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# $\mathscr{P}$-粗积分与函数双向 S-粗集的粗糙度

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**摘要:**在函数双向 S-粗集生成的$\mathscr{P}$-粗积分的基础上提出了函数双向 S-粗集的精度与粗糙度的概念,讨论了函数双向 S-粗集的精度与粗糙度的一系列特性,并得到函数双向 S-粗集的可分辨准则与函数双向 S-粗集的筛选-剩余定理。

**关键词:**函数双向 S-粗集;$\mathscr{P}$-粗积分;粗糙度

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# $\mathscr{P}$- rough integrals and the rough degree of function two-direction S-rough sets

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**Abstract**: The concepts of the precision degree and rough degree of function two-direction S-rough sets are given, which is based on $\mathscr{P}$-rough integrals generated by function two-direction S-rough sets. And a series of their characteristics are discussed. Also, the discernible criterion of function two-direction S-rough sets and the screening-surplus theorem of it are obtained.

**Key words**: function two-direction S-rough sets; $\mathscr{P}$-rough integrals; rough degree

## 0 引言

2005 年,史开泉教授提出了函数 S-粗集[1-5]理论,它包括函数单向 S-粗集、函数双向 S-粗集与函数单向 S-粗集对偶三部分内容,本文是在函数双向 S-粗集的基础上进行的。

事实上,在函数双向 S-粗集$((R,\mathscr{P})_{\circ}(Q^*),(R,\mathscr{P})^{\circ}(Q^*))$中,记$[u]_{\mathscr{P}}=(R,\mathscr{P})_{\circ}(Q^*)$,$[u]^{\mathscr{P}}=(R,\mathscr{P})^{\circ}(Q^*)$,当$[u]_{\mathscr{P}}=[u]^{\mathscr{P}}$时,函数等价关系 $R$ 对 $Q^*$ 认识是清楚的,$Q^*$ 是可定义集。当$[u]_{\mathscr{P}}\subset[u]^{\mathscr{P}}$时,$R$ 对 $Q^*$ 的认识分两部分,对于$[u]_{\mathscr{P}}\in Q^*$部分是清楚的,对于 $Q^*-[u]_{\mathscr{P}}$部分是模糊的、粗糙的,因此 $R$ 对 $Q^*$ 的认识整体上是粗糙的,故 $Q^*$ 是不可定义集,也被称作粗糙集。根据文献[6],$[u]_{\mathscr{P}}$、$[u]^{\mathscr{P}}$生成的多项式函数 $p_{\mathscr{P}}(x)$ 和 $p^{\mathscr{P}}(x)$ 若在区间$[a,b]$上连续,则生成积分对$(\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x,\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x)$,即$\mathscr{P}$-粗积分。根据积分的几何意义 $\lambda_{\mathscr{P}}=(\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x-\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x)\div\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x$ 从面积角度表达了 $R$ 对 $Q^*$ 认识粗糙部分相对于认识整体部分$[u]^{\mathscr{P}}$的比率。本文定义 $\lambda_{\mathscr{P}}$ 为函数双向 S-粗集的粗糙度,并定义 $\gamma_{\mathscr{P}}=\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x/\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x$ 为函数双向 S-粗集的精度。在此基础上讨论了它们的相关特性,且提出了函数双向 S-粗集的筛选–剩余定理。

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**约定** $\zeta(x)$是有限函数论域,$Q(x)=\{u_1(x),u_2(x),\cdots,u_\gamma(x)\}\subset\zeta(x)$是有限函数集,$[u(x)]$是$\zeta(x)$上的$R$-函数等价类,$R$是$\zeta(x)$上的等价关系,$\zeta(x),Q(x),u(x),[u(x)]$分别记做$\zeta,Q,u,[u]$。$F$、$\bar{F}$是元素迁移族,且$f\in F$、$\bar{f}\in\bar{F}$是元素迁移,DIS = discernibility,IND = in discernibility。

## 1 函数双向S-粗集[4]与$\mathscr{P}$-粗积分[6]

**定义1** 称$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$是函数粗集$(R_-(Q),R^-(Q))$生成的函数双向S-粗集,若存在$\mathscr{P}=F\cup\bar{F}$,使$Q^*=Q\cup\{v_i\mid v_i\in\zeta,v_i\overline{\in}Q,f(v_i)=u_i\in Q\}-\{u_j\mid u_j\in Q,\bar{f}(u_j)=v_j\overline{\in}Q\}$,且

$$(R,\mathscr{P})_\circ(Q^*)=\cup[u]=\{u\mid u\in\zeta,[u]\subseteq Q^*\},$$

$$(R,\mathscr{P})^\circ(Q^*)=\cup[u]=\{u\mid u\in\zeta,[u]\cap Q^*\neq\phi\}。$$

**定义2** 称积分对$(\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x,\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x)$是函数双向S-粗集$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$生成的$\mathscr{P}$-粗积分,简称$\mathscr{P}$-粗积分。

