

Logistic regression and Prediction Configural Frequency Analysis – a comparison

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Abstract

Logistic regression (LR) and Prediction Configural Frequency Analysis (PCFA) are compared. First, the underlying statistical models are presented. Second, sample design matrices are created. Third, data are analyzed using both methods. Two data examples are analyzed. The first is artificial, the second uses data from a project on domestic violence. Fourth, the goals of LR, a variable-oriented approach, and PCFA, a person-oriented approach are discussed. One conclusion of the comparisons is that, for researchers who wish to enrich results by employing both methods, the standard model of LR needs to be extended so that it becomes parallel to the base model of PCFA.

Key words: Logistic regression, prediction configural frequency analysis, design matrix, extended models, model comparison

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This article presents a comparison of logistic regression analysis (LR) and prediction configural frequency analysis (PCFA). The comparison has three components. The first focuses on the statistical models of LR and PCFA. The second focuses on the goals of analysis. The third involves a comparative application of LR and PCFA to the same data. The main message is that a complete comparison of the two methods cannot resort exclusively to either the underlying models or to the different goals of analysis alone because both differ. Researchers who intend to employ both methods because they hope to enrich person- and variable-oriented results with each other will have to extend the standard model of LR so that it becomes parallel to the base model of PCFA.

The article begins with a description of the statistical models of LR and PCFA, and with an example of the extended LR model, in Section 1. In Section 2, sample design matrices are used to illustrate similarities and differences between the two methods. Section 3 presents data examples.

1. The statistical models of logistic regression and prediction CFA

Logistic regression is a variant of regression analysis. In its simplest form, it has one or more predictor variables and one dichotomous dependent or criterion variable. The statistical model of logistic regression (LR) can be expressed in at least three ways. One uses the logits of the criterion variable (cf. Agresti, 2002; Lawal, 2003). This approach first performs the logistic transformation, also called *logit*, of a proportion, p , that is,

$$\text{logit}(p) = \ln \frac{p}{1-p},$$

where p is the probability that the dependent variable assumes the value of 1. The logit function is a *sigmoid curve* that is symmetric about 0.5. As p approximates 0, the function approximates $-\infty$, and as p approximates 1, the function approximates $+\infty$. Using logits, the logistic regression equation can be given as

$$\text{logit}(p) = \beta_0 + \beta_1 x_1 + \dots + \beta_q x_q,$$

where the x_i are the predictor variables with $i = 1, \dots, q$, and the β are the regression parameters. A second notation, often used in the context of logistic regression, follows below, in Table 1.

The third way to express LR involves using the terms of hierarchical log-linear modeling (cf. Agresti, 2002). In this article, we use the log-linear representation because PCFA can also be expressed using the hierarchical log-linear model notation (von Eye, 2002). Consider the three variables, X , Y , and Z , with Y being the dependent variable that is predicted from X and Z . For these three variables, Table 1 presents the five possible following logit models and their log-linear equivalents (from Agresti, 2002, p. 332). In general, every logistic regression model can be equivalently expressed in terms of a log-linear model. The inverse is not true.

Table 1:
Logit and log-linear models for the predictors, X and Z , and the criterion variable, Y

log-linear notation	logit model	logit symbol
[Y, XZ]	α	(-)
[XY, XZ]	$\alpha + \lambda_i^X$	(X)
[YZ, XZ]	$\alpha + \lambda_k^Z$	(Z)
[XY, YZ, XZ]	$\alpha + \lambda_i^X + \lambda_k^Z$	(X + Z)
[XYZ]	$\alpha + \lambda_i^X + \lambda_k^Z + \lambda_{ik}^{XZ}$	(X*Z)

The first of these five models is the null model. It proposes independence of the predictor and the criterion variables, and takes into account the association of the two predictor variables. The second of these models relates the criterion, Y , to one of the predictors, X , but not to the other, Z , and the predictors to each other. The third model relates the criterion to predictor Z , but not to X , and X and Z to each other. The fourth model relates the criterion, Y , to both predictors, and the two predictor variables to each other. This is the standard logistic regression model. The fifth model includes the interaction between the triplet of variables, X , Y , and Z above and beyond the terms included in the fourth model. This model is saturated.

