as working memory and spatial ability) in research concerned with developmental aspects of numerical cognition.

No defined neuroanatomical neurophysiological substrates. Thirdly, while the observed performance patterns of patients with acquired calculation disorders can be associated to quite specific neuroanatomical lesions (mostly affecting parietal brain areas in case of quantity processing deficits and perisylvian lesions in case of linguistically mediated calculation skills such as number fact retrieval, for a review, see Dehaene et al., 2003), children's calculation difficulties generally can not be linked to circumscribed lesions.[5] Additionally, developmental dyscalculia has been attributed to different aetiologies, among which one could find rather unspecific cerebral dysfunction such as the right hemisphere, white matter dysfunction hypothesis (dyscalculia being a key symptom of the so-called 'non-verbal learning disability'; Rourke & Strang 1983; see also Weintraub and Mesulam's [1983] concept of developmental right hemisphere disorder), periventricular leucomalacias (as often described in very-low-birth-weight children; Isaacs, Edmonds, Lucas, Gadian, 2001), accompanying symptoms of – anatomically largely unspecificed – metabolic diseases such as phenylketonuria or neuropsychiatric disorders such as attention-deficit hyperactivity-disorder (e.g., Zentall, Smith, Yung-bin, & Wieczorek, 1994) as well as a common functional deficit of genetic

[5] However, a recent brain imaging study correlating behavioral data with voxel-based morphometry was able to demonstrate decreased gray matter density in parietal areas of formerly premature children that exhibited dyscalculia relative to a very well matched control group of premature children with average calculation skills (Isaacs, Edmonds, Lucas, & Gadian, 2001).

diseases such as Turner's Disease (e.g., Bruandet, Molko, Cohen, & Dehaene, 2004; Mazzocco, 1998; Rovet, Szekely, & Hockenberry, 1994; which lead some to conclude that developmental dyscalculia might have a genetic predisposition; Geary, 1993; Shalev, Manor, Kerem, Ayali, Badichi, Friedlander, & Gross-Tsur, 2001). Moreover, dyscalculia has been reported to be frequently associated with dyslexia (e.g., Landerl, Bevan, & Butterworth, 2004; Lewis, Hitch, & Walker, 1994). However, there have been attempts to link behavioural performance patterns in the arithmetic domain to specific neuroanatomical sites of dysfunction.

The latter becomes plausible upon considering that developmental dyscalculia can be quite heterogeneous (e.g., Geary, 1994; Temple, 1991, 1994; von Aster, 2000) and typically is characterised by large interindividual differences (e.g., Dowker, 1998; Siegler, 1987). Consequently, Geary (1994) suggested that specific difficulties in retrieving number facts ('memory subtype', frequently associated with dyslexia/poor reading skills) as well as difficulties to execute arithmetic procedures ('procedural subtype', probably reflecting an insufficient/lacking understanding of arithmetic concepts) can be associated to left-hemisphere dysfunction, whereas difficulties with visual-spatial aspects of number processing such as understanding of place value ('visuo-spatial subtype') are associated with right hemisphere dysfunction. Similar subtyping approaches of developmental dyscalculia relying on behavioural data – i.e., double dissociations between specific components of the calculation system – have been adopted by other authors (Badian, 1983; Kosc, 1974; Temple, 1991; von Aster, 2000). However, up to date a theoretically derived and empirically validated developmental calculation model is still lacking.

Transcoding models

Finally, another class of models are transcoding models (e.g., Power & Dal Martello, 1990, 1997 for more detailed reviews and discussion see also Lochy, Delazer, Domahs, Zoppoth, & Seron, unpublished manuscript; Noël & Turconi, 1999; Seron & Fayol, 1994). Based on a first extensive analysis of children's number transcoding errors, Power and Dal Martello proposed an algorithm how children transform verbal numbers to an Arabic code. When they are asked to write down a number like "four hundred and seventy six" they build up a semantic representation in a first comprehension process where the above number is represented as <C4 x C100> + <C7 x C10> + <C6> where Cn is the semantic representation of the respective quantity n (n being a natural number). It can be seen that the semantic representation is already thought to be linked to the Arabic number system which is a base-10 system without any subpower (as indicated by the multipliers C10 and C100 in the model; for different number systems, see Zhang & Norman, 1995). Two rules or operators are then used to transform the above semantic representation to an Arabic number. First, the concatenation operator concatenates the products: From C4 * C100 to 4 & 00 to 400 and in the same way for the decades. When this operator is not used properly "two hundred" might be transcoded into 2100. The other operator is the overwriting operator which operates the zeros of the semantic representation. Suppose the concatenation operator would be successfully applied. The remaining representation would be < 400> + <70> +<6> in the above example. The overwriting operator or rule overwrites the zeros so that < 400> + <70> +<6> is transformed into < 400> # <70> # <6> (with # being the overwriting operator) and then to 476.

