Fault Detection Observer Design in Low Frequency Domain for Linear Time-delay Systems

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Abstract This paper deals with the robust fault detection (FD) problem in low frequency domain for linear time-delay systems. The H_∞ norm and H_- index are used to measure the robustness to unknown inputs and the sensitivity to faults, respectively. The main results include derivation of a sufficient condition for the existence of a robust FD observer and its construction based on the linear matrix inequality (LMI) solution parameters. Finally, numerical simulations show the effectiveness of the presented methodology.

Key words Fault detection (FD), time-delay system, low frequency range, linear matrix inequality (LMI)

The research and application of robust fault detection (FD) in automated processes have received considerable attention during last decades and a great number of results have been achieved[1−4]. The main challenge in robust FD is to distinguish faults from other disturbances. There have been a number of results using robust control theory to solve this problem, e.g., the H_∞/H_∞ approach[4], multi-objective H_∞ approach[5], H_∞ filter approach[6], robust fault estimation schemes[7−9], and recently developed H_-/H_∞ approach[10−14].

However, most of those works considered the whole frequency spectrum. In practice, however, faults usually emerge in the low frequency domain, e.g., for an incipient signal, the fault information is contained within a low frequency band as the fault development is slow[1], and the actuator stuck failures that occur in the flight control systems just belong to the low frequency domain[15]. This motivated the FD observer design for linear time-invariant (LTI) systems in low frequency[16−17].

On the other hand, time delays are frequently encountered in industry and are often the source of performance degradation of a system. So, this paper focuses on the FD observer design problem in the low frequency domain for linear time-delay systems with unknown inputs.

The proposed design methodology of this paper is based on the following idea: by combining the new results in [18] and H_-/H_∞ observer approach, the FD problem is converted into a detection observer design problem in the low frequency domain, and LMI-based design conditions are then derived.

The following notations are used throughout this paper. For a matrix A, A^* denotes its complex conjugate transpose. The Hermitian part of a square matrix A is denoted by $He(A) = A + A^*$. The symbol H_n stands for the set of $n \times n$ Hermitian matrices. I denotes the identity matrix with an appropriate dimension. For matrices Φ and P, $\Phi \otimes P$ means the Kronecker product. For matrices $G \in \mathbf{C}^{n\times m}$ and $\Pi \in H_{n+m}$, a function $\sigma : \mathbf{C}^{n\times m} \times H_{n+m} \rightarrow H_m$ is defined by

$$\sigma(G,\Pi) = \left[\begin{array}{c} G \\ I_m \end{array}\right]^* \Pi \left[\begin{array}{c} G \\ I_m \end{array}\right] \tag{1}$$

1 Problem formulation and preliminaries

1.1 Problem formulation

Definition 1[17]**.** The H_- index of a transfer function matrix $G(s)$ is defined as

$$\|G(s)\|_-^\Omega = \inf_{\omega\in\Omega} \underline{\sigma}[G(\mathrm{j}\omega)] \tag{2}$$

where $\underline{\sigma}$ denotes the minimum singular value, Ω is a subset of real numbers as shown in Table I in [17].

Faults considered in this paper are assumed to be in the low frequency domain, i.e., $\omega \in \Omega = [-\varpi, \varpi]$, where ϖ is a positive scalar.

In this paper, we consider the following linear time-delay systems:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + A_d\boldsymbol{x}(t-\tau) + B_f\boldsymbol{f}(t) + B_d\boldsymbol{d}(t) \\ \boldsymbol{y}(t) &= C\boldsymbol{x}(t) + D_f\boldsymbol{f}(t) + D_d\boldsymbol{d}(t) \end{aligned} \tag{3}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the state space vector, $\boldsymbol{y}(t) \in \mathbf{R}^{n_y}$ denotes the measurement output vector, $\boldsymbol{d}(t) \in \mathbf{R}^{n_d}$ is the unknown input vector satisfying $\boldsymbol{d}(t) \in L_2$, $\boldsymbol{f}(t) \in \mathbf{R}^{n_f}$ denotes the fault to be detected. A, A_d, B_f, B_d C, D_f, and D_d are known matrices with appropriate dimensions and τ is a known constant time-delay. Without loss of generality, we assume (A, C) to be observable and omit the control input.