Based on the generalized log-linear model, $\log m = X\lambda$, where m is the column vector of expected cell frequencies, and λ is a column vector of model parameters, we can formulate the following log-linear logistic regression model for the criterion variable Y and I binary predictor variables X_{is} for $i = 1, \dots, I$,

$$\log m = \lambda + \lambda_k^Y + \sum_i \lambda_i^{X_i} + \sum_{ki} \lambda_{ki}^{YX_i} + \sum_{i,j,\dots} \lambda_{i,j,\dots}^{X_i X_j \dots}$$

for $i \neq j$ and $i, j \in I$, and where m indicates the cell frequencies.

Note that the logistic regression model makes no assumptions concerning the relations among predictors. Therefore, the model is saturated in the predictors. Note also that higher order predictor-criterion relationships are not part of the standard LR model. They can, however, be considered. This issue will be taken up again later, in the comparison of LR and PCFA.

This last characteristic constitutes an important difference between standard, continuous variable regression models and logistic regression. In standard regression, predictors are assumed to be independent.

In general, the model of LR contains (1) the terms for those predictor-criterion relationships that are of interest to the researcher, and (2) the interactions among the predictor variables. The model is thus saturated in the predictor variables. The *goal of application of LR* is parallel to the goal of application of standard regression. Researchers attempt to identify the degree to which dependent measures can be predicted from independent measures. This degree can be expressed (1) using R^2 or its equivalents, (2) the portion of variance accounted for by individual predictors, (3) goodness-of-fit tests, and (4) the results of significance tests.

1.1 The log-linear base model of prediction configural frequency analysis (PCFA)

The goal of analysis with PCFA differs from the goals of analysis with LR in a fashion parallel to the differences between log-linear modeling and CFA in general. Using PCFA, researchers attempt to establish the relationships between predictors and criteria at the level of individual category patterns, that is, *configurations* (Havránek, Kohnen, & Lienert, 1986; Heilmann, Lienert, & Maly, 1979; Lienert & Krauth, 1973; see also von Eye, 2002). Results are stated in terms of configurations rather than in terms of variable relationships. The goal of PCFA is thus the rejection of local null hypotheses instead of model fitting. If, for individual predictor-criterion configurations, the local null hypothesis is rejected, prediction *types* or *antitypes* result that are interpreted as carrying the predictor-criterion relationship. The base model will not be modified because, locally, null hypotheses are rejected. Instead, prediction types and antitypes will be interpreted with respect to the base model of PCFA.

In contrast to the LR model, the base model of PCFA contains only the terms that are *NOT* of interest to the researcher.

Specifically, the PCFA base model has the following three characteristics:

- a) it makes no assumptions concerning relationships among predictors; thus, it is saturated in the predictors

$$\lambda + \sum_{ij\dots} \lambda_{ij\dots} X_{ij\dots}$$

- b) it makes no assumptions concerning relationships among criterion variables; thus, it is saturated in the criterion variables, and
- c) all terms that relate predictors to criterion variables are set to zero; this includes the higher order predictor-criterion interaction terms (examples follow).

The base model of PCFA can thus be violated, and results in types and antitypes only if predictor-criterion relationships exist, that is, if at least one of the terms that the base model sets to zero is different than zero. This, however, is studied at the level of individual predictor-criterion configurations instead of the level of variable associations.

1.2 Sample design matrices

This section compares LR and PCFA at the level of sample models. The models will be expressed using log-linear model equations. In addition, for each of the sample models, the design matrix will be given.

Consider the criterion variable, Y , and the three predictor variables X_1 , X_2 , and X_3 .