Errors that result from an incorrect application of the overwriting operator have been frequently observed when a zero has to be inserted. When "hundred and six" has to be written down, children often fail to overwrite one zero and instead write down "1006". They just write the numbers behind each other instead of overwriting the zero at the place of the unit position.

However, a recent transcultural study questions the general validity of the model. Lochy (2003) studied transcoding performance in German and French speaking children. In German (and in Dutch) two-letter words are inverted; the verbal representation of 21 is "einundzwanzig" (one-and-twenty). Lochy found that the transcoding errors in Belgium (French speaking) and Austrian (German speaking) children differed considerably with regard to the inversion property in the language. When Austrian first graders committed errors in decade-unit trials, they committed inversion errors ("twenty-one" → "12" in 77% of all errors while for Belgium children only 1,8 % of all errors were inversion errors. What is even more remarkable is that the inversion property was wrongly generalized to other stimuli. Most interestingly were the transcoding differences in the UH structures ("five hundred"): While 19 of 32 errors consisted of an inversion instead of concatenation (i.e., 105 instead of 500), almost no such errors occurred for Belgium children. Lochy suggests the application of a third operator "§" which helps to invert the verbal number representation in the transcoding process. Without such an operator, these transcultural results cannot be explained by the model of Power and Dal Martello (1990).

Transcoding models might be considered as sub-models to other models. For instance, with regard to Dehaene and Cohen's (1995; 1997, see also Dehaene, 1992) triple-code model, transcoding models might describe the relation between the verbal number representation and the other representations. However, transcoding models also demonstrate a clear lack of model building so far in children's work. Too many models explain just one task or phenomenon (e.g., subitizing, transcoding) and are not task-overlapping. In contrast, when models are task-overlapping and more general, often they do not make very specific predictions. While transcoding errors are not generally incompatible with general model frameworks like that of McCloskey or Dehaene, these models do not yet specify the specific qualitative types (and not at all the quantity) of errors that should be observed at a certain point of numerical development.

Other cognitive abilities influencing math performance

There might be several other cognitive and non-cognitive abilities that should be considered to be influencing numerical/arithmetic performance. Among the latter, emotional factors such as math anxiety or attitude towards math, have been reported to be associated with math performance (e.g., Ashcraft & Kirk, 2001; Faust, Ashcraft, & Fleck, 1996; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998). Specifically, Hopko et al. (1998) suggest that poor performance of high math anxious individuals is a consequence of their inability to divert attention away from the worry and not attributable to experiences of worry per se.

It has been repeatedly stressed that a negative attitude towards mathematics and/or math anxiety seems to negatively affect actual math performance, both in the developmental (Gregory, Snell, & Dowker, 1999; Holt, 1966) and in the experimental literature (e.g., Ashcraft & Kirk, 2001; Faust et al., 1996; Hopko et al., 1998). Importantly, the negative effects of math anxiety are not confined to highly advanced and difficult mathematics, but have also been reported for highly familiar and simple arithmetic problems (Faust et al., 1996; Hembree, 1990). According to our own findings, math anxiety can be found even in primary school teachers (22 out of 49 teachers rated themselves as being moderately to highly math anxious; Dowker, Delazer, Nuerk, & Kaufmann, submitted). Further, we found moderate and high levels of math anxiety in school teachers to be associated with low performance levels on simple and complex multiplications as well as math reasoning (the overall error rate being quite high, namely 26.3 %). Finally, in a data subset that is part of a large scale study that aims at collecting data for the German standardization of a mathematical aptitude test for 4 to 8 year-old children we also found a correlation between math anxiety and math performance, third graders reporting higher levels of math anxiety than first graders. Thus, it seems that – even in normally achieving children – increasing exposure to formal school mathematics is likely to result in higher levels of math anxiety that are accompanied by decreasing performance levels. Most interestingly, in all three grades, females yielded higher scores on the math anxiety questionnaire (thus indicating higher levels of math anxiety). However, it remains an open question whether girls are really more anxious than boys or whether the gender differences in math anxiety scores arise because girls tend to report math anxiety more readily.

