L_2-gain Analysis and Control Synthesis for a Class of Uncertain Switched Nonlinear Systems

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Abstract This paper addresses the issue of L_2-gain analysis and control synthesis for a class of uncertain switched nonlinear systems using the average dwell-time method incorporated with piecewise Lyapunov functions. Sufficient conditions for a weighted L_2-gain and the stabilization are derived. A switched state feedback controller is designed to achieve exponential stability of the closed-loop system with the weighted L_2-gain property.

Key words Switched nonlinear systems, average dwell-time, exponential stability, weighted L_2-gain

A switched system consists of a continuous-time and/or discrete-time process interacting with a logical or decision-making process. Each continuous/discrete-time subsystem is represented as a set of differential/difference equations whereas the logic/decision-making subsystem is represented as a finite automation or a more general discrete event system. Due to its success in application and importance in theory, analysis and synthesis of this kind of systems have attracted much attention in recent years. Interests that focus on switched systems are mainly as follows: stability analysis[1−4], stabilization[5−7], controllability[8], observability[9], switching optimal control[10], H_∞ control[11−14], L_2-gain analysis[15−17], and others[18]. Stability is of considerable importance in the study of switched systems, and many researches were devoted to the study of this property. Among these researches, the common Lyapunov function technique was introduced to check the uniform stability or stabilizability of switched systems[1−2, 5]. But a common Lyapunov function may not exist or is too difficult to find. In this case, the multiple Lyapunov functions technique[3], the average dwell-time technique[4], and the switched Lyapunov function approach[19] were proposed to analyze the stability property for switched systems under some designed switching laws.

On the other hand, switched systems with disturbances are commonly found in practice. Thus, the stability and the L_2-gain analysis problem or H_∞ control problem for disturbed switched systems become interesting issues. The H_∞ control problem for switched systems was studied in [11−14]. Reference [15] addressed the L_2-gain analysis and control synthesis problem for a class of discrete-time disturbed uncertain switched linear systems using linear matrix inequalities. Reference [16] investigated the disturbance attenuation problem for a class of disturbed autonomous switched linear systems with the average dwell-time method, and a weighted L_2-gain was achieved. The L_2-gain analysis problem for a class of disturbed switched delay linear systems was addressed in [17]. All the papers mentioned above were about switched linear systems. Few results on the topic for switched nonlinear systems have been available by now.

In this paper, we study the weighted L_2-gain property and control synthesis problem for disturbed uncertain switched nonlinear systems using the average dwell-time method incorporated with piecewise Lyapunov functions. The switched system under consideration is composed of a nonlinear part and a linear part. Both the linear and the nonlinear parts are assumed to be stabilizabled under some average dwell-time based switching laws. Furthermore, we present sufficient conditions for the weighted L_2-gain property and stabilization. We also design a controller to achieve the weighted L_2-gain property and stability of the closed-loop system.

The results obtained in this study are different from the results obtained in previous studies. First, switched nonlinear systems were studied. Second, the average dwell-time method was used to select up a class of switching laws which are time-dependent, while multiple Lyapunov function method is mainly used for designing a state-dependent switching law[11, 18]. In addition, exponential stability was achieved, while [11, 18] only give asymptotic stability. Differently from the existing results handling the same uncertainty[12], we used the average dwell-time method, which is considered to be more feasible than other design methods in many cases.

1 Problem statement and preliminaries

Consider the following disturbed uncertain switched nonlinear system:

$$\begin{cases} \dot{\boldsymbol{x}}_1(t) = \hat{A}_{1\sigma(t)}\boldsymbol{x}_1(t) + A_{2\sigma(t)}\boldsymbol{x}_2(t) + \hat{B}_{\sigma(t)}\boldsymbol{u}_{\sigma(t)}(t) + \\ \qquad G_{\sigma(t)}\boldsymbol{w}(t) \\ \dot{\boldsymbol{x}}_2(t) = \boldsymbol{f}_{2\sigma(t)}(\boldsymbol{x}_2(t)) \\ \boldsymbol{y}(t) = C_{\sigma(t)}\boldsymbol{x}_1(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}_1(t) \in \mathbf{R}^{n-d}$ and $\boldsymbol{x}_2(t) \in \mathbf{R}^d$ are the states, $\boldsymbol{u}_{\sigma(t)}(t) \in \mathbf{R}^m$ is the control input, $\boldsymbol{w}(t) \in L_2[0,\infty)$ is the external disturbance input, and $\boldsymbol{y}(t) \in \mathbf{R}^p$ is the controlled output. $\sigma(t) : [0,\infty] \to I_N = \{1, \cdots, N\}$ is the switching signal, which is a piecewise constant function of time and will be determined later. $\sigma(t) = i$ means that the i-th subsystem is activated. $\hat{A}_{1i} = A_{1i} + \triangle A_{1i}$, $\hat{B}_i = B_i + \triangle B_i(t)$, A_{1i}, A_{2i}, B_i, G_i, and $C_i (i \in I_N)$ are constant matrices of appropriate dimensions. $\boldsymbol{f}_{2i}(\boldsymbol{x}_2)$ are smooth vector fields with $\boldsymbol{f}_{2i}(\mathbf{0}) = \mathbf{0}$. $\triangle A_{1i}(t)$ and $\triangle B_i(t)$ are uncertain time-varying matrices with the form

$$[\triangle A_{1i}(t), \triangle B_i(t)] = E_i\Gamma(t)[F_{1i}, F_{2i}], \qquad i \in I_N \tag{2}$$

where $E_i \in \mathbf{R}^{(n-d)\times l}$, $F_{1i} \in \mathbf{R}^{k\times(n-d)}$, and $F_{2i} \in \mathbf{R}^{k\times m}$ are known constant matrices, which characterize the structure of uncertainties, and F_{2i} are of full column rank. Γ is the norm-bounded time-varying uncertainty, i.e.,

$$\begin{aligned} \Gamma = \Gamma(t) \in \{\Gamma(t) : \Gamma(t)^{\mathrm{T}}\Gamma(t) \le I, \Gamma(t) \in \mathbf{R}^{l\times k}, \text{and} \\ \text{the elements of } \Gamma(t) \text{ are Lebesgue measurable.}\} \end{aligned} \tag{3}$$

Equation (2) is a standard assumption on system uncertainties[20].

