Primary vertex reconstruction based on the Kalman filter technique at BESⅢ*

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Abstract Primary vertex reconstruction is crucial to estimate the beam profile in collision experiments. We study the principle of an iterative process, called the Kalman filter method, and apply it to primary vertex reconstruction at BESⅢ. A Newton procedure to find the zero point of the distance function's gradient is used for primary vertex finding in 3-dimensional space. Results are obtained based on raw data at BESⅢ.

Key words primary vertex, Kalman method, BESⅢ

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1 Introduction

The purpose of primary vertex reconstruction is to determine the interaction points of events, and then further determine the average of the beam profile in high energy physics experiments. Information on a general interaction point is a critical parameter for reconstruction of secondary particles decaying from that point. In addition, it is also a key parameter in the realization of a Monte Carlo simulation since the consistency of the Monte Carlo simulation and raw collision data in physical analysis is very important.

Beijing Spectrometer (BES)Ⅲ [1, 2] has been updated for the Beijing Electron and Positron Collider (BEPC)Ⅱ in the τ-charm region. Exciting raw collision data have been collected at a high luminosity since July 2008. The BES Offline Software System (BOSS) [3] has successfully served the raw data processing.

Two algorithms, the Kalman and global methods [4, 5], are provided for primary and secondary vertex fitting at BESⅢ. The Kalman method is applied to precisely determine the parameters of the beam position, while the global method is generally used for physics analysis. This paper discusses the Kalman method and its application for primary vertex reconstruction. With the enhancement of raw collision data, the primary vertex reconstruction from vertex constraints is essential to check the performance of detector hardware and software of subsystems. It also contributes to the criteria for selecting good charged tracks for physics analysis.

2 Algorithm: the Kalman filter method

The Kalman filter method [6, 7] aims at obtaining the optimal estimation of an unknown variable using known measurements. It provides the smallest possible variance among all linear least-squares estimators if the linear model with Gaussian errors is applicable. In filter language, the parameters to be estimated are called "state vectors". Initially, the state vector consists only of prior information about the vertex position $\boldsymbol{x}_0$, and its covariance matrix C_0. $\boldsymbol{x}_0$ can be set to an arbitrary value and C_0^{-1} is set to zero if

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there is no prior information. For each track k, the state vector is augmented by the 3-momentum vector $\boldsymbol{p}_k$. Without process noise, the "system equation" is simply the identity

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} = \boldsymbol{x}.$$

The functional dependence of the track parameters on the state vector is determined by the equation of motion (track model), and is described by the "measurement equation", which in general is nonlinear,

$$\boldsymbol{q}_k = \tilde{\boldsymbol{\alpha}}(\boldsymbol{x},\boldsymbol{p}_k) + \boldsymbol{\epsilon}_k, \quad \mathrm{cov}(\boldsymbol{\epsilon}_k) = V_k.$$

If there is multiple scattering between the vertex and the reference surface, it has to be included as additional fluctuation in V_k. A linear regression model is obtained by approximating $\tilde{\boldsymbol{\alpha}}$ by a first order Taylor expansion at some point $(\boldsymbol{x}_\mathrm{e},\boldsymbol{p}_\mathrm{e})$,

$$\begin{aligned}\tilde{\boldsymbol{\alpha}}(\boldsymbol{x},\boldsymbol{p}) &\approx \tilde{\boldsymbol{\alpha}}_\mathrm{e}(\boldsymbol{x}_\mathrm{e},\boldsymbol{p}_\mathrm{e}) + A(\boldsymbol{x}-\boldsymbol{x}_\mathrm{e}) + B(\boldsymbol{p}-\boldsymbol{p}_\mathrm{e}) = \\ &\boldsymbol{c}_\mathrm{e} + A\boldsymbol{x} + B\boldsymbol{p},\end{aligned}$$

with $A = \partial\tilde{\boldsymbol{\alpha}}/\partial\boldsymbol{x}|_{\boldsymbol{x}=\boldsymbol{x}_\mathrm{e},\boldsymbol{p}=\boldsymbol{p}_\mathrm{e}}$ and $B = \partial\tilde{\boldsymbol{\alpha}}/\partial\boldsymbol{p}|_{\boldsymbol{x}=\boldsymbol{x}_\mathrm{e},\boldsymbol{p}=\boldsymbol{p}_\mathrm{e}}$ being the two $(n\times3)$ matrices of derivatives of $(\boldsymbol{x}_\mathrm{e},\boldsymbol{p}_\mathrm{e})$, where n is the number of track parameters.