The standard *logistic regression model* for these variables is

$$\begin{aligned} \log m = & \lambda + \lambda^Y + \lambda^{X_1} + \lambda^{X_2} + \lambda^{X_3} \\ & + \lambda^{YX_1} + \lambda^{YX_2} + \lambda^{YX_3} \\ & + \lambda^{X_1X_2} + \lambda^{X_1X_3} + \lambda^{X_2X_3} + \lambda^{X_1X_2X_3}. \end{aligned}$$

Parameters set to zero are λ^{Y, X_1, X_2} , λ^{Y, X_1, X_3} , λ^{Y, X_2, X_3} , and $\lambda^{Y, X_1, X_2, X_3}$.

The *PCFA base model* for these variables is

$$\begin{aligned} \log m = & \lambda + \lambda^Y + \lambda^{X_1} + \lambda^{X_2} + \lambda^{X_3} \\ & + \lambda^{X_1 X_2} + \lambda^{X_1 X_3} + \lambda^{X_2 X_3} + \lambda^{X_1 X_2 X_3}. \end{aligned}$$

Parameters set to zero are λ^{YX_1} , λ^{YX_2} , λ^{YX_3} , λ^{Y, X_1, X_2} , λ^{Y, X_1, X_3} , λ^{Y, X_2, X_3} , and $\lambda^{Y, X_1, X_2, X_3}$, that is, all parameters for that part of the model in which predictors and criteria are related to each other.

The comparison of these two model equations shows that the PCFA base model is more restrictive than the LR model. The number of parameters set to zero is greater in the PCFA base model. Now, PCFA types and antitypes result from deviations from the base model. Any violation of the base model can lead to the emergence of types and antitypes. In the present comparison, we note that the number of parameters set to zero in PCFA is greater than the number of parameters included in the standard model of LR. Thus, the number of possibilities to violate the PCFA model is greater than LR. Therefore, the results of PCFA and LR are comparable only if (1) the violations that lead to the emergence of types and antitypes can be traced back only to those predictor relationships that are modeled by PCFA, or (2) the model of LR is extended such that it also accommodates the terms that are set to zero in the base model of PCFA but are not part of the standard model of LR. This topic will be taken up again in more detail later. At this point, we just present the extended LR model that makes the results of PCFA and LR comparable, for the present example. This model is

$$\begin{aligned} \log m = & \lambda + \lambda^Y + \lambda^{X_1} + \lambda^{X_2} + \lambda^{X_3} \\ & + \lambda^{YX_1} + \lambda^{YX_2} + \lambda^{YX_3} \\ & + \lambda^{Y, X_1, X_2} + \lambda^{Y, X_1, X_3} + \lambda^{Y, X_2, X_3} + \lambda^{Y, X_1, X_2, X_3} \\ & + \lambda^{X_1 X_2} + \lambda^{X_1 X_3} + \lambda^{X_2 X_3} + \lambda^{X_1 X_2 X_3}. \end{aligned}$$

This model is extreme in the sense that it contains all possible predictor-criterion relationships. It allows the researcher to test hypotheses concerning the magnitude of each parameter that relates predictors and criteria. In other words, this model is sensitive to every possible predictor-criterion relationship and is therefore comparable to the PCFA base model. However, in contrast to the PCFA base model, this model is saturated. Researchers therefore tend to prefer more parsimonious models when they perform LR.

In the following paragraphs, we present design matrices for the standard model of LR and the base model of PCFA. For these two design matrices, let Y , X_1 , X_2 , and X_3 be dichotomous. Crossed, they form a $2 \times 2 \times 2 \times 2$ contingency table. The design matrix for the standard LR model appears in Table 2.