Consider the switched system

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{f}_{\sigma(t)}(\boldsymbol{x}(t)) + \boldsymbol{g}_{\sigma(t)}(\boldsymbol{x}(t))\boldsymbol{w}(t) \\ \boldsymbol{y}(t) = \boldsymbol{h}_{\sigma(t)}(\boldsymbol{x}(t)) \end{cases} \tag{4}$$

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where $\boldsymbol{x}(t)\in \mathbf{R}^n$, $\boldsymbol{w}(t)$, $\boldsymbol{y}$, and $\sigma(t)$ are the same as in (1), $\boldsymbol{f}_i(\boldsymbol{x})$, $\boldsymbol{g}_i(\boldsymbol{x})$, and $\boldsymbol{h}_i(\boldsymbol{x})$ ($i \in I_N$) are all smooth with $\boldsymbol{f}_i(\boldsymbol{0}) = \boldsymbol{0}$.

Definition 1. The switched system (4) with $\boldsymbol{w}(t) \equiv \boldsymbol{0}$ is said to be globally exponentially stable with stability degree $\lambda > 0$ under $\sigma(t)$ if $\|\boldsymbol{x}(t)\| \le \mathrm{e}^{\alpha-\lambda(t-t_0)}\|\boldsymbol{x}(t_0)\|$ holds for all $t \ge t_0$ and a constant $\alpha \ge 0$.

Definition 2. System (1) is said to be globally exponentially stabilizable via switching if there exist a switching signal $\sigma(t)$ and an associate switched state feedback $\boldsymbol{u}_{\sigma(t)}(t) = K_{\sigma(t)}\boldsymbol{x}_1(t)$ such that the corresponding closed-loop system (1) with $\boldsymbol{w}(t) \equiv \boldsymbol{0}$ is globally exponentially stable for all admissible uncertainties.

Definition 3. System (4) is said to have a weighted L_2-gain γ, if the following inequality holds for some switching law $\sigma(t)$ and some real-valued function $\beta(t)$ with $\beta(0) = 0$

$$\int_0^{\infty} \mathrm{e}^{-\lambda t}\boldsymbol{y}^{\mathrm{T}}(t)\boldsymbol{y}(t)\mathrm{d}t \le \gamma^2 \int_0^{\infty} \boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)\mathrm{d}t + \beta(\boldsymbol{x}(0)) \quad (5)$$

Our purpose is to establish sufficient conditions for the switched system (1) with $\boldsymbol{u}_{\sigma(t)}(t) \equiv \boldsymbol{0}$ to be globally exponentially stable with a weighted L_2-gain, and to design a controller to make the closed-loop system globally exponentially stable with a weighted L_2-gain.

Since the control input only appears in $\boldsymbol{x}_1$-part, it is natural and reasonable to design a controller of the form

$$\boldsymbol{u}_{\sigma(t)} = K_{\sigma(t)}\boldsymbol{x}_1(t) \quad (6)$$

if $\boldsymbol{x}_2$ can be made exponentially convergent.

The following lemma will be used in the development of the main results.

Lemma 1[1]. Consider the nonlinear switched system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_i(\boldsymbol{x}(t)), \qquad i \in I_N = \{1, \cdots, N\}$$

Assume for each $i \in I_N$ there exists a Lyapunov function V_i such that for some positive constants a_i, b_i, and c,

$$a_i\|\boldsymbol{x}\|^2 \le V_i(\boldsymbol{x}) \le b_i\|\boldsymbol{x}\|^2 \quad (7)$$

and

$$\frac{\partial V_i(\boldsymbol{x})}{\partial \boldsymbol{x}} f_i(\boldsymbol{x}) \le -c\|\boldsymbol{x}\|^2 \quad (8)$$

Then, two positive constants μ and λ can be found such that the switched nonlinear system is globally exponentially stable for any switching signal that has the average dwell-time property with $\tau_a \ge \ln\mu/\lambda$.

Remark 1. It is not difficult to find that $\mu = \sup\{b_p/a_q \mid p, q \in I_N\}$, $\lambda \in [0, \lambda_0)$, and $\lambda_0 = \min\{c/b_i \mid i \in I_N\}$ in Lemma 1[1, 4].

2 L_2-gain analysis

In this section, we derive sufficient conditions for the autonomous switched system (1) to have the weighted L_2-gain property.

Theorem 1. Given any constant $\gamma > 0$, suppose that switched system (1) satisfies the following conditions:

1) There exist constants $\varepsilon_i > 0$, $\lambda_0 > 0$, and $\mu \ge 1$, such that the following inequalities

$$\begin{aligned} A_{1i}^{\mathrm{T}}P_i + P_iA_{1i} + \varepsilon_i^{-2}P_iE_iE_i^{\mathrm{T}}P_i + \gamma^{-2}P_iG_iG_i^{\mathrm{T}}P_i + \\ \varepsilon_i^2F_{1i}^{\mathrm{T}}F_{1i} + C_i^{\mathrm{T}}C_i + \lambda_0P_i + I < 0 \end{aligned} \quad (9)$$

$$P_i \le \mu P_j, \quad i, j = 1, \cdots, N \quad (10)$$

have positive definite solutions P_i;

2) There exist proper, positive definite, and radially unbounded functions $W_i(\boldsymbol{x}_2(t))$ such that

$$\frac{\mathrm{d}W_i(\boldsymbol{x}_2)}{\mathrm{d}\boldsymbol{x}_2} f_{2i}(\boldsymbol{x}_2(t)) \le -\beta_i\|\boldsymbol{x}_2\|^2 \quad (11)$$

$$a_{1i}\|\boldsymbol{x}_2\|^2 \le W_i(\boldsymbol{x}_2) \le a_{2i}\|\boldsymbol{x}_2\|^2 \quad (12)$$

for some constants $\beta_i > 0$, $a_{1i} > 0$, and $a_{2i} > 0$, $i = 1, \cdots, N$.