称$\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x$、$\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x$分别是$\mathscr{P}$-粗积分$(\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x,\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x)$的下近似积分和上近似积分。其中$p_{\mathscr{P}}(x)$、$p^{\mathscr{P}}(x)$分别是$(R,\mathscr{P})_\circ(Q^*)$、$(R,\mathscr{P})^\circ(Q^*)$根据文献[5,6]生成的多项式函数。

**定理1**[6]($\mathscr{P}$-粗积分扩充定理) 若$((R,\mathscr{P}_1)_\circ(Q_1^*),(R,\mathscr{P}_1)^\circ(Q_1^*))$与$((R,\mathscr{P}_2)_\circ(Q_2^*),(R,\mathscr{P}_2)^\circ(Q_2^*))$是函数粗集$(R_-(Q),R^-(Q))$生成的两个函数双向S-粗集,且$((R,\mathscr{P}_2)_\circ(Q_2^*),(R,\mathscr{P}_2)^\circ(Q_2^*))\subseteq((R,\mathscr{P}_1)_\circ(Q_1^*),(R,\mathscr{P}_1)^\circ(Q_1^*))$,则有

$$(\int_a^b p_{\mathscr{P}_2}(x)\mathrm{d}x,\int_a^b p^{\mathscr{P}_2}(x)\mathrm{d}x)\leqslant(\int_a^b p_{\mathscr{P}_1}(x)\mathrm{d}x,\int_a^b p^{\mathscr{P}_1}(x)\mathrm{d}x)。\tag{1}$$

## 2 $\mathscr{P}$-粗积分与函数双向S-粗集的精度与粗糙度

**约定** $(\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x,\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x)$是函数双向S-粗集$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$生成的$\mathscr{P}$-粗积分。

**定义3** 称$\gamma_{\mathscr{P}}$是函数双向S-粗集$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$的精度,如果

$$\gamma_{\mathscr{P}}=\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x/\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x。\tag{2}$$

**定义4** 称$\lambda_{\mathscr{P}}$是函数双向S-粗集$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$的粗糙度,如果

$$\lambda_{\mathscr{P}}=(\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x-\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x)/\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x。\tag{3}$$

对任一函数双向S-粗集$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$,有命题1 ~ 3:

**命题1** $0\leqslant\gamma_{\mathscr{P}}\leqslant1;0\leqslant\lambda_{\mathscr{P}}\leqslant1$。

**命题2** 当$\gamma_{\mathscr{P}}=1$时,$\lambda_{\mathscr{P}}=0$;当$\gamma_{\mathscr{P}}=0$时,$\lambda_{\mathscr{P}}=1$,且$\gamma_{\mathscr{P}}+\lambda_{\mathscr{P}}=1$。

**命题3** 函数双向S-粗集的精度越大,其粗糙度越小,反之亦真。

这里只给出命题1的证明。

**证明** 对任意一个函数双向S-粗集$((R,\mathscr{P})_\circ(Q^*),(R,\mathscr{P})^\circ(Q^*))$,有$(R,\mathscr{P})_\circ(Q^*)\subseteq(R,\mathscr{P})^\circ(Q^*)$,根据文献[6]知$0\leqslant\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x\leqslant\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x$,即$0\leqslant\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x/\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x\leqslant1$从而$0\leqslant\gamma_{\mathscr{P}}\leqslant1$成立。又知$\lambda_{\mathscr{P}}=(\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x-\int_a^b p_{\mathscr{P}}(x)\mathrm{d}x)\div\int_a^b p^{\mathscr{P}}(x)\mathrm{d}x=1-\gamma_{\mathscr{P}}$,所以有$0\leqslant\lambda_{\mathscr{P}}\leqslant1,\gamma_{\mathscr{P}}+\lambda_{\mathscr{P}}=1$成立。