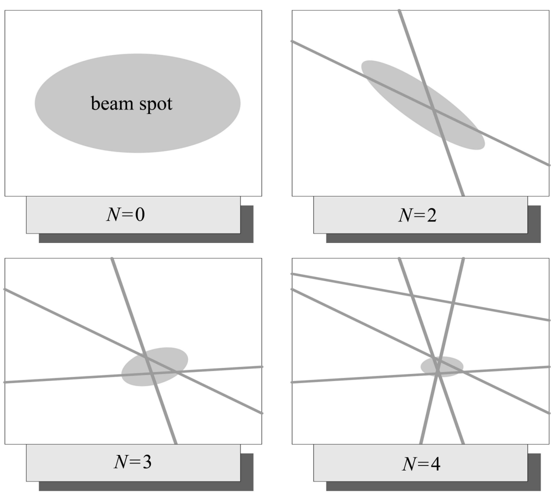

Fig. 1. The schematic of adding a new track into an updated vertex.

Vertex reconstruction using the Kalman filter method is characterized of updating the vertex position and its covariance matrix step by step through adding a new track k (measured by $\boldsymbol{\alpha}_{0k}$). Fig. 1 shows the schematic of updating the vertex by adding new tracks in the filtering procedure. It includes three types of process: filtering, prediction and smoothing. In terms of "time", prediction is the estimation of the state vector at a "future" time; filtering is the estimation of the "present" state vector, based upon all "past" measurements; smoothing is the estimation of the state vector at some time in the "past" based on all measurements taken up to the "present" time.

The χ^2 in the Kalman filter method can be written as a sum of two terms. The total χ^2 of the fit is equal to the sum of the χ^2 in all filter steps.

$$\begin{aligned}\chi^2_\mathrm{KF} &= (\boldsymbol{x}_k-\boldsymbol{x}_{k-1})^\mathrm{T}C^{-1}_{k-1}(\boldsymbol{x}_k-\boldsymbol{x}_{k-1}) + \\ &(\boldsymbol{\alpha}_{0k}-\tilde{\boldsymbol{\alpha}}_k)^\mathrm{T}G_k(\boldsymbol{\alpha}_{0k}-\tilde{\boldsymbol{\alpha}}_k),\end{aligned} \quad (1)$$

where $\tilde{\boldsymbol{\alpha}}_k = \boldsymbol{c}_{\mathrm{e}k} + A_k\boldsymbol{x}_k + B_k\boldsymbol{p}_k$. Minimizing the χ^2 with respect to $\boldsymbol{x}_k$ and $\boldsymbol{p}_k$ yields two vector equations which can be solved for the parameters $\boldsymbol{x}_k$, $\boldsymbol{p}_k$ and their covariance matrices. The results are as follows:

$$\begin{aligned}\boldsymbol{x}_k &= C_k\left[C^{-1}_{k-1}\boldsymbol{x}_{k-1} + A^\mathrm{T}_kG^\mathrm{B}_k(\boldsymbol{\alpha}_{0k}-\boldsymbol{c}_{\mathrm{e}k})\right], \\ \boldsymbol{p}_k &= W_kB^\mathrm{T}_kG_k(\boldsymbol{\alpha}_{0k}-\boldsymbol{c}_{\mathrm{e}k}-A_k\boldsymbol{x}_k), \\ \mathrm{cov}(\boldsymbol{x}_k) &= C_k = (C^{-1}_{k-1} + A^\mathrm{T}_kG^\mathrm{B}_kA_k)^{-1}, \\ \mathrm{cov}(\boldsymbol{p}_k) &= D_k = W_k + W_kB^\mathrm{T}_kG_kA_kC_kA^\mathrm{T}_kG_kB_kW_k = \\ &W_k + E^\mathrm{T}_kC^{-1}_kE_k, \\ \mathrm{cov}(\boldsymbol{x}_k,\boldsymbol{p}_k) &= E_k = -C_kA^\mathrm{T}_kG_kB_kW_k,\end{aligned} \quad (2)$$

with

$$\begin{aligned}G_k &= V^{-1}_k, \\ W_k &= (B^\mathrm{T}_kG_kB_k)^{-1}, \\ G^\mathrm{B}_k &= G_k - G_kB_kW_kB^\mathrm{T}_kG_k.\end{aligned} \quad (3)$$