Table 2:
Design matrix for a logistic regression of the dichotomous variable Y onto the dichotomous variables $X_1, X_2,$ and X_3

λ	main effects				YX interactions			interactions among X			
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	-1	1	1	-1	1	-1	-1	-1
1	1	1	-1	1	1	-1	1	-1	1	-1	-1
1	1	1	-1	-1	1	-1	-1	-1	-1	1	1
1	1	-1	1	1	-1	1	1	-1	-1	1	-1
1	1	-1	1	-1	-1	1	-1	-1	1	-1	1
1	1	-1	-1	1	-1	-1	1	1	-1	-1	1
1	1	-1	-1	-1	-1	-1	-1	1	1	1	-1
1	-1	1	1	1	-1	-1	-1	1	1	1	1
1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1
1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1
1	-1	1	-1	-1	-1	1	1	-1	-1	1	1
1	-1	-1	1	1	1	-1	-1	-1	-1	1	-1
1	-1	-1	1	-1	1	-1	1	-1	1	-1	1
1	-1	-1	-1	1	1	1	-1	1	-1	-1	1
1	-1	-1	-1	-1	1	1	1	1	1	1	-1

This LR model has $df = 2 \times 2 \times 2 \times 2 - 1 - 4 - 3 - 3 - 1 = 4$. The remaining degrees of freedom result because four parameters were set to zero. Table 3 presents the design matrix for the PCFA base model.

Table 3:
Design matrix for PCFA base model

λ	main effects				interactions among X			
1	1	1	1	1	1	1	1	1
1	1	1	1	-1	1	-1	-1	-1
1	1	1	-1	1	-1	1	-1	-1
1	1	1	-1	-1	-1	-1	1	1
1	1	-1	1	1	-1	-1	1	-1
1	1	-1	1	-1	-1	1	-1	1
1	1	-1	-1	1	1	-1	-1	1
1	1	-1	-1	-1	1	1	1	-1
1	-1	1	1	1	1	1	1	1
1	-1	1	1	-1	1	-1	-1	-1
1	-1	1	-1	1	-1	1	-1	-1
1	-1	1	-1	-1	-1	-1	1	1
1	-1	-1	1	1	-1	-1	1	-1
1	-1	-1	1	-1	-1	1	-1	1
1	-1	-1	-1	1	1	-1	-1	1
1	-1	-1	-1	-1	1	1	1	-1

This model has $df = 2 \times 2 \times 2 \times 2 - 1 - 4 - 3 - 1 = 7$ because seven parameters were set to zero.

2. Sample data structures

This section presents data examples. We begin with artificial data examples that were constructed to illustrate the comparative characteristics of LR and PCFA. Examples using empirical data follow.

2.1 An artificial example

The following artificial data example involves the four variables Y , X_1 , X_2 , and X_3 . Y is the criterion variable. The output protocol was created using SPSS 10.0. It displays the observed and the expected cell frequencies, estimated using the LR model given in section 1.2 and below.

Factor	Value	Observed Count	%	Expected Count	%
Y	1.00				
X1	1.00				
X2	1.00				
X3	1.00	7.00 (.07)	7.26 (.07)
X3	2.00	22.00 (.22)	22.17 (.22)
X2	2.00				
X3	1.00	22.00 (.22)	22.17 (.22)
X3	2.00	122.00 (1.20)	121.40 (1.20)
X1	2.00				
X2	1.00				
X3	1.00	22.00 (.22)	22.17 (.22)
X3	2.00	122.00 (1.20)	121.40 (1.20)
X2	2.00				
X3	1.00	122.00 (1.20)	121.40 (1.20)
X3	2.00	1808.00 (17.80)	1809.02 (17.81)
Y	2.00				
X1	1.00				
X2	1.00				
X3	1.00	15.00 (.15)	14.74 (.15)
X3	2.00	55.00 (.54)	54.83 (.54)
X2	2.00				
X3	1.00	55.00 (.54)	54.83 (.54)
X3	2.00	365.00 (3.59)	365.60 (3.60)
X1	2.00				
X2	1.00				
X3	1.00	55.00 (.54)	54.83 (.54)
X3	2.00	365.00 (3.59)	365.60 (3.60)
X2	2.00				
X3	1.00	365.00 (3.59)	365.60 (3.60)
X3	2.00	6634.00 (65.32)	6632.98 (65.31)