We propose to use the following FD observer:

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}}(t) &= A\hat{\boldsymbol{x}}(t) + A_d\hat{\boldsymbol{x}}(t-\tau) + H(\boldsymbol{y} - \hat{\boldsymbol{y}}) \\ \hat{\boldsymbol{y}}(t) &= C\hat{\boldsymbol{x}}(t) \\ \boldsymbol{r}(t) &= \boldsymbol{y} - \hat{\boldsymbol{y}} \end{aligned} \tag{4}$$

where $\hat{\boldsymbol{x}}(t) \in \mathbf{R}^n$ and $\hat{\boldsymbol{y}}(t) \in \mathbf{R}^{n_y}$ represent the state and output estimation vectors, respectively. $\boldsymbol{r}(t) \in \mathbf{R}^{n_r}$ is the so-called residual signal. The design parameter is observer gain matrix H.

Remark 1. The disturbances considered in this paper are assumed to be in the same frequency range as that of faults because disturbances that belong to the high frequency domain can be decoupled by designing a low-pass filter after the residual outputs.

Denoting $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$ and augmenting the model of system (3) to include the states of FD observer (4), we obtain the following augmented system:

$$\begin{aligned} \dot{\boldsymbol{e}}(t) &= \bar{A}\boldsymbol{e}(t) + A_d\boldsymbol{e}(t-\tau) + \bar{B}_d\boldsymbol{d}(t) + \bar{B}_f\boldsymbol{f}(t) \\ \boldsymbol{r}(t) &= C\boldsymbol{e}(t) + D_d\boldsymbol{d}(t) + D_f\boldsymbol{f}(t) \end{aligned} \tag{5}$$

where $\bar{A} = A - HC$, $\bar{B}_d = B_d - HD_d$, and $\bar{B}_f = B_f - HD_f$.

Then, the FD observer design problem can now be formulated as follows:

1) System (5) is asymptotically stable;
2) $\| G_{rf}(\mathrm{j}\omega) \|_-^{[-\varpi,\varpi]} > \beta_1$;
3) $\| G_{rd}(\mathrm{j}\omega) \|_\infty^{[-\varpi,\varpi]} < \beta_2$, where

$$G_{rf}(s) = C(sI - \bar{A} - \mathrm{e}^{-ds}A_d)^{-1}\bar{B}_f + D_f \tag{6}$$

$$G_{rd}(s) = C(sI - \bar{A} - \mathrm{e}^{-ds}A_d)^{-1}\bar{B}_d + D_d \tag{7}$$

Received April 29, 2008; in revised form October 27, 2008
Supported by the State Key Program of National Natural Science of China (60534010), the National Basic Research Program of China (973 Program) (2009CB320604), the National Natural Science foundation of China (60674021, 60804024, 60974043), the Funds for Creative Research Groups of China (60821063), the 111 Project (B08015), and Research Fund for the Doctoral Program of Higher Education of China (20060145019)
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DOI: 10.3724/SP.J.1004.2009.01465

and β_1, β_2 are two given positive scalars.

1.2 Preliminaries

In this subsection, some useful lemmas are given.

We introduce the main results of [18]. Given a linear time-delay system

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + A_d\boldsymbol{x}(t-\tau) + B\boldsymbol{d}(t)\\ \boldsymbol{y}(t) &= C\boldsymbol{x}(t) + D\boldsymbol{d}(t)\end{aligned} \tag{8}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the state space vector, $\boldsymbol{y}(t) \in \mathbf{R}^{n_y}$ denotes the measurement output vector, and $\boldsymbol{d}(t) \in \mathbf{R}^{n_\varpi}$ is the disturbance input vector. A, A_d, B, C, and D are known matrices with appropriate dimensions, and τ is a constant time-delay. The transfer function matrix $G(\lambda)$ from $\boldsymbol{d}$ to $\boldsymbol{y}$ is denoted by

$$G(s) = C(sI - A - \mathrm{e}^{-\tau s}A_d)^{-1}B + D \tag{9}$$

Given a Hermitian matrix Π, the specification can be described by

$$\sigma(G(\lambda), \Pi) < 0, \quad \forall\lambda \in \bar{\Lambda}(\Phi, \Psi) \tag{10}$$

where

$$\Lambda(\Phi, \Psi) = \{\lambda \in \mathbf{C} | \sigma(\lambda, \Phi) = 0, \sigma(\lambda, \Psi) \geq 0\} \tag{11}$$

and $\bar{\Lambda} = \Lambda$ if Λ is bounded, $\bar{\Lambda} = \Lambda \bigcup \{\infty\}$ if is unbounded.