设$((R,\mathscr{P}_1)_\circ(Q_1^*),(R,\mathscr{P}_1)^\circ(Q_1^*))$与$((R,\mathscr{P}_2)_\circ(Q_2^*),(R,\mathscr{P}_2)^\circ(Q_2^*))$是函数粗集$(R_-(Q),R^-(Q))$生成的两个函数双向S-粗集,其粗糙度分别是$\gamma_{\mathscr{P}_1}$、$\gamma_{\mathscr{P}_2}$,其精度分别是$\lambda_{\mathscr{P}_1}$、$\lambda_{\mathscr{P}_2}$,有下列定理与推论。

**定理2** 当$(R,\mathscr{P}_1)_\circ(Q_1^*)=(R,\mathscr{P}_2)_\circ(Q_2^*)$且$(R,\mathscr{P}_1)^\circ(Q_1^*)\supseteq(R,\mathscr{P}_2)^\circ(Q_2^*)$,则有

$$\gamma_{\mathscr{P}_1} \leqslant \gamma_{\mathscr{P}_2}。 \tag{4}$$

**证明** 设$(\int_a^b p_{\mathscr{P}_i}(x)\mathrm{d}x, \int_a^b p^{\mathscr{P}_i}(x)\mathrm{d}x)$分别是$((R,\mathscr{P}_i)_{\circ}(Q_i^*),(R,\mathscr{P}_i)^{\circ}(Q_i^*))$生成的$\mathscr{P}_i$- 粗积分,$i = 1,2$。因$(R,\mathscr{P}_1)_{\circ}(Q_1^*) = (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) \supseteq (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,根据文献[6]可知$\int_a^b p_{\mathscr{P}_1}(x)\mathrm{d}x = \int_a^b p_{\mathscr{P}_2}(x)\mathrm{d}x$、$\int_a^b p^{\mathscr{P}_1}(x)\mathrm{d}x \geqslant \int_a^b p^{\mathscr{P}_2}(x)\mathrm{d}x$,由定义 3 得 $\gamma_{\mathscr{P}_1} \leqslant \gamma_{\mathscr{P}_2}$。

**推论 1** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) = (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) \supseteq (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\lambda_{\mathscr{P}_1} \geqslant \lambda_{\mathscr{P}_2}。 \tag{5}$$

**定理 3** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) = (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) \subseteq (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\gamma_{\mathscr{P}_1} \geqslant \gamma_{\mathscr{P}_2}。 \tag{6}$$

**推论 2** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) = (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) \subseteq (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\lambda_{\mathscr{P}_1} \leqslant \lambda_{\mathscr{P}_2}。 \tag{7}$$

**定理 4** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) \subseteq (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) = (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\gamma_{\mathscr{P}_1} \leqslant \gamma_{\mathscr{P}_2}。 \tag{8}$$

**推论 3** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) \subseteq (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) = (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\lambda_{\mathscr{P}_1} \geqslant \lambda_{\mathscr{P}_2}。 \tag{9}$$

**定理 5** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) \supseteq (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) = (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\gamma_{\mathscr{P}_1} \geqslant \gamma_{\mathscr{P}_2}。 \tag{10}$$

**推论 4** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) \supseteq (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_1)^{\circ}(Q_1^*) = (R,\mathscr{P}_2)^{\circ}(Q_2^*)$,则

$$\lambda_{\mathscr{P}_1} \leqslant \lambda_{\mathscr{P}_2}。 \tag{11}$$

**定理 6** 当$(R,\mathscr{P}_1)_{\circ}(Q_1^*) = (R,\mathscr{P}_2)_{\circ}(Q_2^*)$且$(R,\mathscr{P}_2)^{\circ}(Q_2^*) = (R,\mathscr{P}_1)^{\circ}(Q_1^*)$,则

$$\mathrm{IND}\{\lambda_{\mathscr{P}_2}, \lambda_{\mathscr{P}_2}\}。 \tag{12}$$

定理 3 ~ 6、推论 1 ~ 4的证明与定理 2类似,略。

**定理 7** (函数双向 S- 粗集可分辨准则) 若 $\mathrm{DIS}\{\lambda_{\mathscr{P}_2}, \lambda_{\mathscr{P}_2}\}$或$\mathrm{DIS}\{\gamma_{\mathscr{P}_2}, \gamma_{\mathscr{P}_2}\}$,则

$$\mathrm{DIS}\{((R,\mathscr{P}_1)_{\circ}(Q_1^*),(R,\mathscr{P}_1)^{\circ}(Q_1^*)),((R,\mathscr{P}_2)_{\circ}(Q_2^*),(R,\mathscr{P}_2)^{\circ}(Q_2^*))\}。 \tag{13}$$