These data were constructed to conform with a logistic regression such that Y is close to perfectly predictable from X_1 , X_2 , and X_3 . The estimated log-linear model is thus

$$\begin{aligned} \log m = & \lambda + \lambda^Y + \lambda^{X_1} + \lambda^{X_2} + \lambda^{X_3} \\ & + \lambda^{YX_1} + \lambda^{YX_2} + \lambda^{YX_3} \\ & + \lambda^{X_1X_2} + \lambda^{X_1X_3} + \lambda^{X_2X_3} + \lambda^{X_1X_2X_3}. \end{aligned}$$

This model is the same as the first logistic regression model in Section 1.2. For the overall model fit, one obtains the Likelihood Ratio $\chi^2 = 0.0316$ with $df = 4$ and $p = 0.9999$, and the Pearson $\chi^2 = 0.0315$ with $df = 4$ and $p = 0.9999$, which is rather acceptable. For the individual model parameters, one obtains

Asymptotic 95% CI

Parameter	Estimate	SE	Z-value	Lower	Upper	Effect
2	-2.8983	.0519	-55.80	-3.00	-2.80	Y
4	-2.8983	.0519	-55.80	-3.00	-2.80	X1
6	-2.8983	.0519	-55.80	-3.00	-2.80	X2
8	-1.2993	.0262	-49.61	-1.35	-1.25	X3
10	1.0010	.1313	7.62	.74	1.26	X1 x X2
14	1.0010	.1313	7.62	.74	1.26	X1 x X3
18	.1969	.0927	2.12	.02	.38	Y x X1
22	1.0010	.1313	7.62	.74	1.26	X2 x X3
26	.1969	.0927	2.12	.02	.38	Y x X2
30	.1969	.0927	2.12	.02	.38	Y x X3
34	-.4172	.3011	-1.39	-1.01	.17	X1 x X2 x X3

With the exception of the $X_1 \times X_2 \times X_3$ effect, all effects are significant. The residuals are all very small. Thus, Y is nicely predictable from X_1 , X_2 , and X_3 .

We now ask whether PCFA can detect types and antitypes in a data set constructed to reflect the characteristics of LR. The PCFA base model for the present data is

$$\begin{aligned} \log m = & \lambda + \lambda^Y + \lambda^{X_1} + \lambda^{X_2} + \lambda^{X_3} \\ & + \lambda^{X_1X_2} + \lambda^{X_1X_3} + \lambda^{X_2X_3} + \lambda^{X_1X_2X_3}. \end{aligned}$$

As for the LR run, this is the same base model as given for the example in Section 1.2. In other words, the X - Y relationships are not part of the model. If these relationships exist, they can surface in the form of types and antitypes. We obtain, again using SPSS 10.0:

Factor	Value	Observed Count	%	Expected Count	%
Y	1.00				
X1	1.00				
X2	1.00				
X3	1.00	7.00 (.07)	4.87 (.05)
X3	2.00	22.00 (.22)	17.04 (.17)
X2	2.00				
X3	1.00	22.00 (.22)	17.04 (.17)
X3	2.00	122.00 (1.20)	107.75 (1.06)
X1	2.00				
X2	1.00				
X3	1.00	22.00 (.22)	17.04 (.17)
X3	2.00	122.00 (1.20)	107.75 (1.06)
X2	2.00				
X3	1.00	122.00 (1.20)	107.75 (1.06)
X3	2.00	1808.00 (17.80)	1867.78 (18.39)
Y	2.00				
X1	1.00				
X2	1.00				
X3	1.00	15.00 (.15)	17.13 (.17)
X3	2.00	55.00 (.54)	59.96 (.59)
X2	2.00				
X3	1.00	55.00 (.54)	59.96 (.59)
X3	2.00	365.00 (3.59)	379.25 (3.73)
X1	2.00				
X2	1.00				
X3	1.00	55.00 (.54)	59.96 (.59)
X3	2.00	365.00 (3.59)	379.25 (3.73)
X2	2.00				
X3	1.00	365.00 (3.59)	379.25 (3.73)
X3	2.00	6634.00 (65.32)	6574.22 (64.73)