Lemma 1[18]**.** Let matrices $A \in \mathbf{C}^{n\times n}$, $A_d \in \mathbf{C}^{n\times n}$, $B \in \mathbf{C}^{n\times n_{\bar{\omega}}}$, $C \in \mathbf{C}^{n_y\times n}$, $D \in \mathbf{C}^{n_y\times n_{\bar{\omega}}}$, $\Pi \in H_{n_y+n_{\bar{\omega}}}$, and $\Phi, \Psi \in H_2$ be given and define Λ by (11). Suppose Λ represents curves on the complex plane. Then $\sigma(G(\lambda), \Pi) < 0$ holds for all $\lambda \in \bar{\Lambda}(\Phi, \Psi)$ if there exist $P = P^*$, $Q = Q^* > 0$, and $X = X^* > 0$ such that

$$\begin{bmatrix} A & B & A_d \\ I & 0 & 0\end{bmatrix}^*(\Phi\otimes P + \Psi \otimes Q)\begin{bmatrix} A & B & A_d \\ I & 0 & 0\end{bmatrix} + \begin{bmatrix} \begin{bmatrix} C & D \\ 0 & I\end{bmatrix}^* \Pi \begin{bmatrix} C & D \\ 0 & I\end{bmatrix} + \begin{bmatrix} X & 0 \\ 0 & 0\end{bmatrix} & 0 \\ 0 & -X\end{bmatrix} < 0 \tag{12}$$

Remark 2. In the rest of this paper we choose $\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0\end{bmatrix}$ and $\Psi = \begin{bmatrix} -1 & 0 \\ 0 & \varpi^2\end{bmatrix}$, then $\lambda \in \bar{\Lambda}(\Phi, \Psi)$ is equivalent to $\omega \in [-\varpi, \varpi]$, where $\lambda = \mathrm{j}\omega$.

Furthermore, for the later development, the following Lemmas are required.

Lemma 2 (Finsler's Lemma)**.** Let $\boldsymbol{\xi} \in \mathbf{C}^n$, $\mathcal{P} \in \mathbf{C}^{n\times n}$ and $\mathcal{H} \in \mathbf{C}^{n\times m}$. Let $\mathcal{H}^\perp$ be any matrix such that $\mathcal{H}^\perp\mathcal{H} = 0$. The following statements are equivalent:

1) $\boldsymbol{\xi}^*\mathcal{P}\boldsymbol{\xi} < 0$, $\forall \mathcal{H}^*\boldsymbol{\xi} = 0$, $\boldsymbol{\xi} \neq 0$;
2) $\mathcal{H}^\perp\mathcal{P}\mathcal{H}^{\perp^*} < 0$;
3) $\exists\mu \in \mathbf{R} : \mathcal{P} - \mu\mathcal{H}\mathcal{H}^* < 0$;
4) $\exists\mathcal{X} \in \mathbf{R}^{m\times n} : \mathcal{P} + \mathcal{H}\mathcal{X} + \mathcal{X}^*\mathcal{H}^* < 0$

Lemma 3 (Elimination Lemma)**.** Let Γ, Λ, and $\Theta = \Theta^*$ be given matrices. There exists a matrix F to solve the matrix inequality

$$\Gamma F\Lambda + (\Gamma F\Lambda)^* + \Theta < 0$$

if and only if the following conditions are satisfied

$$\Gamma^\perp\Theta\Gamma^{\perp^*} < 0, \quad \text{and} \quad \Lambda^{*\perp}\Theta\Lambda^{*\perp^*} < 0$$