Since there is no process noise in the vertex fit, the smoother is extremely simple. The momentum vectors and covariance matrices with the final estimate of the vertex position can be recomputed:

$$\begin{aligned}\boldsymbol{x}_n &= \boldsymbol{x}_k, \quad C_n = C_k, \\ \boldsymbol{p}^n_k &= W_kB^\mathrm{T}_kG_k(\boldsymbol{\alpha}_{0k}-\boldsymbol{c}_{\mathrm{e}k}-A_k\boldsymbol{x}_n), \\ \mathrm{cov}(\boldsymbol{p}^n_k) &= D^n_k = W_k + W_kB^\mathrm{T}_kG_kA_kC_nA^\mathrm{T}_kG_kB_kW_k = \\ &W_k + E^{n\mathrm{T}}_kC^{-1}_nE^n_k, \\ \mathrm{cov}(\boldsymbol{x}_n,\boldsymbol{p}^n_k) &= E^n_k = -C_nA^\mathrm{T}_kG_kB_kW_k, \\ \mathrm{cov}(\tilde{\boldsymbol{\alpha}}^n_k) &= A_kC_nA^\mathrm{T}_k + A_kE^n_kB^\mathrm{T}_k + \\ &(A_kE^n_kB^\mathrm{T}_k)^\mathrm{T} + B_kD^n_kB^\mathrm{T}_k.\end{aligned} \quad (4)$$

If there is a significant change in the smoothed vertex position, it may be worthwhile to recompute the derivative matrices A_k and B_k.

Suppose that only a few tracks originate possibly from a secondary vertex; the estimated position of the primary vertex has no noticeable bias. The filtered or smoothed residuals can be used to decide whether or not a particular track really does belong to the primary vertex. The residuals and their covariance matrices have the following forms:

$$
\begin{aligned}
\boldsymbol{r}_k &= \boldsymbol{\alpha}_{0k} - \boldsymbol{c}_{\mathrm{e}k} - A_k \boldsymbol{x}_k - B_k \boldsymbol{p}_k, \\
\mathrm{cov}(\boldsymbol{r}_k) &= R_k = V_k(G_k^{\mathrm{B}} - G_k^{\mathrm{B}} A_k C_k A_k^{\mathrm{T}} G_k^{\mathrm{B}}) V_k, \\
\boldsymbol{r}_k^n &= \boldsymbol{\alpha}_{0k} - \boldsymbol{c}_{\mathrm{e}k} - A_k \boldsymbol{x}_n - B_k \boldsymbol{p}_k^n, \\
\mathrm{cov}(\boldsymbol{r}_k^n) &= R_k^n = V_k(G_k^{\mathrm{B}} - G_k^{\mathrm{B}} A_k C_n A_k^{\mathrm{T}} G_k^{\mathrm{B}}) V_k.
\end{aligned} \tag{5}
$$

Since R_k and R_k^n are singular the filtered chi-square χ^2_{F} and the smoothed chi-square χ^2_{S} have to be computed in the following way:

$$
\begin{aligned}
\chi^2_{\mathrm{F}} &= \boldsymbol{r}_k^{\mathrm{T}} G_k \boldsymbol{r}_k + (\boldsymbol{x}_k - \boldsymbol{x}_{k-1})^{\mathrm{T}} C_{k-1}^{-1} (\boldsymbol{x}_k - \boldsymbol{x}_{k-1}), \\
\chi^2_{\mathrm{S}} &= \boldsymbol{r}_k^{n\mathrm{T}} G_k \boldsymbol{r}_k^n + (\boldsymbol{x}_n - \boldsymbol{x}_k^{n*})^{\mathrm{T}} (C_k^{n*})^{-1} (\boldsymbol{x}_n - \boldsymbol{x}_k^{n*}),
\end{aligned} \tag{6}
$$

where $\boldsymbol{x}_k^{n*}$ is the smoothed estimate $\boldsymbol{x}_n$ with the track $\boldsymbol{\alpha}_{0k}$ removed. It is obtained by the inverse Kalman filter:

$$
\begin{aligned}
C_k^{n*} &= (C_n^{-1} - A_k^{\mathrm{T}} G_k^{\mathrm{B}} A_k)^{-1}, \\
\boldsymbol{x}_k^{n*} &= C_k^{n*} \left[C_n^{-1} \boldsymbol{x}_n - A_k^{\mathrm{T}} G_k^{\mathrm{B}} (\boldsymbol{\alpha}_{0k} - \boldsymbol{c}_{\mathrm{e}k}) \right].
\end{aligned} \tag{7}
$$

If $\boldsymbol{\alpha}_{0k}$ belongs to the primary vertex, χ^2_{F} and χ^2_{S} are χ^2-distributed with 2 degrees of freedom.