Then, the autonomous switched system (1) is globally exponentially stable when $\boldsymbol{w}(t) = \boldsymbol{0}$ and has the weighted L_2-gain γ under arbitrary switching laws satisfying the average dwell-time

$$\tau_a \ge \tau_a^* = \frac{\ln\hat{\mu}}{\lambda} \quad (13)$$

where $\hat{\mu} = \max\{\mu, \mathrm{a}_{2i}/\mathrm{a}_{1j} \mid i, j \in I_N\}$, $\lambda \in [0, \hat{\lambda}_0)$, and $\hat{\lambda}_0 = \min\{\lambda_0, \beta_i/\mathrm{a}_{2i} \mid i \in I_N\}$.

Proof. For arbitrary $t > 0$, denote $t_0 \le t_1 \le t_2 \le \cdots \le t_k \le \cdots \le t_{N_\sigma(0,t)}$ as the switching instants of $\sigma(t)$ over the interval $(0, t)$ and $N_\sigma(0, t)$ as the number of switchings of $\sigma(t)$ in the interval $(0, t)$.

Define the following piecewise Lyapunov function candidate

$$V(\boldsymbol{x}) = V_{\sigma(t)}(\boldsymbol{x}) = \boldsymbol{x}_1^{\mathrm{T}}P_{\sigma(t)}\boldsymbol{x}_1 + kW_{\sigma(t)}(\boldsymbol{x}_2) \quad (14)$$

where P_i are the solutions of (9) and (10), and k is a positive scalar to be defined later.

Then, when $\sigma(t) = i$, differentiating $V(\boldsymbol{x})$ along the trajectory of the autonomous switched system (1) gives rise to

$$\begin{aligned} \dot{V} \ = \ & \boldsymbol{x}_1^{\mathrm{T}}(\hat{A}_{1i}^{\mathrm{T}}P_i + \hat{A}_{1i}P_i)\boldsymbol{x}_1 + 2\boldsymbol{x}_1^{\mathrm{T}}P_iA_{2i}\boldsymbol{x}_2 + 2\boldsymbol{x}_1^{\mathrm{T}}P_iG_i\boldsymbol{w} + \\ & k\frac{\mathrm{d}W_i(\boldsymbol{x}_2)}{\mathrm{d}\boldsymbol{x}_2} f_{2i}(\boldsymbol{x}_2) = \\ & \boldsymbol{x}_1^{\mathrm{T}}(A_{1i}^{\mathrm{T}}P_i + A_{1i}P_i)\boldsymbol{x}_1 + 2\boldsymbol{x}_1^{\mathrm{T}}P_i\Delta A_{1i}\boldsymbol{x}_1 + \\ & 2\boldsymbol{x}_1^{\mathrm{T}}P_iA_{2i}\boldsymbol{x}_2 + 2\boldsymbol{x}_1^{\mathrm{T}}P_iG_i\boldsymbol{w} + k\frac{\mathrm{d}W_i(\boldsymbol{x}_2)}{\mathrm{d}\boldsymbol{x}_2} f_{2i}(\boldsymbol{x}_2) \le \\ & \boldsymbol{x}_1^{\mathrm{T}}(A_{1i}^{\mathrm{T}}P_i + A_{1i}P_i + \varepsilon_i^{-2}P_iE_iE_i^{\mathrm{T}}P_i + \varepsilon_i^2F_{1i}^{\mathrm{T}}F_{1i})\boldsymbol{x}_1 + \\ & 2\boldsymbol{x}_1^{\mathrm{T}}P_iA_{2i}\boldsymbol{x}_2 + 2\boldsymbol{x}_1^{\mathrm{T}}P_iG_i\boldsymbol{w} - k\beta_i\|\boldsymbol{x}_2\|^2 \end{aligned}$$

It is easy to see that there exist constants $l_i > 0$, $q_i > 0$, $i \in I_N$, such that

$$\|A_{2i}\boldsymbol{x}_2\| \le l_i\|\boldsymbol{x}_2\|, \qquad \|\boldsymbol{x}_1^T P_i\| \le q_i\|\boldsymbol{x}_1\|$$

Let $l = \max\{l_iq_i \mid i \in I_N\}$. Then, we have

$$\begin{aligned} & \dot{V} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} - \gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} \le \\ & \boldsymbol{x}_1^{\mathrm{T}}(A_{1i}^{\mathrm{T}}P_i + A_{1i}P_i + \varepsilon_i^{-2}P_iE_iE_i^{\mathrm{T}}P_i + \varepsilon_i^2F_{1i}^{\mathrm{T}}F_{1i})\boldsymbol{x}_1 + \\ & 2l\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| - k\beta_i\|\boldsymbol{x}_2\|^2 + 2\boldsymbol{x}_1^{\mathrm{T}}P_iG_i\boldsymbol{w} + \boldsymbol{x}_1^{\mathrm{T}}C_i^{\mathrm{T}}C_i\boldsymbol{x}_1 - \\ & \gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} \le \\ & \boldsymbol{x}_1^{\mathrm{T}}(A_{1i}^{\mathrm{T}}P_i + A_{1i}P_i + \varepsilon_i^{-2}P_iE_iE_i^{\mathrm{T}}P_i + \gamma^{-2}P_iG_iG_i^{\mathrm{T}}P_i + \\ & \varepsilon_i^2F_{1i}^{\mathrm{T}}F_{1i} + C_i^{\mathrm{T}}C_i)\boldsymbol{x}_1 + 2l\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| - k\beta_i\|\boldsymbol{x}_2\|^2 \end{aligned}$$