Current issues on the role of parietal brain areas in quantity processing

How specific is the relation between parietal cortex and number processing?

It is widely acknowledged that quantity processing is preferentially subserved by parietal brain regions. In particular, the intraparietal sulcus (IPS) seems to support the manipulation of abstract representations of numerical magnitudes (e.g., Dehaene et al., 1999; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Pesenti et al., 2000; Kaufmann, Koppelstaetter, Delazer, et al., 2005; Pinel, Dehaene, Riviere, & LeBihan, 2001). Dehaene and colleagues (2003) proposed that number processing is mediated by three parietal circuits, namely a) the horizontal segment of the intraparietal sulcus (HIPS; mediating numerical quantity processing in itself), b) the left angular gyrus (as well as adjacent perisylvian structures; being activated upon language-based number processing such as in verbal counting, verbal retrieval of number facts) and c) the bilateral posterior superior parietal system (PSPS, subserving attentional and spatial orientation on the mental number line).

There are several studies outside the domain of mathematical cognition that reported cerebral activations in and around the IPS. Such as, the IPS has been found to be activated also during attentional processing (Gruber et al., 2001). Most interestingly, upon training monkeys in time discrimination tasks, Onoe, Komori, Onoe, Takechi, Tsukada, and Watanabe (2001) found increased blood flow in inferior parietal regions (as well as in dorsolateral prefrontal cortex [DLPC]; see also Leon & Shadlen, 2003). Thus, parietal regions seem to play a key role in both time and magnitude perception.

On the contrary, magnitude processing does not seem to be an exclusive domain of parietal cortex. Interestingly, number selective neurons (‘numerons’) have been demonstrated in primate’s prefrontal brain areas (time: Onoe et al., 2001; number: Nieder et al., 2002; Nieder & Miller, 2003). In addition, there is evidence supporting the assumption that also the cerebellum and the basal ganglia might be involved in magnitude processing (for a review, see Walsh, 2003a, 2003b). Considering the close link between time and magnitude perception (as purported by Walsh, 2003a, 2003b; see also the animal studies of Meck & Church, 1983)[6], and the repeatedly reported key role of the cerebellum in temporal cognition (e.g., Ivry & Keele, 1989; Ivry, 1996), the absence of studies that systematically investigate its role in numerical cognition is rather surprising. It is thus not implausible to speculate a possible link between the timing function of the cerebellum – as it pertains to motor, sensory, and cognitive tasks – and the development of numerical cognition. In the following, we will try to further outline the potential relational nature between the cerebellum and mental magnitude representations.

As Butterworth (1999) correctly states, the acquisition of the counting sequence (learning the verbal counting sequence, learning to associate this verbal sequence to quantity representations and finally learning to map these count words onto the mental number line) has been found to strongly depend on the use of finger counting, and children seem to readily rely on their fingers upon performing counting and computations, even if they already master the verbal counting sequence. However, we are skeptical regarding the conclusion drawn by Butterworth ('Neither do we know whether it's the feel of the hand shape or the look of it that is critical in calculation'; p 234, Butterworth, 1999). Alternatively, we suggest that finger counting, which requires motor coordination as well as the coordination of verbal output (namely the counting sequence) and motor action (adjusting the number of fingers to the increment or decrement of counted words/items), is likely to be critically supported by cerebral networks that include cerebellar areas as well (the latter idea has been elaborated by a personal communication between X. Seron and L.K. some time ago).