2 Main result

2.1 Fault sensitivity condition

In this section, the fault sensitivity condition is considered. Let $d(t) = 0$ in (5), we have

$$\begin{aligned}\dot{\boldsymbol{e}}(t) &= \bar{A}\boldsymbol{e}(t) + A_d\boldsymbol{e}(t-\tau) + \bar{B}_f\boldsymbol{f}(t)\\ \boldsymbol{r}(t) &= C\boldsymbol{e}(t) + D_f\boldsymbol{f}(t)\end{aligned} \tag{13}$$

If we choose $\Pi = \begin{bmatrix} -I & \\ & \beta_1^2 I\end{bmatrix}$ and Φ, Ψ as given in Remark 2, then for system (13), the performance (10) becomes

$$\| G_{rf}(\mathrm{j}\omega) \|_{-}^{[-\varpi,\varpi]} > \beta_1, \forall\omega \in [-\varpi, \varpi]$$

where $G_{rf}(s) = C(sI - \bar{A} - \mathrm{e}^{-ds}A_d)^{-1}\bar{B}_f + D_f$.

Theorem 1. For system (13), let a symmetric matrix $\Pi_1 = \begin{bmatrix} -I & \\ & \beta_1^2 I\end{bmatrix} \in \mathbf{R}^{(n_r+n_f)\times(n_r+n_f)}$ and Φ, Ψ, $\beta_1 > 0$ are given, then there is an FD observer satisfying $\| G_{rf}(j\omega) \|_{-}^{[-\varpi,\varpi]} > \beta_1$, if there exist $P_1 = P_1^*$, $Q_1 = Q_1^* > 0$, and $X_1 = X_1^* > 0$, W, V_{f1}, V_{f2}, and $\mathcal{K}$ such that the following inequality

$$T\begin{bmatrix} \Phi\otimes P_1 + \Psi\otimes Q_1 & 0 & 0 & 0 \\ 0 & \Pi_1 & 0 & 0 \\ 0 & 0 & X_1 & 0 \\ 0 & 0 & 0 & -X_1\end{bmatrix} T^* < He\begin{bmatrix} WR_1 \\ V_{f1} \\ V_{f2} \\ -A^*WR_1 + C^*\mathcal{K}R_1 - V_{f1} - C^*V_{f2} - \\ B_f{}^*WR_1 + D_f^*\mathcal{K}R_1 - D_f^*V_{f2} - \\ A_d^*WR_1\end{bmatrix} \tag{14}$$

holds with the assumption that $R_1 \in \mathbf{C}^{n\times(4n+n_f+n_r)}$ satisfies

$$YT\begin{bmatrix} \Phi\otimes P_1 + \Psi\otimes Q_1 & 0 & 0 & 0 \\ 0 & \Pi_1 & 0 & 0 \\ 0 & 0 & X_1 & 0 \\ 0 & 0 & 0 & -X_1\end{bmatrix} T^*Y^* - \mu_1 YR_1^*R_1Y^* < 0 \tag{15}$$

$$Y = \begin{bmatrix} A^* - C^*H^* & I & C^* & I & 0 & 0 \\ B_f^* - D_f^*H^* & 0 & D_f^* & 0 & I & 0 \\ A_d^* & 0 & 0 & 0 & 0 & I \\ I & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{16}$$

where $\mu_1 > 0$ is a real scalar and T is the permutation matrix such that

$$[M_1, M_2, M_3, M_4, M_5, M_6]T = [M_1, M_2, M_3, M_5, M_4, M_6] \tag{17}$$

for arbitrary matrices M_1, M_2, M_3, M_4, M_5, and M_6 with column dimensions n, n, n_r, n, n_f and n, respectively. The observer gain matrix is given by

$$\mathcal{K} = H^*W \tag{18}$$

Proof. By Lemma 1, the performance $\| G_{rf}(j\omega) \|_{-}^{[-\varpi,\varpi]} > \beta_1$ is satisfied if the following inequality

$$\begin{bmatrix} \Xi & I \end{bmatrix} T\Delta T^* \begin{bmatrix} \Xi & I \end{bmatrix}^* < 0 \tag{19}$$

$$\Xi = \begin{bmatrix} A^* - C^*H^* & I & C^* \\ B_f^* - D_f^*H^* & 0 & D_f^* \\ A_d^* & 0 & 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \Phi \otimes P_1 + \Psi \otimes Q_1 & 0 & 0 & 0 \\ 0 & \Pi_1 & 0 & 0 \\ 0 & 0 & X_1 & 0 \\ 0 & 0 & 0 & -X_1 \end{bmatrix} \tag{20}$$

holds, where T is defined by (17).