Let $b = \min\{\beta_i/a_{2i} \mid i \in I_N\}$, from (9) and (12), we can obtain

$$\begin{aligned} & \dot{V} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} - \gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} \le \\ & -\lambda_0\boldsymbol{x}_1^{\mathrm{T}}P_i\boldsymbol{x}_1 - kbW_i(\boldsymbol{x}_2) + kbW_i(\boldsymbol{x}_2) - \boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{x}_1 + \\ & 2l\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| - k\beta_i\|\boldsymbol{x}_2\|^2 \le \\ & -\hat{\lambda}_0V + ka_{2i}b\|\boldsymbol{x}_2\|^2 - \boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{x}_1 + 2l\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| - k\beta_i\|\boldsymbol{x}_2\|^2 \le \\ & -\hat{\lambda}_0V - (k\beta_i - ka_{2i}b - l^2)\|\boldsymbol{x}_2\|^2 \end{aligned}$$

where $\hat{\lambda}_0 = \min\{\lambda_0, b\} = \min\{\lambda_0, \beta_i/a_{2i} \mid i \in I_N\}$.

Choosing $k \geq \max\{\frac{l^2}{\beta_i - a_{2i}b} \mid i \in I_N\}$, we have

$$\dot{V} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} - \gamma^2\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w} \leq -\hat{\lambda}_0 V \tag{15}$$

When $\boldsymbol{w}(t) = \boldsymbol{0}$, from the above inequality, we obtain

$$\dot{V} \leq -\hat{\lambda}_0 V \tag{16}$$

According to (10), (12), and (14), it holds that

$$V_i(t) \leq \hat{\mu} V_j(t) \tag{17}$$

where $\hat{\mu} = \max\{\mu, a_{2i}/a_{1j} \mid i,j \in I_N\}$.

Then, from (16) and (17), we have

$$V(t) \leq \hat{\mu}^{N_\sigma(0,t)}\mathrm{e}^{-\hat{\lambda}_0 t}V(0) = \mathrm{e}^{N_\sigma(0,t)\ln\hat{\mu} - \hat{\lambda}_0 t}V(0)$$

where $V(t_k)$ is the value of the Lyapunov function V at the switching instant t_k, $k = 0, 1, 2, \cdots, N_\sigma(0,t)$. $V(t^-_{N_\sigma(0,t)})$ is the value of the Lyapunov function V at the former instant of the switching instant $t_{N_\sigma(0,t)}$.

Furthermore, in view of $N_\sigma(0,\tau) \leq \tau/\tau_a^*$, for $\forall\tau > 0$, (13) implies that

$$N_\sigma(0,\tau)\ln\hat{\mu} \leq \lambda\tau \tag{18}$$

Thus,

$$V(t) \leq \mathrm{e}^{-(\hat{\lambda}_0 - \lambda)t}V(0) \tag{19}$$

From (12), we know that there exist constants $\lambda_1 > 0$, $\lambda_2 > 0$, $a_1 > 0$, and $a_2 > 0$, such that

$$\lambda_1\|\boldsymbol{x}_1\|^2 + a_1\|\boldsymbol{x}_2\|^2 \leq V(t) \leq \lambda_2\|\boldsymbol{x}_1\|^2 + a_2\|\boldsymbol{x}_2\|^2 \tag{20}$$

where $\lambda_1 = \min\{\lambda_{\min}(P_i) \mid i \in I_N\}$, $\lambda_2 = \max\{\lambda_{\max}(P_i) \mid i \in I_N\}$, $a_1 = \min\{a_{1i} \mid i \in I_N\}$, and $a_2 = \max\{a_{2i} \mid i \in I_N\}$.

Let $b_1 = \min\{\lambda_1, a_1\}$ and $b_2 = \max\{\lambda_2, a_2\}$. We have

$$b_1(\|\boldsymbol{x}_1\|^2 + \|\boldsymbol{x}_2\|^2) \leq V(t) \leq b_2(\|\boldsymbol{x}_1\|^2 + \|\boldsymbol{x}_2\|^2) \tag{21}$$

Combining (19) $\sim$ (21) gives

$$\|\boldsymbol{x}(t)\|^2 \leq \frac{1}{b_1}\mathrm{e}^{-(\hat{\lambda}_0-\lambda)t}V(0) \leq \frac{b_2}{b_1}\mathrm{e}^{-(\hat{\lambda}_0-\lambda)t}\|\boldsymbol{x}(0)\|^2$$

Thus,

$$\|\boldsymbol{x}(t)\| \leq \sqrt{\frac{b_2}{b_1}}\mathrm{e}^{-\frac{(\hat{\lambda}_0-\lambda)}{2}t}\|\boldsymbol{x}(0)\| \tag{22}$$

which implies global exponential stability of the autonomous switched system (1) when $\boldsymbol{w}(t) = \boldsymbol{0}$.

Integrating both sides of (15) and from (17), we obtain

$$\begin{aligned}
V(t) \leq\ & V(t_{N_\sigma(0,t)})\mathrm{e}^{-\hat{\lambda}_0(t - t_{N_\sigma(0,t)})} - \int_{t_{N_\sigma(0,t)}}^{t} \mathrm{e}^{-\hat{\lambda}_0(t-\tau)} \times \\
& [\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau \leq \\
& \hat{\mu}\big[V(t_{N_\sigma(0,t)-1})\mathrm{e}^{-\hat{\lambda}_0(t_{N_\sigma(0,t)} - t_{N_\sigma(0,t)-1})} - \\
& \int_{t_{N_\sigma(0,t)-1}}^{t_{N_\sigma(0,t)}} \mathrm{e}^{-\hat{\lambda}_0(t_{N_\sigma(0,t)}-\tau)}[\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \\
& \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau\big]\mathrm{e}^{-\hat{\lambda}_0(t-t_{N_\sigma(0,t)})} - \\
& \int_{t_{N_\sigma(0,t)}}^{t} \mathrm{e}^{-\hat{\lambda}_0(t-\tau)}[\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau \\
& \vdots \\
\leq\ & \hat{\mu}^{N_\sigma(0,t)}\mathrm{e}^{-\hat{\lambda}_0 t}V(0) - \hat{\mu}^{N_\sigma(0,t)}\int_0^{t_1}\mathrm{e}^{-\hat{\lambda}_0(t-\tau)} \times \\
& [\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau - \hat{\mu}^{N_\sigma(0,t)-1} \times \\
& \int_{t_1}^{t_2}\mathrm{e}^{-\hat{\lambda}_0(t-\tau)}[\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau - \cdots \\
& -\hat{\mu}^0\int_{t_{N_\sigma(0,t)}}^{t}\mathrm{e}^{-\hat{\lambda}_0(t-\tau)}[\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau = \\
& \mathrm{e}^{-\hat{\lambda}_0 t + N_\sigma(0,t)\ln\hat{\mu}}V(0) - \int_0^t \mathrm{e}^{-\hat{\lambda}_0(t-\tau)+N_\sigma(\tau,t)\ln\hat{\mu}} \times \\
& [\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)]\mathrm{d}\tau
\end{aligned}$$