3 Measurement equation and derivative matrices

At BESIII the helix is determined by a 5-component parameter vector $\boldsymbol{\alpha} = (d_\rho, \phi_0, \kappa, d_z, \lambda)^{\mathrm{T}}$, where d_ρ is the distance of the helix from the pivotal point (0,0,0) in the x-y plane, ϕ_0 is the azimuthal angle to specify the pivotal point with respect to the helix center, κ is the signed reciprocal transverse momentum, d_z is the distance of the helix from the pivot point in the z direction, and $\lambda = \cot\theta$, where θ is the polar angle measured from the $+z$ axis.

Assume that a particle has charge Q and is moving along a helix in a magnetic field of strength B. The trajectory of a helix is governed by the following equations, valid when B is along $\vec{z}$.

$$
\begin{aligned}
p_x &= p_{0x} \cos\rho s_\perp - p_{0y} \sin\rho s_\perp, \\
p_y &= p_{0y} \cos\rho s_\perp + p_{0x} \sin\rho s_\perp, \\
p_z &= p_{0z}, \\
E &= E_0, \\
x &= x_0 + \frac{p_{0x}}{a} \sin\rho s_\perp - \frac{p_{0y}}{a} (1 - \cos\rho s_\perp), \\
y &= y_0 + \frac{p_{0y}}{a} \sin\rho s_\perp + \frac{p_{0x}}{a} (1 - \cos\rho s_\perp), \\
z &= z_0 + \lambda s_\perp.
\end{aligned} \tag{8}
$$

where (x_0, y_0, z_0) is a known point on the helix, $(p_{0x}, p_{0y}, p_{0z}, E_0)$ is its 4-momentum vector there and $\rho = a/p_\perp$, $a = -cBQ$. They are functions of $s_\perp$, the arc length in the x-y plane from (x_0, y_0, z_0) to the point (x, y, z).

The 7-component parameter vector $\boldsymbol{\alpha}$=(p_x, p_y, p_z, E, x, y, z)$^{\mathrm{T}}$ is preferred to be the track representation since it is much simpler to transport in a magnetic field. The input parameters to primary vertex reconstruction are a set of tracks, which are parameterized at the closest point approaching the origin from the main drift chamber (MDC) track fitting.

The choice of expansion point $(\boldsymbol{x}_{\mathrm{e}}, \boldsymbol{p}_{\mathrm{e}})$ is in principle arbitrary; it is natural to choose $(\boldsymbol{x}_{\mathrm{e}}, \boldsymbol{p}_e) = (\boldsymbol{x}_{k-1}, \boldsymbol{p}_{0k})$ as a start point, where $\boldsymbol{x}_{k-1}$ is the vertex already fitted with $k-1$ tracks and $\boldsymbol{p}_{0k}$ is the 3-momentum vector of the k-th track at the closest point approaching the origin; however, the approximation error of the linear expansion should be small compared with the measurement errors. If necessary, the linear expansion can now be repeated at the new point $(\boldsymbol{x}_{\mathrm{e}}, \boldsymbol{p}_e) = (\boldsymbol{x}_k, \boldsymbol{p}_k)$ and the filter can be recomputed, until there is no significant change whether in χ^2 or in the estimation. One more iteration should be sufficient. The estimation of track k's parameter can be obtained from the updated vertex parameters and its 3-momentum vector at the vertex position. This procedure is called virtual measurement, as shown in Fig. 2. The k-th track parameter should be extrapolated from the vertex to the closest point approaching the origin in Eq. (1).

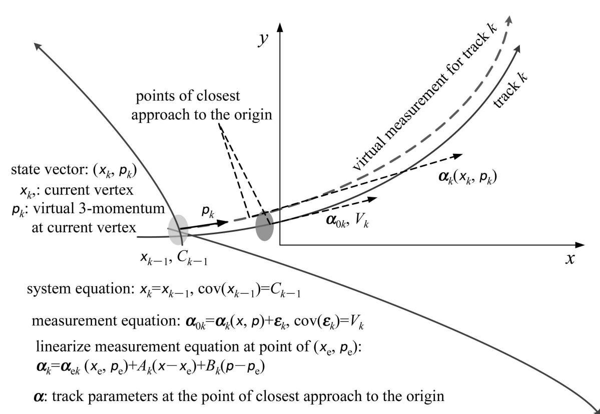

Fig. 2. The schematic illustration of "virtual measurement" in Kalman vertex fitting.

Before deriving the measurement equation and the corresponding Jacobian, let us follow the notations:

1) $\boldsymbol{x} = (x, y, z)$ — the vertex position;

2) $\boldsymbol{p}_k = (p_x, p_y, p_z)$ — the 3-momentum of the k-th track, originating from the vertex $\boldsymbol{x}$;

3) $\boldsymbol{\alpha}_{0k}$ — the k-th track measurement, parameterized at the closest point approaching the origin;

4) $\tilde{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}}(\boldsymbol{x},\boldsymbol{p})$ — parameters of the k-th track, extrapolated from the vertex to the closest point approaching the origin, the measurement equation can be expressed in the helix format.