**定理 8** ($\mathscr{P}$- 粗积分可分辨定理) 若 $\mathrm{DIS}\{\lambda_{\mathscr{P}_2}, \lambda_{\mathscr{P}_2}\}$或$\mathrm{DIS}\{\gamma_{\mathscr{P}_2}, \gamma_{\mathscr{P}_2}\}$,则

$$\mathrm{DIS}\{(\int_a^b p_{\mathscr{P}_1}(x)\mathrm{d}x, \int_a^b p^{\mathscr{P}_1}(x)\mathrm{d}x),(\int_a^b p_{\mathscr{P}_2}(x)\mathrm{d}x, \int_a^b p^{\mathscr{P}_2}(x)\mathrm{d}x)\}。 \tag{14}$$

定理 7、8 的证明可以根据函数双向 S- 粗集的粗糙度与精度的定义得出,略。

## 3 函数双向 S- 粗集集中函数双向 S- 粗集的筛选 – 剩余

**定义 5** 称由函数粗集$(R_-(Q),R^-(Q))$生成的所有函数双向 S- 粗集构成的集合$\{((R,\mathscr{P}_i)_{\circ}(Q^{i,*}),(R,\mathscr{P}_i)^{\circ}(Q^{i,*})) \mid i = 1,2,\cdots,n\}$是由函数粗集$(R_-(Q),R^-(Q))$生成的函数双向 S- 粗集集,简称函数双向 S- 粗集集。

**定理 9** 在函数双向 S- 粗集集$\{((R,\mathscr{P}_i)_{\circ}(Q^{i,*}),(R,\mathscr{P}_i)^{\circ}(Q^{i,*})) \mid i = 1,2,\cdots,n\}$中,若存在 $k$ 个函数双向 S- 粗集,满足$Q^{j_1} \subseteq Q^{j_2} \subseteq,\cdots, \subseteq Q^{j_k}$, $j_h \in \{1,2,\cdots,n\}$, $h = 1,2,\cdots,k$,则

$$(\int_a^b p_{j_h}(x)\mathrm{d}x, \int_a^b p^{j_h}(x)\mathrm{d}x) \leqslant (\int_a^b p_{j_{h+1}}(x)\mathrm{d}x, \int_a^b p^{j_{h+1}}(x)\mathrm{d}x), h = 1,2,\cdots,k-1, \tag{15}$$

其中$(\int_a^b p_{j_h}(x)\mathrm{d}x, \int_a^b p^{j_h}(x)\mathrm{d}x)$是$((R,\mathscr{P}_{j_h})_{\circ}(Q^{j_h,*}),(R,\mathscr{P}_{j_h})^{\circ}(Q^{j_h,*}))$生成的$\mathscr{P}_{j_h}$- 粗积分,$h = 1,2,\cdots,k-1$。

**定理 10** 对于函数双向 S- 粗集集$\{((R,\mathscr{P}_i)_\circ(Q^{i,*}),(R,\mathscr{P}_i)^\circ(Q^{i,*})\mid i=1,2,\cdots,n\}$,$\{\lambda_{\mathscr{P}_i}\mid i=1,2,\cdots,n\}$中存在最小值与最大值。

其中 $\lambda_{\mathscr{P}_i}$ 是$((R,\mathscr{P}_i)_\circ(Q^{i,*}),(R,\mathscr{P}_i)^\circ(Q^{i,*}))$的粗糙度。

**定理 11**(函数双向 S- 粗集筛选 – 剩余定理) 对于某一个函数双向 S- 粗集集$\{((R,\mathscr{P}_i)_\circ(Q^{i,*}),(R,\mathscr{P}_i)^\circ(Q^{i,*})\mid i=1,2,\cdots,n\}$, 给定一个阈值 $c$, 当 $\lambda_{\mathscr{P}_t}\geqslant c, t=j_1,j_2,\cdots,j_l\in\{1,2,\cdots,n\}$ 时, $((R,\mathscr{P}_t)_\circ(Q^{t,*}),(R,\mathscr{P}_t)^\circ(Q^{t,*})(t=j_1,j_2,\cdots,j_l)$被筛选出来,函数双向 S- 粗集集中其它的函数双向 S- 粗集成为筛选剩余。

## 4 结束语

函数双向 S- 粗集已广泛应用于系统分析、规律挖掘、生物医学等领域,$\mathscr{P}$- 粗积分的概念刚刚被提出,函数双向 S- 粗集的精度与粗糙度就是利用$\mathscr{P}$- 粗积分从面积角度反应了函数双向 S- 粗集特征的,这为函数双向 S- 粗集的应用领域的拓宽和深入提供了理论依据。

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