It is further interesting to note, that the cerebellum has extensive connections to prefrontal brain areas (especially the dorsolateral prefrontal cortex), as evidenced by both brain-imaging and behavioral studies (Diamond, 2000). In particular, the cerebellum seems to be most active when the task is novel or when conditions change, while the cerebellar participation decreases with increasing task familiarity. Patients with cerebellar lesions have been found to perform poorly on cognitive tasks mediated by prefrontal cortical network such as verbal fluency, set-shifting and working memory tasks (Fiez, Petersen, Cheney, & Raichle, 1992; Schmahman & Sherman, 1998; for a review, see Diamond, 2000). Considering the recent literature that proposes a key role of prefrontal brain areas in magnitude processing (as demonstrated by the identification of neurons that are sensitive to symbolic magnitude [numerical quantity; Nieder et al., 2002, Nieder & Miller, 2003] as well as non-symbolic magnitude [time duration; Onoe et al., 2001], the latter notion of a close interlink – both at a functional and anatomical level – between cerebellar and prefrontal networks clearly deserves a systematic investigation of researchers concerned with numerical cognition.

Accordingly, Fias, Dupont, Reynvoet, and Orban (2002; see also Walsh, 2003a, 2003b) argue that the anatomo-functional link between parietal regions and number processing is less specific than previously assumed. By reviewing experimental human and non-human

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[6] Meck and Church (1983) were able to train rats to discriminate two sets of numerosities (2 versus 8 events), and moreover, were able to show that rats that were trained on duration discrimination only, spontaneously generalised their discrimination behaviour to non-trained numerical differences (Meck, Church, & Gibbon, 1985). This lead them to conclude that in animals, both aspects of magnitude (durational and numerical) are likely to be represented by the same mental magnitudes (metaphor of the so-called "accumulator model"; see also Gallistel & Gelman, 2000).

primate studies, Walsh (2003a, 2003b) provides us with an excellent and critical overview of the existing respective literature. The latter author argues that concepts such as time, space and quantity – all of which require magnitude processing - share a common parietal cerebral network that largely overlaps. Specifically, regions in the lateral inferior parietal cortex (LIP) seem to contain neurons that are selectively responsive to both spatial (numerical) and temporal information.

Temple and Posner (1998) have been among the first to investigate directly the neural networks mediating quantity processing in young children by employing an ERP (event-related potentials) design. The latter authors asked their participants to decide whether a given magnitude (either one-digit Arabic numeral or respective dot patterns) was smaller or larger than a standard of 5. According to their results, both 5-year old children and adults exhibit a distance effect for both notations (increasing response latency with decreasing numerical distance) as regards reaction times. Moreover, Temple and Posner (1998) partly replicated the results of Dehaene et al. (1998, 2003) by identifying the inferior parietal cortex as being crucial for quantity processing (independent of notation: Arabic digits vs. dot patterns). Importantly, these brain regions have been found to be activated in both 5-year olds and adults, the only difference being that the former displayed slightly delayed waves (while the components of the waveform affected by distance were found to be comparable in 5-year-olds and adults). Similarly, by correlating behavioral data with voxel-based morphometry, a recent study found that formerly very low birth weight (VLBW) children with dyscalculia exhibited a significant decrease in gray matter density in parietal brain areas relative to VLBW children without dyscalculia (Isaacs et al., 2001).

Likewise, a recent meta-analysis that aimed at comparing infant and animal studies regarding number processing abilities stressed that in both human infants and primates parietal cortical networks seem to be crucially involved in quantity processing (Brannon & Roitman, 2003). Interestingly, the latter authors report that time perception seems to be supported by the same parietal regions as well. Thus, Brannon and Roitman (2003) conclude that number and time perception might share a common cerebral network that seems to subserve a variety of magnitude-related tasks. Hence, the latter view corroborates the notion put forward by Gallistel and Gelman (2000) who propose that so-called mental magnitudes are the common underlying representational system for countable and uncountable quantities (such as numerosity and amount, duration, etc.). In line with this hypothesis is the argument that the division between time, space and quantity in every-day life (such as action planning) is an artificial one and in the course of development, an individual has to learn to process these dimensions separately (Bryant & Squire, 2001; Droit-Volet, Clément, & Fayol, 2003; see also Walsh, 2003a, 2003b). In particular, Bryant and Squires (2001) claim that in cognitive psychology, the possible relation between spatial and mathematical understanding (i.e., symbolic quantity processing) has been rather neglected. Similarly, Droit-Volet et al. (2003) found that 5-year olds were not able to process numerical and durational information separately, but also stated that in young children '...numerical sensitivity is greater than temporal sensitivity...' (p. 74). The latter authors suggest that this difference is due to the fact that number resembles a 'discrete' information while duration is a 'continuous' information (as proposed by Gallistel & Gelman, 2000) and hence, the application of verbal counting leads to improved duration judgements.