The measurement equation $\tilde{\boldsymbol{\alpha}}(\boldsymbol{x},\boldsymbol{p})$ is determined by computing the track parameters from instantaneous position and momentum [8].

$$\tilde{\boldsymbol{\alpha}}(\boldsymbol{x},\boldsymbol{p}) = \begin{pmatrix} \tilde{d}_\rho \\ \tilde{\phi}_0 \\ \tilde{\kappa} \\ \tilde{d}_z \\ \tilde{\lambda} \end{pmatrix} = \begin{pmatrix} -\dfrac{T-p_\perp}{a} \\ \tan^{-1}\left[-\dfrac{p_x+ay}{p_y-ax}\right] \\ \dfrac{Q}{p_\perp} \\ z-\dfrac{p_z}{a}\sin^{-1}J \\ \dfrac{p_z}{p_\perp} \end{pmatrix}, \tag{9}$$

where

$$\begin{aligned} p_\perp &= \sqrt{p_x^2+p_y^2}, \\ T &= \sqrt{(p_x+ay)^2+(p_y-ax)^2}, \\ J &= \sin\rho s_\perp = \frac{p_{0x}p_y-p_{0y}p_x}{p_\perp^2} = \\ & \frac{p_y}{p_\perp}\cdot\frac{p_x+ay}{T}-\frac{p_x}{p_\perp}\cdot\frac{p_y-ax}{T} = \\ & \frac{a}{Tp_\perp}(xp_x+yp_y). \end{aligned} \tag{10}$$

$p_\perp$, T and J are intermediate variables in calculation. The two 5×3 derivative matrices $A=\partial\tilde{\boldsymbol{\alpha}}/\partial\boldsymbol{x}$ and $B=\partial\tilde{\boldsymbol{\alpha}}/\partial\boldsymbol{p}$ can be easily calculated [8].

4 Procedure of primary vertex reconstruction

The actual inputs of primary vertex reconstruction are all the reconstructed tracks. For raw data, the particle identification information still needs further calibration and reconstruction optimization for the dE/dx and TOF system [9, 10]. Currently, all the charged particles are treated as pions in the primary vertex reconstruction process. Since the improper particle hypothesis is assigned during the Kalman track fitting program, the improper track parameters and covariant matrices are obtained for primary vertex reconstruction. This may cause the resolution to be somewhat worse.

The selection criteria for good charged tracks are as follows. All charged tracks are required to be well measured by MDC within $|\cos\theta| < 0.93$, where θ is the polar angle. The track parameters are required to satisfy $|d_z| < 20$ cm, where d_z is the coordinate of the closest point approaching the origin along the z direction.

Primary vertex reconstruction at BESIII includes vertex finding and vertex fitting. The task of vertex finding is to sort a set of tracks into subsets that share points of origin. The vertex finding algorithm operates on geometric knowledge only. The primary vertex locates at the interaction region, and its distribution depends on the beam profile.

A very fast method of finding the minimal distance between two helices is essential for the clustering algorithms in primary vertex reconstruction. As a problem of finding a local minimal, the Newton procedure is applied to find the minimal distance and the corresponding closest points between each helix pair. Fig. 3(a) and (b) show the distribution of minimal distance and the coordinate of the closest point in the x-y plane between two helices. A loose cut, such as 3.5σ on the coordinates of x and y of the closest point, can be set to remove the obviously unqualified tracks.

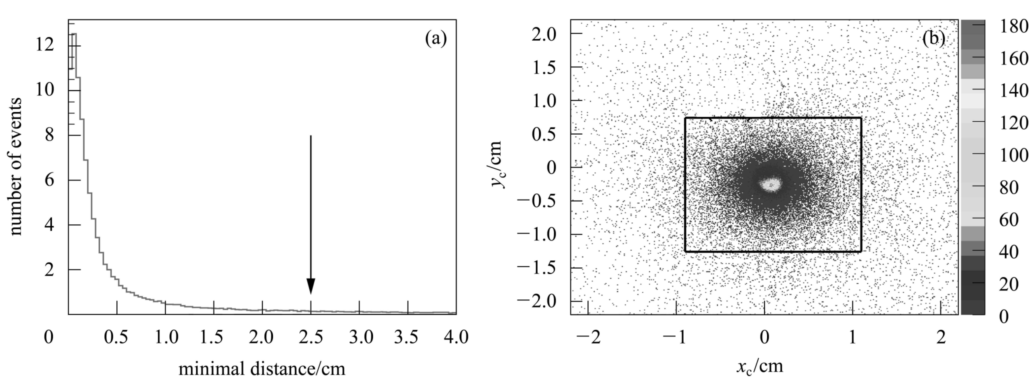

Fig. 3. (a) The distribution of minimal distance and (b) the coordinates of the closest point in the x-y plane between two helices.