Multiplying both sides of the above inequality by $\mathrm{e}^{-N_\sigma(0,t)\ln\hat{\mu}}$ and considering (18) leads to

$$\begin{aligned}
&\int_0^t \mathrm{e}^{-\hat{\lambda}_0(t-\tau)-\lambda\tau}\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau \leq \\
&\quad \mathrm{e}^{-\hat{\lambda}_0 t}V(0) + \gamma^2\int_0^t \mathrm{e}^{-\hat{\lambda}_0(t-\tau)}\boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)\mathrm{d}\tau
\end{aligned} \tag{23}$$

Integrating both sides of the foregoing inequality from $t = 0$ to ∞ and rearranging the double-integral area, we obtain

$$\int_0^\infty \mathrm{e}^{-\lambda\tau}\boldsymbol{y}^{\mathrm{T}}(\tau)\boldsymbol{y}(\tau)\mathrm{d}\tau \leq V(0) + \gamma^2\int_0^\infty \boldsymbol{w}^{\mathrm{T}}(\tau)\boldsymbol{w}(\tau)\mathrm{d}\tau \tag{24}$$

From Definition 3, we know that the autonomous switched system (1) has the weighted L_2-gain γ. □

Remark 2. Applying Schur complement formula, the matrix inequality (9) can be easily transformed into the LMIs form. The second inequality of condition 1) is trivial, as long as $\mu = \sup_{i,j\in I_N}\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_j)}$.

Remark 3. When $\boldsymbol{x}_2(t) = \boldsymbol{0}$, it is easy to verify that the $\boldsymbol{x}_1$-part of the autonomous system (1) is exponentially stable with a weighted L_2-gain under arbitrary switching laws satisfying the average dwell-time $\tau_1 \geq \tau_{a1}^* = (\ln\mu_1)/\lambda_1$, $\lambda_1 \in [0, \lambda_0)$, and μ_1 equals to μ in (10). From Lemma 1, we know that the $\boldsymbol{x}_2$-part of the autonomous system (1) is exponentially stable under arbitrary switching laws satisfying the average dwell-time $\tau_2 \geq \tau_{a2}^* = (\ln\mu_2)/\lambda_2$, $\mu_2 = \max\{a_{2i}/a_{1j} \mid i,j \in I_N\}$, $\lambda_2 \in [0, \tilde{\lambda}_0)$, and $\tilde{\lambda}_0 = \min\{\beta_i/a_{2i} \mid i \in I_N\}$. From (13), we know that the average dwell-time for the whole cascade switched system satisfies $\tau_a \geq \tau_a^* \geq \max\{\tau_{a1}^*, \tau_{a2}^*\}$.

3 Control synthesis

In this section, we design a state feedback controller to make the corresponding closed-loop system (1) globally exponentially stable with a weighted L_2-gain. By Theorem 1, this problem reduces to finding $\boldsymbol{u}_{\sigma(t)} = K_{\sigma(t)}\boldsymbol{x}_1$ such that

$$\begin{cases}
\dot{\boldsymbol{x}}_1(t) = (\hat{A}_{1\sigma(t)} + \hat{B}_{\sigma(t)}K_{\sigma(t)})\boldsymbol{x}_1(t) + A_{2\sigma(t)}\boldsymbol{x}_2(t) + \\
\qquad G_{\sigma(t)}\boldsymbol{w}(t) \\
\dot{\boldsymbol{x}}_2(t) = \boldsymbol{f}_{2\sigma(t)}(\boldsymbol{x}_2(t)) \\
\boldsymbol{y}(t) = C_{\sigma(t)}\boldsymbol{x}_1(t)
\end{cases} \tag{25}$$

is globally exponentially stable with a weighted L_2-gain.

Theorem 2. Given any constant $\gamma > 0$, suppose that switched system (1) satisfies the following conditions:

1) There exist constants $\varepsilon_i > 0$, $\lambda_0 > 0$, and $\mu \geq 1$, such

that the following inequalities

$$A_{1i}^{\rm T}P_i + P_iA_{1i} + \varepsilon_i^{-2}P_iE_iE_i^{\rm T}P_i + \gamma^{-2}P_iG_iG_i^{\rm T}P_i + \varepsilon_i^2F_{1i}^{\rm T}F_{1i} + C_i^{\rm T}C_i + \lambda_0P_i + I - (\varepsilon_i^{-1}P_iB_i + \varepsilon_iF_{1i}^{\rm T}F_{2i})(F_{2i}^{\rm T}F_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i + \varepsilon_iF_{1i}^{\rm T}F_{2i})^{\rm T} < 0 \quad (26)$$

$$P_i \le \mu P_j, \quad i, j = 1, \cdots, N \quad (27)$$

have positive definite solutions P_i;

2) There exist proper, positive definite, and radially unbounded functions $W_i(\boldsymbol{x}_2(t))$ such that

$$\frac{{\rm d}W_i(\boldsymbol{x}_2)}{{\rm d}\boldsymbol{x}_2}f_{2i}(\boldsymbol{x}_2(t)) \le -\beta_i\|\boldsymbol{x}_2\|^2 \quad (28)$$

$$a_{1i}\|\boldsymbol{x}_2\|^2 \le W_i(\boldsymbol{x}_2) \le a_{2i}\|\boldsymbol{x}_2\|^2 \quad (29)$$

for some constants $\beta_i > 0$, $a_{1i} > 0$, and $a_{2i} > 0$, $i = 1, \cdots, N$.