Future perspectives

(A) About potential links between working memory, spatial and numerical cognition with special reference to developmental issues

Working memory and the acquisition of numerical/arithmetic knowledge

There is converging evidence that the negative effects of poor working memory resources are not confined to complex arithmetic skills but are also likely to hamper fact retrieval (Ashcraft, 1995; Kaufmann, 2002; Lemaire, Hervé, & Fayol, 1996). Kaufmann (2002) suggested that working memory resources (particularly the central executive component) might play a critical role in fact retrieval abilities as displayed by a boy suffering from developmental dyscalculia. Moreover, efficient fact retrieval depends on the interplay between intact working memory resources, long-term memory and inhibition mechanisms (Kaufmann, 2002; see Figure 1, p. 303). Even in tasks as simple as transcoding of two-digit numbers, working memory demands might affect performance. In languages with verbal inversion ("one-and-twenty") like German and Dutch, many children and some adults write two-digit numbers from right-to-left, i.e., in the order of verbal comprehension. Lochy (2003) hypothesizes that this might be a compensation strategy for low working memory resources which might later (for multi-digit numbers) hinder efficient processing of verbal number representation.

Spatial cognition and numerical/arithmetic skills or: is the male advantage in math-related tasks attributable to a spatial superiority

Interestingly, the link between spatial and math-related skills has been stressed by the so-called 'spatial cognition hypothesis' that has been formulated to provide an explanatory framework for the frequently observed male superiority in math-related tasks such as math reasoning (word problems) and geometry (Casey et al., 1992; Geary, 1996). According to the spatial cognition hypothesis, the cognitive factor that most likely contributes to these observed gender differences is a male advantage in (three-dimensional) spatial skills.

Specifically, Casey et al. (1992) argue that good spatial abilities should have a beneficial effect on many types of math problems because the latter generally can be solved by using algorithms that depend on analytical or pictorial approaches. Their findings showed that spatial abilities measured in high school students were highly predictive of later performance on a math aptitude test as senior students, which lead them to conclude that spatial cognition and math test performance are correlated. Likewise, Geary (1996) notes that the male superiority in math test performance seems to be due to men's superior spatial skills that are plausibly explained by sexual selection (and probably its differential hormonal environments) that lead men to develop better spatial navigation systems. Consequently, men are more likely than women to draw upon their superior spatial skills in novel situations, such as word problems and geometry (however, for a contrary view, see Dowker, 1996; Friedman, 1995;

Lubinski & Humphreys, 1990).[7] Likewise, females seem to display larger performance increases relative to males after being explicitly trained to represent mathematical relationships spatially (Johnson, 1984). Thus, it has been suggested that gender differences favoring men in spatial skills are likely to result in gender differences in strategic approaches that in turn are reflected in male advantages in math reasoning (word problem-solving) and other math-related tasks (for a review, see Geary, 1996).

Interestingly, while some authors found that spatial ability has a higher predictive power of mathematical performance in males than in females (e.g., Casey et al., 1992), others found the opposite pattern to be true (Friedman, 1995). Hence, the evidence to date regarding the relationship between spatial ability and mathematical ability remains rather controversial.

Interestingly, disturbances in the mental number line representation of adult neglect patients were reported by Zorzi et al. (2002) who found that neglect patients were unable to perform a number bisection task. Upon being requested to indicate the numerical middle of two Arabic numerals, they tended to make right-shifted errors (a result which was interpreted by the authors as reflecting the patient's lack or distortion of left-hemifield perception, presumably including the left side of the mental number line).

(B) Study of patient groups that are at risk for developing dyscalculia

Over the last decade, there has been an increasing interest in studying children and adult patient groups that are known to have a high risk of developing arithmetic difficulties. Risk groups are mostly determined with respect to phenomenological characteristics and thus incorporate a variety of developmental disorders, some of which are of genetic origin. Hence, risk group studies could aid in identifying causal relationships between specific genetic conditions or neurophysiological features (like in VLBW children) and the cognitive domain of interest. A detailed review of the respective literature is beyond the scope of this article. Nonetheless, we will mention some influential studies that were able to contribute significantly to the current state of the art in the field of numerical cognition by studying risk groups such as VLBW children, suffering from genetic disorders like Turner syndrome or fragile X syndrome.