After primary vertex finding, tracks which fulfill the requirements are regarded as the seed tracks for primary vertex fitting. It is important which tracks are selected as the first two candidates for vertex fitting. The track pair with the least minimal distance is a good choice when the initial primary vertex is unknown. The remaining tracks can be added to the pre-determined vertex one by one. The vertex position and updated track parameters can be calculated in the filtering procedure described in Sec. 2 and Sec. 3, where the 3-momentum vector of each track is determined by the reconstructed helix parameter $\boldsymbol{\alpha}=(d_\rho,\phi_0,\kappa,d_z,\lambda)^{\mathrm{T}}$, the 3-position vector $(x,\ y,\ z)$ is taken as the value of the vertex position in the last filter step. The χ^2 of the filter for each track is calculated according to Eq. (6). A track will be removed if the χ^2_{F} is too large during filtering. Since the charged track number in the BESIII experiment is quite low, some of bias may be raised if we test the χ^2_{F} only. To resolve this problem, the χ^2 of smoothing (χ^2_{S}) for each track is calculated when the filtering process is finished. Fig. 4(a) and (b) show the distribution of χ^2 in the filtering and smoothing procedures respectively. The χ^2_{S} is larger than the χ^2_{F} since the former is obtained based on the estimated vertex that takes all filtered tracks and their errors into account. The combined filtering-smoothing algorithm allows the computation of optimal estimates of the vertex position formed by any track sets in an event. Test on both the χ^2_{F} and the χ^2_{S} can effectively remove the ghost tracks and accurately calculate the beam position.

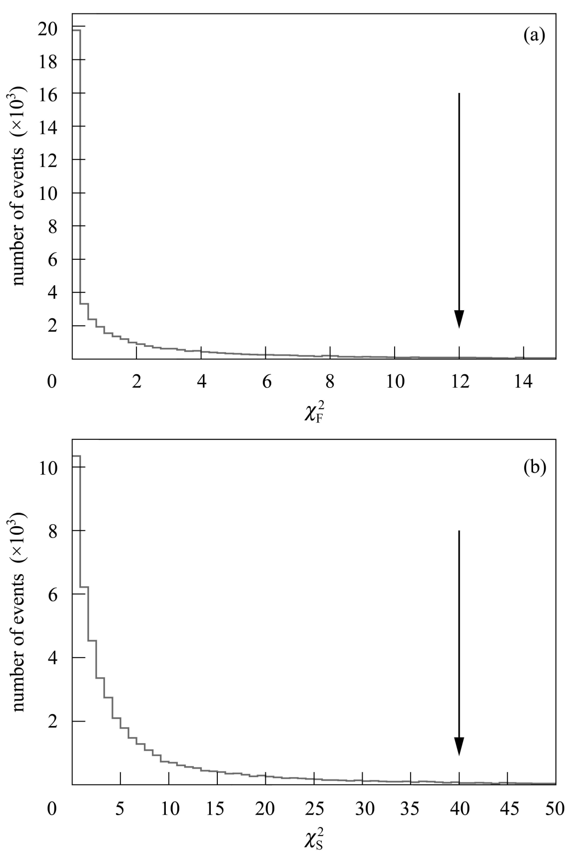

Fig. 4. The distribution of (a) χ^2_{F} and (b) χ^2_{S}.

5 Determination of the beam position

Initial vertex positions are unknown and the freedom of vertex fitting is $2n$–3, n is the number of tracks in an event, while beam positions are given by the Accelerator Group, the freedom of fixed vertex fitting is $2n$. However, Z of beam bunch size from our offline vertex fitting is more precise than the measurements from the Accelerator Group. This is significant for the realization of Monte Carlo simulation.