We let $\mathcal{P} = T\Delta T^*$, $\mathcal{H}^{\perp^*} = \begin{bmatrix} \Xi & I \end{bmatrix}^*$, and $\mathcal{H} = \begin{bmatrix} I \\ -\Xi \end{bmatrix}$. Then, condition (19) is equivalent to 2) of Lemma 2 and the following inequality

$$T \begin{bmatrix} \Phi \otimes P_1 + \Psi \otimes Q_1 & 0 & 0 & 0 \\ 0 & \Pi_1 & 0 & 0 \\ 0 & 0 & X_1 & 0 \\ 0 & 0 & 0 & -X_1 \end{bmatrix} T^* < He\left(\begin{bmatrix} I \\ -\Xi \end{bmatrix} \mathcal{X}\right) \tag{21}$$

is equivalent to 4) of Lemma 2, where $\mathcal{X}$ is a multiplier. So, by Lemma 2, condition (19) is equivalent to condition (21).

However, in Lemma 2, we should notice the equivalence between 4) and other items needs that the structure of $\mathcal{X}$ in 4) has no constraint; once we add an additional constraint to $\mathcal{X}$, 4) will be a sufficient condition for other items.

To make the problem tractable, similar to that of [19], we restrict the class of multiplier $\mathcal{X}$ to be

$$\mathcal{X} = \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} WR_1 + \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} V_f \tag{22}$$

where $W \in \mathbf{C}^{n\times n}$, $\det(W) \neq 0$, $V_f \in \mathbf{C}^{(n+n_r)\times(4n+n_f+n_r)}$ and $R_1 \in \mathbf{C}^{n\times(4n+n_f+n_r)}$ is a multiplier to be chosen.

Then, (21) will be held if

$$T \begin{bmatrix} \Phi \otimes P_1 + \Psi \otimes Q_1 & 0 & 0 & 0 \\ 0 & \Pi_1 & 0 & 0 \\ 0 & 0 & X_1 & 0 \\ 0 & 0 & 0 & -X_1 \end{bmatrix} T^* < He\left(\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -A^* + C^*H^* & -I & -C^* \\ -B_f^* + D_f^*H^* & 0 & -D_f^* \\ -A_d^* & 0 & 0 \end{bmatrix} \begin{bmatrix} WR_1 \\ V_f \end{bmatrix}\right) \tag{23}$$

holds. Defining $\mathcal{K} = H^*W$ and $V_f = \begin{bmatrix} V_{f1} \\ V_{f2} \end{bmatrix}$, with some matrix manipulations, we have that (23) is equivalent to (14), so we can conclude that (14) provides a sufficient condition for performance index $\| G_{rf}(j\omega) \|_{-}^{[-\varpi,\varpi]} > \beta_1$. □

Remark 3. As pointed out in [18 – 19], we can choose R_1 to satisfy (15) with Y defined by (16). If R_1 is given, condition (14) is an LMI in P_1, Q_1, X_1, W, V_{f1}, V_{f2}, and $\mathcal{K}$.

Remark 4. By the condition (17), we can get

$$T = \begin{bmatrix} I_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & I_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_6 \end{bmatrix}$$

where $I_n (n = 1, 2, \cdots, 6)$ denote identity matrices with appropriate dimensions.

2.2 Robustness condition

Here, we study the robustness requirement of system (5). Let $\boldsymbol{f}(t) = 0$ in (5), we have

$$\begin{aligned} \dot{\boldsymbol{e}}(t) &= \bar{A}\boldsymbol{e}(t) + A_d\boldsymbol{e}(t-\tau) + \bar{B}_d\boldsymbol{d}(t) \\ \boldsymbol{r}(t) &= C\boldsymbol{e}(t) + D_d\boldsymbol{d}(t) \end{aligned} \tag{24}$$

To attenuate the disturbance influence, we give the following theorem.