Then, switched system (1) is globally exponentially stable when $\boldsymbol{w}(t) = \boldsymbol{0}$ and has a weighted L_2-gain γ with the switched state feedback:

$$\boldsymbol{u}_i = -(F_{2i}^{\rm T}F_{2i})^{-1}(\varepsilon_i^{-2}B_i^{\rm T}P_i + F_{2i}^{\rm T}F_{1i})\boldsymbol{x}_1(t) \quad (30)$$

under arbitrary switching laws satisfying the average dwell-time

$$\tau_a \ge \tau_a^* = \frac{\ln\hat{\mu}}{\lambda} \quad (31)$$

where $\hat{\mu} = \max\{\mu, a_{2i}/a_{1j} \mid i, j \in I_N\}$, $\lambda \in [0, \hat{\lambda}_0)$, $\hat{\lambda}_0 = \min\{\lambda_0, \beta_i/a_{2i} \mid i \in I_N\}$.

Proof. Similar to the proof of Theorem 1, we define the Lyapunov function

$$V(\boldsymbol{x}) = V_{\sigma(t)}(\boldsymbol{x}) = \boldsymbol{x}_1^{\rm T}P_{\sigma(t)}\boldsymbol{x}_1 + \hat{k}W_{\sigma(t)}(\boldsymbol{x}_2) \quad (32)$$

where P_i are the solutions of (26) and (27), and $\hat{k}$ is a positive scalar to be defined later.

Then, when the i-th switched subsystem is activated, based on (30), we can get

$$\begin{aligned}\dot{V} = \ & \boldsymbol{x}_1^{\rm T}(A_{1i}^{\rm T}P_i + A_{1i}P_i)\boldsymbol{x}_1 + 2\boldsymbol{x}_1^{\rm T}P_i\Delta A_{1i}\boldsymbol{x}_1 + 2\boldsymbol{x}_1^{\rm T}P_iB_i\boldsymbol{u}_i + 2\boldsymbol{x}_1^{\rm T}P_i\Delta B_i\boldsymbol{u}_i + 2\boldsymbol{x}_1^{\rm T}P_iA_{2i}\boldsymbol{x}_2 + \\ & \hat{k}\frac{{\rm d}W_i(\boldsymbol{x}_2)}{{\rm d}\boldsymbol{x}_2}f_{2i}(\boldsymbol{x}_2) + 2\boldsymbol{x}_1^{\rm T}P_iG_i\boldsymbol{w} \le \\ & \boldsymbol{x}_1^{\rm T}(A_{1i}^{\rm T}P_i + A_{1i}P_i + \varepsilon_i^{-2}P_iE_iE_i^{\rm T}P_i)\boldsymbol{x}_1 + \varepsilon_i^2(F_{1i}\boldsymbol{x}_1 + F_{2i}\boldsymbol{u}_i)^{\rm T}(F_{1i}\boldsymbol{x}_1 + F_{2i}\boldsymbol{u}_i) + 2\boldsymbol{x}_1^{\rm T}P_iB_i\boldsymbol{u}_i - \hat{k}\beta_i\|\boldsymbol{x}_2\|^2 + \\ & 2\boldsymbol{x}_1^{\rm T}P_iA_{2i}\boldsymbol{x}_2 + 2\boldsymbol{x}_1^{\rm T}P_iG_i\boldsymbol{w} \le \\ & \boldsymbol{x}_1^{\rm T}\Big\{A_{1i}P_i + A_{1i}P_i + \varepsilon_i^{-2}P_iE_iE_i^{\rm T}P_i + \varepsilon_i^2F_{1i}^{\rm T}F_{1i} - \big[(\varepsilon_i^{-1}P_iB_i + \varepsilon_iF_{1i}^{\rm T}F_{2i})(F_{2i}^{\rm T}F_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i + \\ & \varepsilon_iF_{1i}^{\rm T}F_{2i})^{\rm T}\big]\Big\}\boldsymbol{x}_1 + 2\boldsymbol{x}_1^{\rm T}P_iA_{2i}\boldsymbol{x}_2 - \hat{k}\beta_i\|\boldsymbol{x}_2\|^2 + 2\boldsymbol{x}_1^{\rm T}P_iG_i\boldsymbol{w}\end{aligned}$$

It is easy to know that there exist constants $m_i > 0$, $n_i > 0$, $i \in I_N$, such that

$$\|\boldsymbol{x}_1^{\rm T}P_i\| \le m_i\|\boldsymbol{x}_1\|, \qquad \|A_i\boldsymbol{x}_1\| \le n_i\|\boldsymbol{x}_2\|$$

Let $p = \max\{m_in_i \mid i \in I_N\}$. Taking (26) into consideration gives rise to