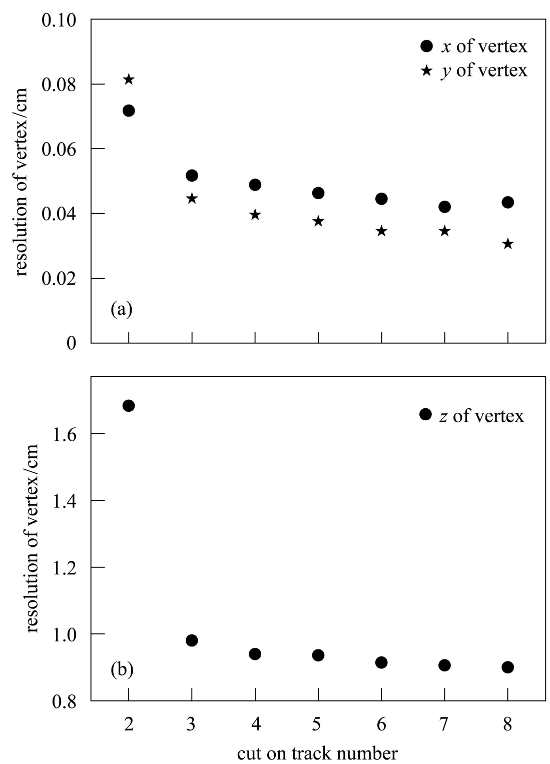

Fig. 5. Track number vs. vertex resolution (a) for the x and y and (b) for the z directions.

In each run, we reconstruct primary vertices for all events and finally get an average vertex position. According to Eq. (2), the primary vertex resolution depends on the number of seed track candidates in the vertex fitting procedure. The correlative relationship between them is shown in Fig. 5, (a) for the x and y and (b) for the z directions of the primary vertex resolution. From the figure, we can see that the greater the number of selected tracks contributing to the vertex fitting, the better the vertex resolution. Especially, the resolution improves significantly from two tracks to three tracks. In order to achieve better vertex resolution, at BESIII, only events with at least three charged tracks are selected for primary vertex reconstruction.

The distribution of the mean of beam bunch position is shown in Fig. 6(a) for raw data from Run 5463 to Run 6201. The results are given using the Kalman vertex fitting method discussed in Sec. 2. Since the samples include all types of events, it is appropriate to fit the distribution of the primary vertex with Gaussian and polynomial functions, which can effectively eliminate background interference such as beam gas. Fig. 7 shows the primary vertex distribution and the fit results in a certain run. In Fig. 6(a), the mean vertex positions in the x and y directions fluctuate slightly and the mean vertex position in the z direction varies from −0.3 cm to 0.4 cm. In detail, the x positions are about 0.1 cm away from the origin while the y positions about −0.25 cm.

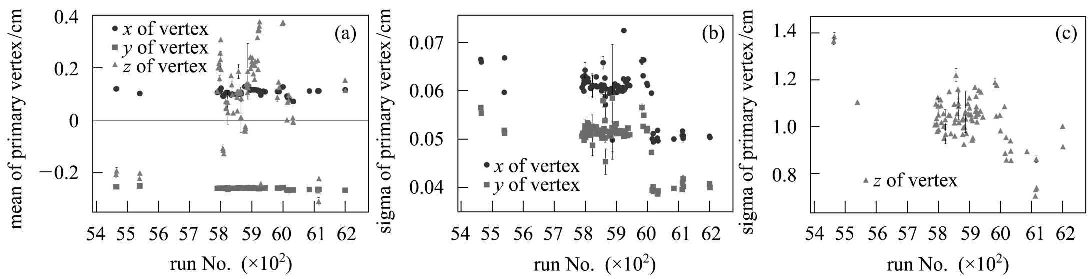

Fig. 6. The vertex positions.

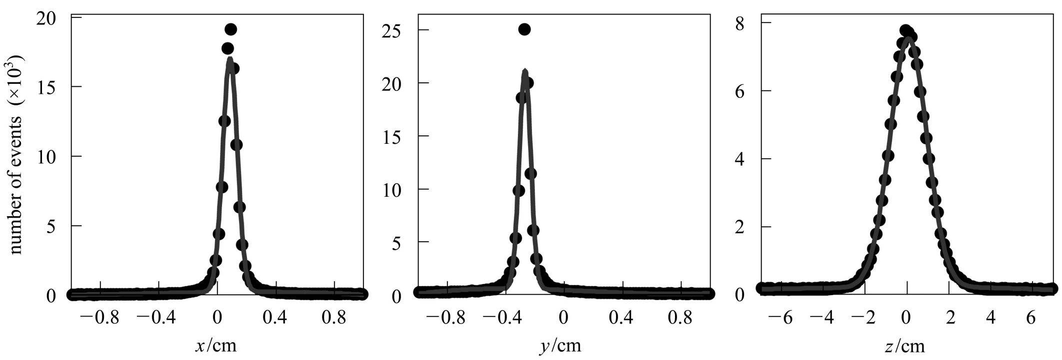

Fig. 7. The fitting results of the primary vertex.