Theorem 2. For system (24), let a symmetric matrix $\Pi_2 = \begin{bmatrix} I & \\ & -\beta_2^2 I \end{bmatrix} \in \mathbf{R}^{(n_r+n_d)\times(n_r+n_d)}$ and $\Phi, \Psi, \beta_2 > 0$ are given, then there is an FD observer satisfying $\| G_{rd}(\mathrm{j}\omega) \|_{\infty}^{[-\varpi,\varpi]} < \beta_2$, if there exist $P_2 = P_2^*$, $Q_2 = Q_2^* > 0$, and $X_2 = X_2^* > 0$, W, V_{d1}, V_{d2}, and $\mathcal{K}$ such that the following inequality

$$T \begin{bmatrix} \Phi \otimes P_2 + \Psi \otimes Q_2 & 0 & 0 & 0 \\ 0 & \Pi_2 & 0 & 0 \\ 0 & 0 & X_2 & 0 \\ 0 & 0 & 0 & -X_2 \end{bmatrix} T^* < He \begin{bmatrix} WR_2 \\ V_{d1} \\ V_{d2} \\ -A^*WR_2 + C^*\mathcal{K}R_2 - V_{d1} - C^*V_{d2} \\ -B_d{}^*WR_2 + D_d^*\mathcal{K}R_2 - D_d^*V_{d2} \\ -A_d^*WR_2 \end{bmatrix} \tag{25}$$

holds with the assumption that $R_2 \in \mathbf{C}^{n\times(4n+n_{\boldsymbol{d}}+n_{\boldsymbol{r}})}$ satisfies

$$YT \begin{bmatrix} \Phi \otimes P_2 + \Psi \otimes Q_2 & 0 & 0 & 0 \\ 0 & \Pi_2 & 0 & 0 \\ 0 & 0 & X_2 & 0 \\ 0 & 0 & 0 & -X_2 \end{bmatrix} T^*Y^* - \mu_2 Y R_2^* R_2 Y^* < 0 \tag{26}$$

$$Y = \begin{bmatrix} A^* - C^*H^* & I & C^* & I & 0 & 0 \\ B_d^* - D_d^*H^* & 0 & D_d^* & 0 & I & 0 \\ A_d^* & 0 & 0 & 0 & 0 & I \\ I & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

where $\mu_2 > 0$ is a real scalar and T is defined by (17). And the observer gain matrix is given by

$$\mathcal{K} = H^*W \tag{28}$$

Proof. The proof is similar to Theorem 1, so it is omitted.

2.3 Stability condition

Conditions (14) and (25) do not ensure a stable observer, so we wish to add an additional constraint to guarantee the stability of system (5).

Lemma 4. System (5) is asymptotically stable if there exist matrices $W, \mathcal{K}, P_3 = P_3^* > 0$, and $X_3 = X_3^* > 0$ such that

$$\begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} (\Phi \otimes P_3) \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}^* + \begin{bmatrix} 0 & 0 & 0 \\ 0 & X_3 & 0 \\ 0 & 0 & -X_3 \end{bmatrix} < He\left(\begin{bmatrix} W \\ -A^*W + C^*\mathcal{K} \\ -A_d^*W \end{bmatrix} \begin{bmatrix} -qI & pI & 0 \end{bmatrix} \right) \tag{29}$$

where $\boldsymbol{r} = [p^* \quad q^*] \in \mathbf{C}^2$ is an arbitrary fixed vector satisfying $\boldsymbol{r}\Phi\boldsymbol{r}^* < 0$.