As mentioned already above, Isaacs et al. (2001) were the first to show that in a group of formerly VLBW children – at the time of the assessment they were about 15 years old – without major neurological complications, dyscalculia was associated with structural brain abnormalities. In particular, the latter authors report decreased gray matter density in parietal brain areas of VLBW children with dyscalculia relative to a well-matched group of VLBW children without calculation difficulties.

With respect to genetic disorders, it has been frequently reported that individuals suffering from Turner syndrome often exhibit mathematics difficulties (Mazzocco, 1998; Rovet et al., 1994). Recently, it has been found that young adults diagnosed with Turner syndrome not only were impaired in computational skills and cognitive estimation (e.g., "What is the length of a bus?") but also displayed difficulties in quantity processing such as subitizing (quick enumeration of small object sets; Bruandet et al., 2004). Furthermore, the latter au-

[7] The latter authors propose that the relationship between spatial abilities and performance on mathematical achievement tests is a correlational one.

thor's interpretation is not confined to implications regarding a possible genetic contribution to dyscalculia/number-specific modulations of parietal neural networks. Moreover, by reviewing previous findings of beneficial effects of estrogen replacement therapy on specific cognitive functions like working memory (menopausal women) and visual-spatial memory (ovariectomized rhesus monkeys and rats), Bruandet and colleagues (2004) claim that "the results suggest that parietal impairment may be partially, but not totally compensated by estrogen replacement therapy, perhaps pointing to an early effect of estrogen on parietal lobe organization."

Another genetic condition that is frequently associated with dyscalculia is fragile X syndrome. Interestingly, the cognitive profiles differ somewhat in males and females. Males with the full mutation usually display mental retardation, while this is not true for full mutation females (Cornish, Munir, & Cross, 1998). Moreover, it has been reported that females often display poor spatial abilities, non-verbal performance, short-term memory and attentional skills (Cornish, Munir, & Cross, 2001), while the cognitive deficiencies of males seem to be rather task-specific than global (Munir, Cornish, & Wilding, 2000). Regarding arithmetic performance, Mazzocco (1998) found that fragile X females had lower math than reading cluster scores in the Woodcock Johnson-Revised achievement test. Moreover, the latter author reports that arithmetic performance levels of fragile X and Turner females were qualitatively different.[8] Interestingly, upon performing a correlational analysis between math achievement and neuropsychological test scores, Mazzocco (1998) found that for fragile X females, the strongest predictor of math performance was the perceptual/organizational factor score of the WISC-R (Wechsler Intelligence Scale for Children-Revised), whereas for the Turner females, the strongest predictor was performance on the jugdement of line orientation.

Finally, individuals with Prader-Willi syndrome (a chromosomal defect concerning pair 15) seem to have specific math difficulties, too (Bertella, Marchi, Molinari, Grugni, & Semenza, 2001) which led the authors to propose a key role of chromosome pair 15 in the genetic transmission of mathematical abilities.

Overall, prospective studies of risk groups (e.g., individuals suffering from Turner syndrome, fragile-X syndrome, Prader-Willi, VLBW), could be beneficial as regards the tracking of arithmetical abilities and disabilities in individuals with specific neuroanatomical, neurophysiological and/or functional characteristics. Such as, risk groups allow us to study the developmental course of specific cognitive abilities under certain conditions (e.g., abnormal X chromosome in Turner syndrome and fragile X syndrome) and thus could facilitate the identification of potentially contributing factors (either facilitating or interfering ones) to arithmetical development and/or performance.

However, it has to be kept in mind, that risk groups very rarely are homogeneous groups of well-matched individuals. This is because even in genetic diseases the phenotype might vary substantially with the genotype and mosaicity. Thus, the high probability of considerable group heterogeneity in genetic disease – as regards neurophysiological and neuropsychological characteristics – requires the careful matching of individuals and/or controlling of confounding variables. In our view more desirable are approaches that focus on single-case

[8] For instance, females diagnosed with fragile X syndrome committed less operation and alignment errors in complex calculation, but – relative to Turner females – about equal table, (close-miss) calculation, procedural and other errors.

studies because only by employing the latter, one is able to avoid averaging data over (supposedly quite heterogeneous) groups (Siegler, 1987). Hence, the most favorable study design is a longitudinal prospective investigation that would enable us to compare performance changes within and between individuals and moreover, to study interactions between cognitive performance and therapeutic interventions (pharmaceutical, hormon replacement therapy, cognitive-behavioral therapy etc.).