and the overall goodness-of-fit Likelihood Ratio $X^2 = 15.86$ with $df = 7$ and $p = 0.026$, and the Pearson $X^2 = 16.49$ with $df = 7$ and $p = 0.021$. The PCFA base model is thus rejected, and we can expect types and antitypes to surface. We inspect the residuals, given in the next output protocol,

Factor	Value	Resid.	Adj. Resid.	Dev. Resid.	
Y	1.00				
X1	1.00				
X2	1.00				
X3	1.00	2.13	1.10	2.26	
X3	2.00	4.96	1.37	3.35	
X2	2.00				
X3	1.00	4.96	1.37	3.35	
X3	2.00	14.25	1.59	5.51	
X1	2.00				
X2	1.00				
X3	1.00	4.96	1.37	3.35	
X3	2.00	14.25	1.59	5.51	
X2	2.00				
X3	1.00	14.25	1.59	5.51	
X3	2.00	-59.78	-3.82	-10.85	Antitype
Y	2.00				
X1	1.00				
X2	1.00				
X3	1.00	-2.13	-1.10	-2.00	
X3	2.00	-4.96	-1.37	-3.08	
X2	2.00				
X3	1.00	-4.96	-1.37	-3.08	
X3	2.00	-14.25	-1.59	-5.29	
X1	2.00				
X2	1.00				
X3	1.00	-4.96	-1.37	-3.08	
X3	2.00	-14.25	-1.59	-5.29	
X2	2.00				
X3	1.00	-14.25	-1.59	-5.29	
X3	2.00	59.78	3.82	10.96	Type

One type and one antitype result. We thus conclude that LR and PCFA are sensitive to the same data characteristics. However, we also know from the discussion in Section 1.2 that PCFA is capable of responding to data characteristics that would be covered in terms that are typically set to zero in standard applications of LR. In the next section, we discuss an example with empirical data.

2.2 *An empirical data example*

The following example re-analyzes data published by Bogat, Levendosky, Davidson, DeJonghe, and von Eye (2005). In a study on domestic violence, the degree of depression of 207 women was assessed on three occasions, one during and two after a pregnancy, one and two years following the birth of the child². 54 of these women had never experienced domestic violence, and 153 had been victimized at one or more occasions. Depression was assessed using the Beck’s depression inventory (BDI; Beck, Ward, Mendelson, Mock, & Erbaugh, 1961). For the following analyses, the raw depression scores were dichotomized at the median. A score of 1 suggests below average depression relative to the total sample, and a score of 2 suggests above average depression. Domestic violence was scored as 1 = no experience of violence ever, and 2 = victimized at least once.

For analysis, the four dichotomous variables were crossed. The resulting 2 (depression during pregnancy; D1) x 2 (depression 1 year post partum; D2) x 2 (depression 2 years post partum; D3) x 2 (domestic violence status; DV) cross-classification is now analyzed using three models. The first is the standard LR model, in which the three measures of depression serve as predictors of DV. This model is

$$\begin{aligned} \log m = & \lambda + \lambda^{D1} + \lambda^{D2} + \lambda^{D3} + \lambda^{DV} \\ & + \lambda^{D1, D2} + \lambda^{D1, D3} + \lambda^{D2, D3} + \lambda^{D1, DV} + \lambda^{D2, DV} + \lambda^{D3, DV} \\ & + \lambda^{D1, D2, D3}. \end{aligned}$$

This approach is analogous to a discriminant analysis of the two DV groups based on the three measures of depression. The second analysis is a PCFA in which DV status is predicted. The third analysis involves estimating a saturated model. Table 4 displays the parameter estimates for the log-linear logistic regression model of the data (the raw frequencies are displayed in the context of the configural analysis, in Table 5). These results were obtained using SYSTAT 10.2.