Proof. From the Lyaponov stability conditions for time-delay systems, system (5) is stable if there exist symmetric matrices $P_3 > 0$ and $X_3 > 0$ such that

$$\begin{bmatrix} \bar{A} & A_d \\ I & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} 0 & P_3 & 0 \\ P_3 & X_3 & 0 \\ 0 & 0 & -X_3 \end{bmatrix} \begin{bmatrix} \bar{A} & A_d \\ I & 0 \\ 0 & I \end{bmatrix} < 0 \tag{30}$$

Notice that $\begin{bmatrix} \bar{A}^* & I & 0 \\ A_d^* & 0 & I \end{bmatrix}$ is the null space of $\begin{bmatrix} -I \\ \bar{A}^* \\ A_d^* \end{bmatrix}$. According to Lemma 3, (30) will be held if the following inequality

$$\begin{bmatrix} 0 & P_3 & 0 \\ * & X_3 & 0 \\ * & * & -X_3 \end{bmatrix} < He \begin{bmatrix} -I \\ \bar{A}^* \\ A_d^* \end{bmatrix} WR \tag{31}$$

holds, where W is chosen as that in Theorem 1 and Theorem 2, and $R = \begin{bmatrix} -qI & pI & 0 \end{bmatrix}$, where $r = [p^* \quad q^*] \in \mathbf{C}^2$ is an arbitrary fixed vector satisfying $\boldsymbol{r}\Phi\boldsymbol{r}^* < 0$. If $\mathcal{K} = H^*W$, then Lemma 4 is completed. □

Remark 5. If we choose $\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\Psi = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then we get the LMI conditions for fault detection in the full frequency domain.

2.4 Detection observer design

By combining Theorems 1 and 2, and Lemma 4, conditions 1)∼3) given in Section 1 will be satisfied if LMIs (14), (25), and (29) hold simultaneously.

Theorem 3. System (5) is asymptotically stable and conditions $\| G_{rf}(\mathrm{j}\omega) \|_-^{[-\varpi,\varpi]} > \beta_1$, $\| G_{rd}(\mathrm{j}\omega) \|_\infty^{[-\varpi,\varpi]} < \beta_2$ are satisfied if there exist symmetric matrices P_1, P_2, $P_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $X_1 > 0$, $X_2 > 0$, $X_3 > 0$, and matrices W, V_{f1}, V_{f2}, V_{d1}, V_{d2}, $\mathcal{K}$ such that (14), (25) and (29) hold, where $\mathcal{K} = H^*W$.

Given $\beta_2 > 0$, the observer gain matrix H can be determined through the following optimization:

$$\begin{aligned} &\max \quad \beta_1 \\ &\text{s.t. } (14), (25), (29) \end{aligned} \tag{32}$$

3 Numerical simulations

Consider a linear time-delay system of the form in (3) with the following parameters

$$A = \begin{bmatrix} -0.9231 & 0.5422 \\ -0.9442 & -0.6764 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.4141 \\ -0.3287 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0.6264 & -0.7227 \\ 0.0117 & -0.1610 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.2093 \\ 0.1224 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5432 & 0.4595 \end{bmatrix}, \ D_f = 0.7525, \quad D_d = 0.0834$$

The frequency range is restricted in $(-0.01, 0.01)$. We let

$$q = -1, \ p = 1, \ \beta_2 = 0.2$$

$$R_1 = \begin{bmatrix} I_2 & I_2 & R_{13} & I_2 & R_{15} & 0 \end{bmatrix}$$

$$R_{13} = \begin{bmatrix} -4 \\ 0 \end{bmatrix}, \quad R_{15} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} R_{21} & R_{22} & 0 & I_2 & 0 & 0 \end{bmatrix}$$

$$R_{21} = R_{22} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

and T be given in Remark 4.

Solving the optimization problem (32), we obtain the observer gain matrix $H_{\text{low}} = \begin{bmatrix} 0.6557 \\ -0.2478 \end{bmatrix}$ and $\beta_{1\text{optlow}} = 0.5297$. The actual achieved value of β_1 in the low frequency domain is 0.6328.

In the full frequency domain, the observer gain matrix $H_{\text{full}} = \begin{bmatrix} 1.3882 \\ 0.2243 \end{bmatrix}$ and $\beta_{1\text{optfull}} = 0.3578$. The actual achieved value of β_1 in the full frequency domain is 0.3744.