(C) Intervention studies

Intervention studies can provide a better understanding of the interplay between components of number processing and arithmetic as well as between numerical and non-numerical cognitive domains by analysing student's learning process and the likelihood of the acquisition of compensatory strategies. Furthermore, the thorough and componential investigation of students' educational progress should enable us to shed some light on facilitating/inhibiting effects of specific learning strategies and/or remaining cognitive resources (including abilities and disabilities). Such as, Censabella and Noel (this issue) found that previously learnt information might negatively affect the acquisition of new information (i.e., addition knowledge interfered with the learning of multiplication skills).

Intervention studies with large samples are needed in order to draw firm conclusions about the extent and nature of the effects of early training on specific components on subsequent performance on the same and other components. Some forms of procedural training can lead to better conceptual understanding; and conceptual training can lead to more accurate and efficient use of procedures (Baroody, 2003). There is converging evidence indicating that fact retrieval and automatization of procedures are important in arithmetic, not so much for their own sake, as because they free up time and cognitive resources for deeper-level reflection and reasoning about arithmetical relationships (Geary, Hamson & Hoard, 2000; Ostad, 1998; Siegler, 1988; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). In addition, conceptually based learning can lead to greater procedural efficiency as well as conceptual understanding (Baroody, 2003; Brownell, 1938; Hiebert & LeFevre, 1986; Delazer, 2003).

Interestingly, the results of Kaufmann, Handl, & Thony (2003) indicate that knowledge of counting sequences and mental calculations (advanced computational strategies such as problem decomposition as well as direct fact retrieval) facilitates the application and integration of other components of numerical/arithmetical knowledge (see also Kaufmann, Pohl, Delazer, Semenza, & Dator, in press). According to Kaufmann et al. (2003, in press), significant training effects can be obtained after a relatively short training period of about 5 month, which is in line with other studies (Dowker, 2001; Wright, Martland & Stafford, 2000). Importantly, these positive learning effects can be obtained in preschool children who have had little formal arithmetical training, and thus may be useful in preventing numerical difficulties from arising later on.

A potentially important consequence of early positive experiences with numbers may be in developing a positive attitude toward mathematics (Gregory et al., 1999) and in preventing the development of mathematics anxiety, which in its turn appears to have a negative impact on arithmetical performance in children and adults alike (Holt, 1966; Hembree, 1990;

Ashcraft & Kirk, 2001). Perhaps enjoyable and successful preschool experiences with arithmetic may help to encourage a positive attitude to later school arithmetic.

Concluding remarks

One problem this review might well have illustrated is that research about numerical development comes from very different research communities that sometimes seem to study numerical development in parallel universes. Only in the last 5 years or so, more researchers started to link adult research and children's research. This can also be illustrated in the goal to understand numerical disabilities and disorders. Rarely, the endeavour to understand developmental dyscalculia and the endeavour to understand acquired dyscalculia have been well integrated although everybody believes that we are looking at the same – though more or less developed number representations. However, even within the research of numerical development, there exist quite separate literatures and models about calculation, elementary number processing, disturbed number processing and intervention studies. The reason for these separate literatures is that we are concerned with separate age groups and that numerical development is yet again a field in which researchers from many different domains (developmental psychology, pedagogics, pediatry, child and adolescent psychiatry, and applied cognitive neuroscience) meet.

The needs for the future are then straightforward. We may not need more models about number development, but we may need more general models: we may need more horizontal generality in that models account for more than one task for a given age group as well as more vertical generality in that models account for more than one developmental age, because we may not understand numerical processing development as an integrated development of different numerical representations and processes well with a subitizing model for infants, a transcoding model for early school years and a calculation model for somewhat later school years etc. etc.

To sum up, in the last years, we have teased apart the development of numerical representations and shown that there are quite many distinct representations to develop. The task of the next years may be to bring them together again. There may be numerous ways to represent and manipulate a given number, but in the end – if they function correctly – they all converge again to represent one number.

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