The overall goodness-of-fit for the logistic regression model is $LR-X^2 = 0.796$ ($df = 4$; $p = 0.958$; Pearson $X^2 = 0.641$; $df = 4$; $p = 0.939$), which indicates excellent fit. We thus conclude that the model reflects data characteristics very well. The parameters in Table 4 indicate that two of the three predictors are significantly related to the criterion, DV status (parameter estimates and test statistics for these effects are printed in bold face in Table 4). Specifically, we find that depression during pregnancy and depression at one year post partum are significantly associated with DV status. The relationships are as expected. DV status as victim is predicted from above average depression.

² Notice the sample size difference to the data published in the article by Bogat et al. (2004). The explanation for this difference is that, for the present analyses, the missing data were estimated for all woman who began the study.

Table 4:
Parameter estimates for logistic regression of DV status on depression scores

Parameter	se (Param)	Param/se	Effect
0.471	0.126	3.728	D1
-0.231	0.117	-1.974	D2
0.235	0.120	1.963	D3
-0.687	0.105	-6.551	DV
0.514	0.108	4.773	D2 *D1
-0.034	0.109	-0.316	D3 *D1
0.412	0.109	3.782	DV *D1
0.660	0.106	6.213	D3 *D2
-0.041	0.107	-0.383	DV *D2
0.316	0.105	3.015	DV *D3
-0.162	0.104	-1.556	D3 *D2 *D1
1.897	0.130	14.555	CONSTANT

The second analysis involves a PCFA of the four variables of the DV study. The base model of this PCFA is

$$\log m = \lambda + \lambda^{D1} + \lambda^{D2} + \lambda^{D3} + \lambda^{DV} \\ + \lambda^{D1,D2} + \lambda^{D1,D3} + \lambda^{D2,D3} + \lambda^{D1,D2,D3}.$$

Table 5 displays the results of PCFA. These results were obtained using the CFA 2000 program (von Eye, 2001). The results were calculated using Lehman's test with Küchenhoff's continuity correction, under the Bonferroni-adjusted $\alpha^* = 0.003125$.

The overall goodness-of-fit for the PCFA base model is poor. We obtain a LR- $\chi^2 = 39.853$ ($df = 7$; $p < 0.01$; Pearson $\chi^2 = 37.553$; $df = 7$; $p < 0.01$), and therefore anticipate that types and antitypes emerge. Table 5 indicates that two types and two antitypes exist. The first type is constituted by Configuration 1111. These are women who never suffered from above average depression and were never victims of DV. Based on the PCFA model, fewer than 15 women had been expected to show this pattern, but 29 were found. The second type is constituted by Configuration 2222. This configuration describes the 60 women who were victims of DV at one or more points in time. Only 47 had been expected to show this pattern.

Because the outcome variable for this PCFA is dichotomous, the antitypes complement the types (and exist, thus, by default). The first antitype, constituted by Configuration 1112, indicates that it is less likely than expected by chance that women who experienced domestic violence never report above average depression. The second antitype, constituted by Configuration 2221, indicates that it is less likely than predicted by the base model that women who never experienced DV report above average depression at all three points in time.

Table 5:
PCFA of DV status on three depression scores

Configuration	fo	fe	statistic	p	
1111	29	14.870	4.818	.00000073	Type
1112	28	42.130	-4.818	.00000073	Antitype
1121	4	4.435	.038	.48503995	
1122	13	12.565	-.038	.48503995	
1211	6	3.130	1.601	.05467590	
1212	6	8.870	-1.601	.05467590	
1221	6	6.522	-.011	.49579754	
1222	19	18.478	.011	.49579754	
2111	2	4.174	-.990	.16115260	
2112	14	11.826	.990	.16115260	
2121	0	.783	-.373	.35442647	
2122	3	2.217	.373	.35442647	
2211	3	3.391	.071	.47179993	
2212	10	9.609	-.071	.47179993	
2221	4	16.696	-4.167	.00001545	Antitype
2222	60	47.304	4.167	.00001545	Type

We thus note that the relationships between the three depression scores and DV status are carried mostly by the frequency distributions at the extremes. Women who suffered DV report consistent above average depression but not consistent below average depression. Accordingly, women who never experienced DV report consistent below average depression but not consistent above average depression. None of the other configurations comes even close to constituting a type or an antitype.