The primary vertex resolutions along the x, y and z directions are shown in Fig. 6 (b) and (c). The resolution of x is about 500 μm–600 μm, y is about 400 μm–500 μm while z is about 1 cm. The data in Run 6000–6201 have 100% high voltage of all layers. It is clear that the resolutions of the data collected with all high voltage in the inner layer are better than the other data. Besides high voltage, the differences between the runs mainly result from the instability of beam bunches from the accelerator.

For the decay particles, the offset is especially useful for the case that its production point closely approaches the interaction point of the event. For physics analysis, it is inappropriate to select d_ρ, the first component of track parameters, as the selection criteria for good charged tracks, as shown in Fig. 8(a). At BESIII, track parameter d_ρ is given from the origin as the pivot. Since the primary vertex has a clear offset from the origin, the distribution of d_ρ is not single-peaked. We suggest substituting the following variable

$$R_{xy} = (x_i - x_v)\cos\phi + (y_i - y_v)\sin\phi \qquad (11)$$

for d_ρ, where (x_v, y_v) is the primary vertex position, (x_i, y_i) is the position on track i closest to the origin, and ϕ is the second component of track parameters mentioned in Sec. 3.1. The distribution of R_{xy} is single-peaked, as shown in Fig. 8(b). In the BEPC energy region and with a detector magnetic field of 1 Tesla, R_{xy} denotes approximately the signed distance of the helix from the pivotal point, that is, the interaction point, in the x-y plane. The cuts on R_{xy}

are more reasonable than d_ρ.

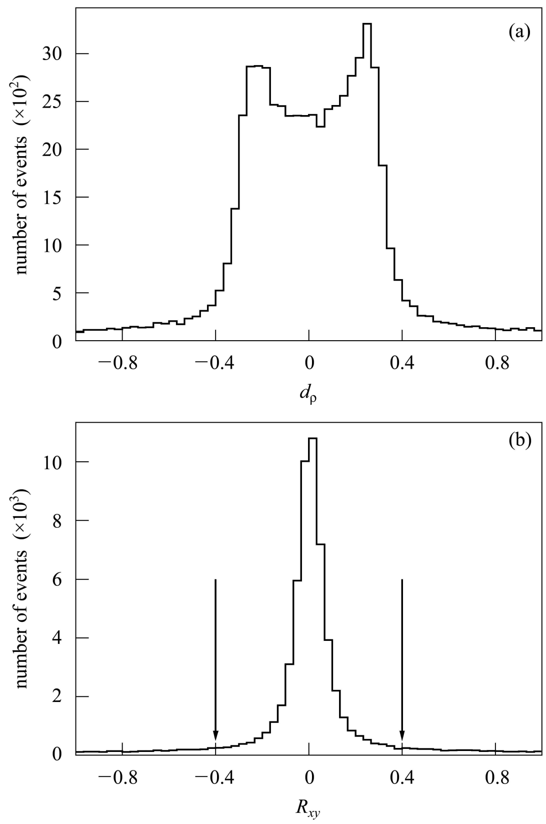

Fig. 8. The distance of the helix from the pivotal point of (a) the origin, and (b) the primary vertex.

5.1 Discussion of pull distribution

The pull of 5-component track parameters (α_i) is defined as

$$\text{pull} = \frac{(\alpha_i - \alpha_{i0})}{\sqrt{V_{\alpha_i} - V_{\alpha_{i0}}}}, \quad (12)$$

where "0" denotes the parameters before performing primary vertex fitting and "V_{α_i}" denotes the ith diagonal element of the covariance matrix. If the pull distribution can be fitted by the Gaussian function $N(0,1)$, it validates the calculation of error matrices. Table 1 gives the fitted resolution of the pull distribution of 5-component track parameters using the Gaussian function.

Table 1. Pull of 5-component track parameters.

parameter	pull
d_ρ	~1.14
ϕ^0	~1.09
κ	~1.50
d_z	~1.78
λ	~1.78

Because of the existence of serious noise in the Main Draft Chamber (MDC) in 2008 BESIII data taking, the high voltage for the inner chamber is not always turned to its full value. This partly spoils the resolution of particle momentum for less good hits in the inner 8 layers. The differences between the results and the expectations are mostly caused by two reasons. One reason is the calibration constants of detector alignment are not considered in the event reconstruction yet. From the results of primary vertex position, it is important to align the detector parameters in the software [11]. The other is that the error matrices of tracking are filled with smaller values than the truth. This is related to the function we choose for fitting. This work is in progress.

6 Conclusion

Primary vertex reconstruction based on the Kalman filter method has been developed. According to raw collision data at BESIII, vertex resolutions for multi-prong events agree with the expectations in principle. Updated calibration constants of dE/dx and alignment are expected. The results can be improved after particle identification.

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