$$\begin{aligned}& \dot{V} + \boldsymbol{y}^{\rm T}\boldsymbol{y} - \gamma^2\boldsymbol{w}^{\rm T}\boldsymbol{w} \le \\ & \boldsymbol{x}_1^{\rm T}\Big\{A_{1i}^{\rm T}P_i + A_{1i}P_i + \varepsilon_i^{-2}P_iE_iE_i^{\rm T}P_i + \varepsilon_i^2F_{1i}^{\rm T}F_{1i} + \gamma^{-2}P_iG_iG_i^{\rm T}P_i + C_i^{\rm T}C_i - \big[(\varepsilon_i^{-1}P_iB_i + \varepsilon_iF_{1i}^{\rm T}F_{2i}) \times \\ & (F_{2i}^{\rm T}F_{2i})^{-1}(\varepsilon_i^{-1}P_iB_i + \varepsilon_iF_{1i}^{\rm T}F_{2i})^{\rm T}\big]\Big\}\boldsymbol{x}_1 - \hat{k}\beta_i\|\boldsymbol{x}_2\|^2 + 2p\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| \le \\ & -\lambda_0\boldsymbol{x}_1^{\rm T}P_i\boldsymbol{x}_1 - \hat{k}bW_i(\boldsymbol{x}_2) + \hat{k}bW_i(\boldsymbol{x}_2) - \boldsymbol{x}_1^{\rm T}\boldsymbol{x}_1 + 2p\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| - \hat{k}\beta_i\|\boldsymbol{x}_2\|^2 \le \\ & -\hat{\lambda}_0V + \hat{k}a_{2i}b\|\boldsymbol{x}_2\|^2 - \boldsymbol{x}_1^{\rm T}\boldsymbol{x}_1 + 2p\|\boldsymbol{x}_1\|\|\boldsymbol{x}_2\| - \hat{k}\beta_i\|\boldsymbol{x}_2\|^2 \le \\ & -\hat{\lambda}_0V - (\hat{k}\beta - \hat{k}a_{2i}b - p^2)\|\boldsymbol{x}_2\|^2\end{aligned}$$

where $b = \min\{\beta_i/a_{2i} \mid i \in I_N\}$, $\hat{\lambda}_0 = \min\{\lambda_0, b\} = \min\{\lambda_0, \beta_i/a_{2i} \mid i \in I_N\}$.

Let $\hat{k} \ge \max\{\frac{p^2}{\beta_i - a_{2i}b} \mid i \in I_N\}$. We have

$$\dot{V} + \boldsymbol{y}^{\rm T}(\tau)\boldsymbol{y} - \gamma^2\boldsymbol{w}^{\rm T}(\tau)\boldsymbol{w} \le -\hat{\lambda}_0V \quad (33)$$

The remainder of the proof is the similar to that of Theorem 1. □

4 Example

Consider the switched system (1) with $I_N = \{1, 2\}$, $n = 4$, $d = 2$, and

$$A_{11} = \begin{bmatrix} -4 & 0 \\ 2 & 1 \end{bmatrix}, \ A_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ \boldsymbol{B}_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \ E_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \ F_{11} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{F}_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \boldsymbol{G}_1 = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \ \boldsymbol{f}_{21} = \begin{bmatrix} -x_3 \\ -3x_3 - 4.4x_4 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} -5 & -2 \\ 3 & -4 \end{bmatrix}, \ A_{22} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \ \boldsymbol{B}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \ E_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \ F_{12} = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\boldsymbol{F}_{22} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \boldsymbol{G}_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \ \boldsymbol{f}_{22} = \begin{bmatrix} -2x_3(1 + x_4^2) \\ -x_4 \end{bmatrix}$$

$$\Gamma(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}$$

Let $\gamma = 1$ and $\varepsilon_1 = \varepsilon_2 = 1$. Solving (26) gives

$$P_1 = \begin{bmatrix} 2.6179 & 0.3342 \\ 0.3342 & 2.3329 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 2.9598 & 0.1577 \\ 0.1577 & 2.0921 \end{bmatrix}$$

It is easy to verify that P_1 and P_2 are positive definite matrices, which indicate that condition 1) in Theorem 2 is satisfied. Choose

$$W_1 = 1.5x_3^2 + 0.8x_3x_4 + 1.5x_4^2, \quad W_2 = x_3^2 + 2x_4^2$$

We have $a_{11} = 1.1$, $a_{21} = 1.9$, $a_{12} = 1$, $a_{22} = 2$, $\dot{W}_1 \le -1.6(x_3^2 + x_4^2)$, and $\dot{W}_2 \le -4(x_3^2 + x_4^2)$. This implies that condition 2) in Theorem 2 is satisfied. Using Theorem 2, we now find the average dwell-time and design the switched state feedback. Let $\mu = 1.4$, $\lambda_0 = 0.8$, and $\lambda = 0.7$. We get $\hat{\mu} = 1.9$ and $\tau_a^* = 0.9$. Design the switched state feedback as follows:

$$u_1 = -1.6x_1 - 3.5x_2, \quad u_2 = -21.8x_1 - 0.6x_2 \tag{34}$$

A simple calculation shows that the average dwell-time for the linear switched part of the system is $\tau_{a1} \geq \tau_{a1}^* = (\ln \mu)/\lambda = 0.5$, and the average dwell-time for the nonlinear switched part is $\tau_{a2} \geq \tau_{a2}^* = 0.7$. Thus, $\tau_a \geq \tau_a^* \geq \max\{\tau_{a1}^*, \tau_{a2}^*\}$ is obvious.

Figs. 1 $\sim$ 4 give the state responses and the switching signals for the linear part and the nonlinear part of the switched system. Figs. 5 and 6 are the state response and the switching signal of the whole switched system, respectively.