In the third analysis, we would have reported the results from the saturated log-linear model. This model contains all parameters that can be estimated for the present data. The difference between this model and the model reported above is that the second and third order associations between the depression scores and DV status are also included in the model. For the present data, we can expect the parameters that were set to zero for the standard LR model to be of minor impact. The reason for this expectation is that the goodness-of-fit of the above LR model is so good that there is practically no room for improvement. Unfortunately, both SPSS and SYSTAT had problems fitting the saturated model without invoking the Δ option. Therefore, we decided to omit the interaction among all four variables. The estimated model parameters for this model appear in Table 6.

Table 6:
Extended LR model for the DV data in Table 5

Parameter	se (Param)	Param/se	Effect
0.519	0.152	3.410	D1
-0.300	0.159	-1.887	D2
0.274	0.146	1.879	D3
-0.743	0.147	-5.058	DV
0.589	0.147	3.997	D2 *D1
-0.074	0.143	-0.516	D3 *D1
0.452	0.129	3.505	DV *D1
0.675	0.129	5.218	D3 *D2
-0.110	0.149	-0.742	DV *D2
0.350	0.126	2.766	DV *D3
-0.184	0.113	-1.635	D3 *D2 *D1
0.110	0.142	0.774	DV *D2 *D1
-0.055	0.134	-0.413	DV *D3 *D1
0.016	0.118	0.138	DV *D3 *D2
1.851	0.155	11.930	CONSTANT

As expected, the extended LR model describes the data even better than the standard LR model ($LR-X^2 = 0.152$; $df = 1$; $p = 0.697$; Pearson $X^2 = 0.086$; $df = 1$; $p = 0.769$). However, this improvement cannot be significant because the standard LR model was already extremely good. The parameters in Table 6 are very close to the ones in Table 4. Those parameters that are significant in the standard LR model, are also significant in the extended model. None of the parameters that was non-significant in Table 4 is significant in Table 6. In addition, none of the parameters for the second and third order associations among the depression scores and DV status turned out significant. We thus conclude that (1) the results from the standard LR model can be retained, and (2) the types and antitypes result from the significant variable interactions indicated in Table 4.

3. Discussion

From the above analyses and the artificial and the empirical data examples, we conclude

- 1) Although LR and PCFA focus on the same data characteristics, specifically, on the relations among the X and the Y variables standard models of LR and PCFA differ. Therefore, results from LR and results from PCFA cannot be compared or interpreted as complementing each other without further analysis. This analysis has to demonstrate that the terms not considered in the standard model of LR are non-significant. If they are significant, the LR model must be extended by these terms to make results comparable.

- 2) If these terms are included (or omitted) in standard LR, they may surface in the form of significant effect parameters (LR) and/or in the form of types and antitypes (PCFA).
- 3) The differences between LR and PCFA lie also in the goals of application. LR is applied to examine variable relationships from an aggregate level or variable-oriented perspective. In contrast, PCFA is applied with the goal of identifying those patterns of predictor-criterion categories that specifically contradict the assumption of independence of predictors and criteria. More specifically, the differences between LR and PCFA are parallel to those between log-linear modeling and CFA in general:
 - LR expresses results in the form of variable relationships that are assumed to be valid across the entire range of admissible variable scores
 - PCFA expresses results in the form of types and antitypes, that is, in the form of **local deviations** from the base model
 - PCFA is sensitive to higher order interactions among predictors and criteria. In standard LR, these terms are set to zero.

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