To show the effectiveness of our method more clearly, some simulations are also given. The system is assumed to be affected by stuck faults such that $f(t) = 5$, $t \geq 6\,\text{s}$, and $f(t) = 0$ elsewhere.

As clearly seen from Fig. 1, the residual is more sensitive to faults in the low frequency domain than in the full range.

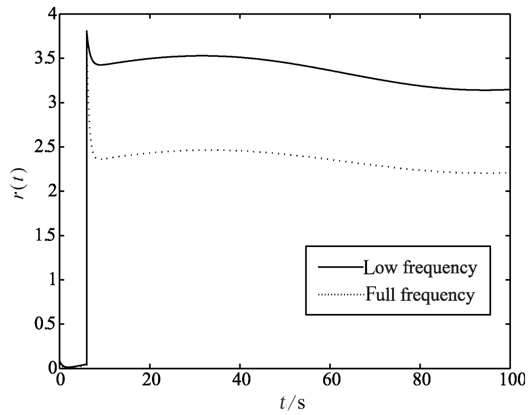

Fig. 1 The residual outputs with the disturbance $d(t) = \sin(0.05t)$

After designing the FD observer, the important task is the evaluation of the generated residual. A threshold J_{th} and residual evaluation function $J_{\boldsymbol{r}}$ can be determined by

$$J_{\boldsymbol{r}} = \sqrt{\frac{1}{t}\int_0^t \boldsymbol{r}^{\mathrm{T}}(\tau)\boldsymbol{r}(\tau)\mathrm{d}\tau}$$

$$J_{\text{th}} = \sup_{f=0,\, d \in L_2,\, \omega \in (-0.01, 0.01)} J_{\boldsymbol{r}}$$

Based on this, the occurrence of faults can be detected by the following logic rules:

$$J_r > J_{\text{th}} \Rightarrow \text{with faults} \Rightarrow \text{alarm}$$
$$J_r \leq J_{\text{th}} \Rightarrow \text{no faults}$$

Using Matlab, we can obtain $J_{\text{th}} = 0.1469$. The residual evaluation function J_r and threshold J_{th} are reported in Figs. 2 and 3. From Fig. 2, we can conclude that faults can be effectively detected by using finite frequency FD observer and Fig. 3 illustrates that the finite frequency FD observer can receive better results than the full frequency FD observer.

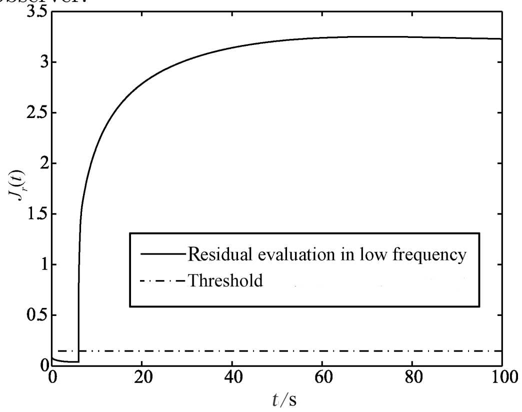

Fig. 2 The residual evaluation and the threshold

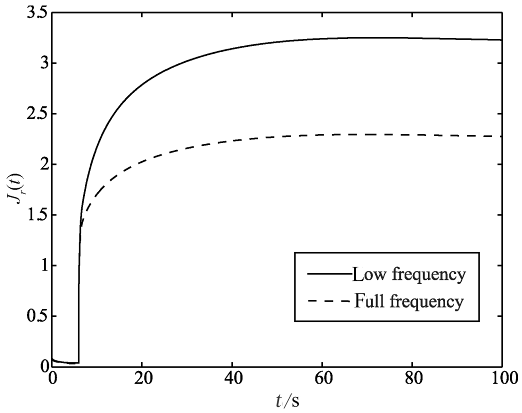

Fig. 3 The residual evaluation

4 Conclusion

In this paper, we have investigated the problem of FD for linear time-delay systems in the low frequency domain. The H_∞ norm and H_- index have been used to measure the robustness to unknown inputs and the sensitivity to fault, respectively. A design method has been presented in terms of solutions to a set of LMIs and a numerical example has been given to illustrate the effectiveness of the proposed method.

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