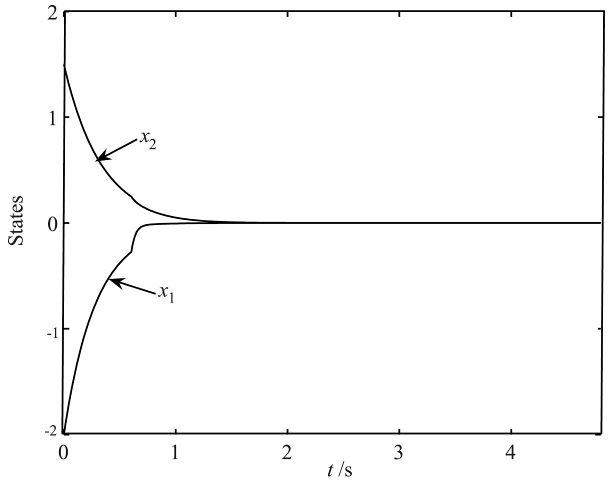

Fig. 1 The state responses of the linear part

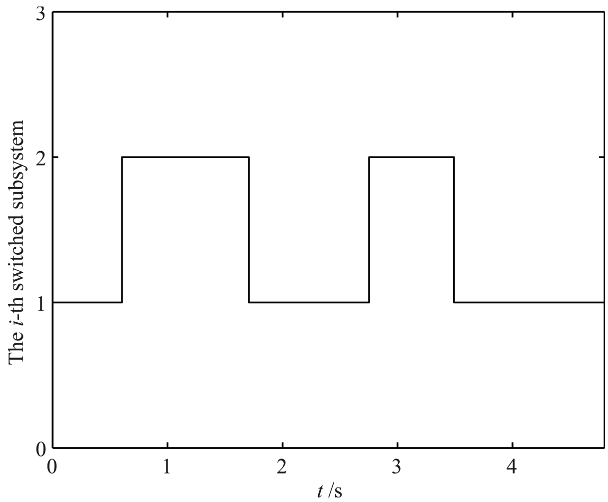

Fig. 2 The switching signal for the linear part

5 Conclusions

In this paper, we have studied the L_2-gain property and control synthesis problem for a class of uncertain switched nonlinear cascade systems with external disturbances. Sufficient conditions for exponential stability and the weighted L_2-gain property are proposed. The switched state feedback controller is designed to achieve global exponential stability with the weighted L_2-gain property. Moreover, the average dwell-time and the state decay have been calculated explicitly.

Due to the difficulty of coping with the duel design of continuous controllers and switching laws, the matching condition is adopted to simplify the analysis and design. How to remove the matching condition is a challenging issue that deserves further study for switched systems.

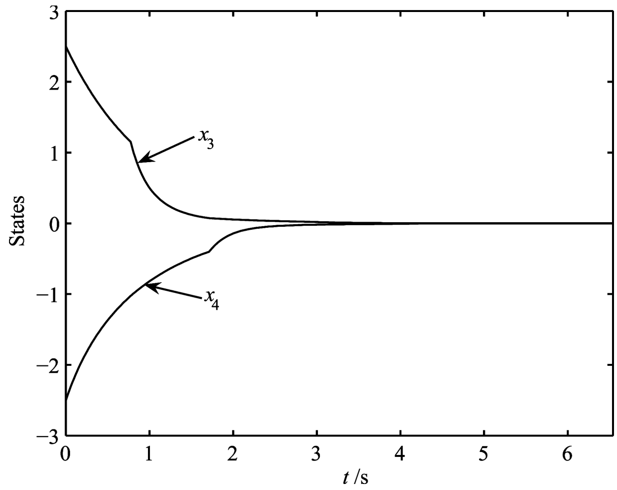

Fig. 3 The state responses of the nonlinear part

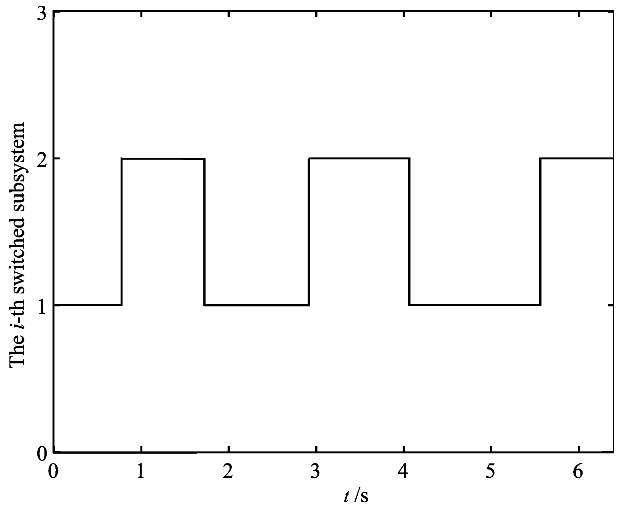

Fig. 4 The switching signal for the nonlinear part

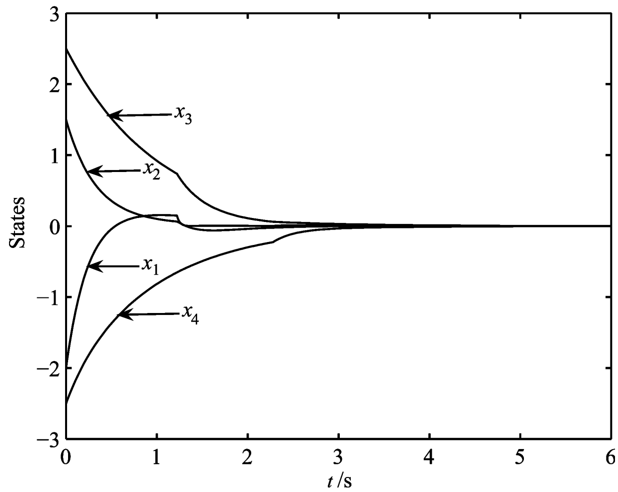

Fig. 5 The state responses of the switched system

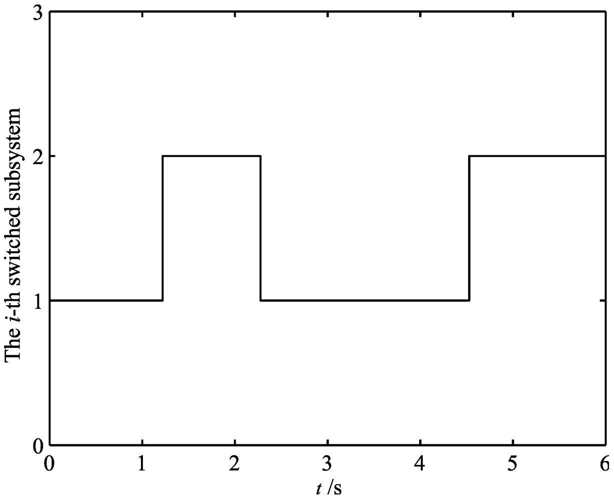

Fig. 6 The switching signals for the switched system

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