The Fictitious Domain Method for Patient-Specific Biomechanical Modeling: Promise and Prospects

S. V. Shah[1], L. F. Kallivokas[2], B. Jaramaz[2,3], O. Ghattas[1,3], A.M. DiGioia[2,3]
sameer@cs.cmu.edu, loukas@cs.cmu.edu, branko@cmu.edu, ghattas@cs.cmu.edu, digioia@ri.cmu.edu

[1]Computational Mechanics Laboratory
Department of Civil and Environmental Engineering
Carnegie Mellon University
Pittsburgh, PA 15213

[2]Center for Orthopaedic Research
Shadyside Hospital
Pittsburgh, PA 15232

[3]Center for Medical Robotics and
Computer Assisted Surgery
Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 15213

Abstract

In this paper we discuss the application of the fictitious domain method to the numerical simulation of the mechanical processes induced by press-fitting cementless femoral implants in total hip replacement surgeries. The immediate concern is to demonstrate the feasibility of the method, its advantages over other competing numerical techniques and its applicability to a wide range of analysis problems in biomechanics whose primary input originates from CT- or MRI-collected data. We consider the implantation of a press-fit femoral prosthesis as a prototype problem for sketching the application path of the proposed methodology. Of concern is also the assessment of the robustness and speed of the methodology, for both factors are critical if one were to consider patient-specific modeling. The latter is part of a thrust to develop pre-operative planners enhanced with robust and fast biomechanical feedback. In this communication we present the concepts and report on initial numerical results.

Keywords*: Finite element method, fictitious domain, biomechanics.*

Categories*: Preoperative planners and surgical simulators.*

1 Overview

Typical pre-operative planning systems in total hip replacement surgeries include today only geometric templating capabilities that are used, invariably, to find an appropriate match between the femoral and acetabular implants and their respective receiving bones. If a biomechanical feedback mechanism were to exist that would present, in a way meaningful to the planning surgeon, the potential mechanical effects of a specific implant choice, it is then conceivable that such a mechanism would act as a desirable safety feature. Its introduction might prevent selections leading to either short- or long-term failures of the chosen prosthesis and thus to a possible improvement of the clinical outcome. The recent introduction of robotic systems in the operating room [1] -responsible for preparing the bone canal receiving the implant- only augmented the need for pre-operative planners enhanced with such a biomechanical feedback. However, the inclusion of a feedback module into a pre-operative system imposes a severe demand for computational speed and robustness of the underlying geometry and analysis tools. It is within the above framework that we explore in this paper the applicability and suitability of the fictitious domain method as part of an analysis tool of a biomechanical feedback mechanism for a pre-operative planner.

Typical analysis modules are based on finite element methods that employ unstructured mesh techniques; that is, techniques that are based on the meshing of solids by elements of variable size that conform to the geometry of the solid. For example, in our prototype problem, the conventional approach to the finite element modeling would require meshing the femur and the femoral implant. Towards this end, the first step would involve resolving the geometry. Typically, the input originates from CT-scan data and the standard pipeline that leads to the finite element model includes (Fig. 1a):

i) Patient CT-scans resulting in a series of planar tomographic images;

ii) Noise-filtering of the images;

iii) Extraction of boundaries by edge detection algorithms yielding contours on each tomographic cross-section;

iv) Connection of the contours across consecutive cross-sections typically achieved by triangulation (surface reconstruction step);

v) Identification of bone and canal volumes;

vi) Canal preparation and femoral neck osteotomy - both operations result in the intersection of the bone volumes identified in the previous step;

vii) Volume meshing using finite elements and assignment of material properties (Fig. 1a)

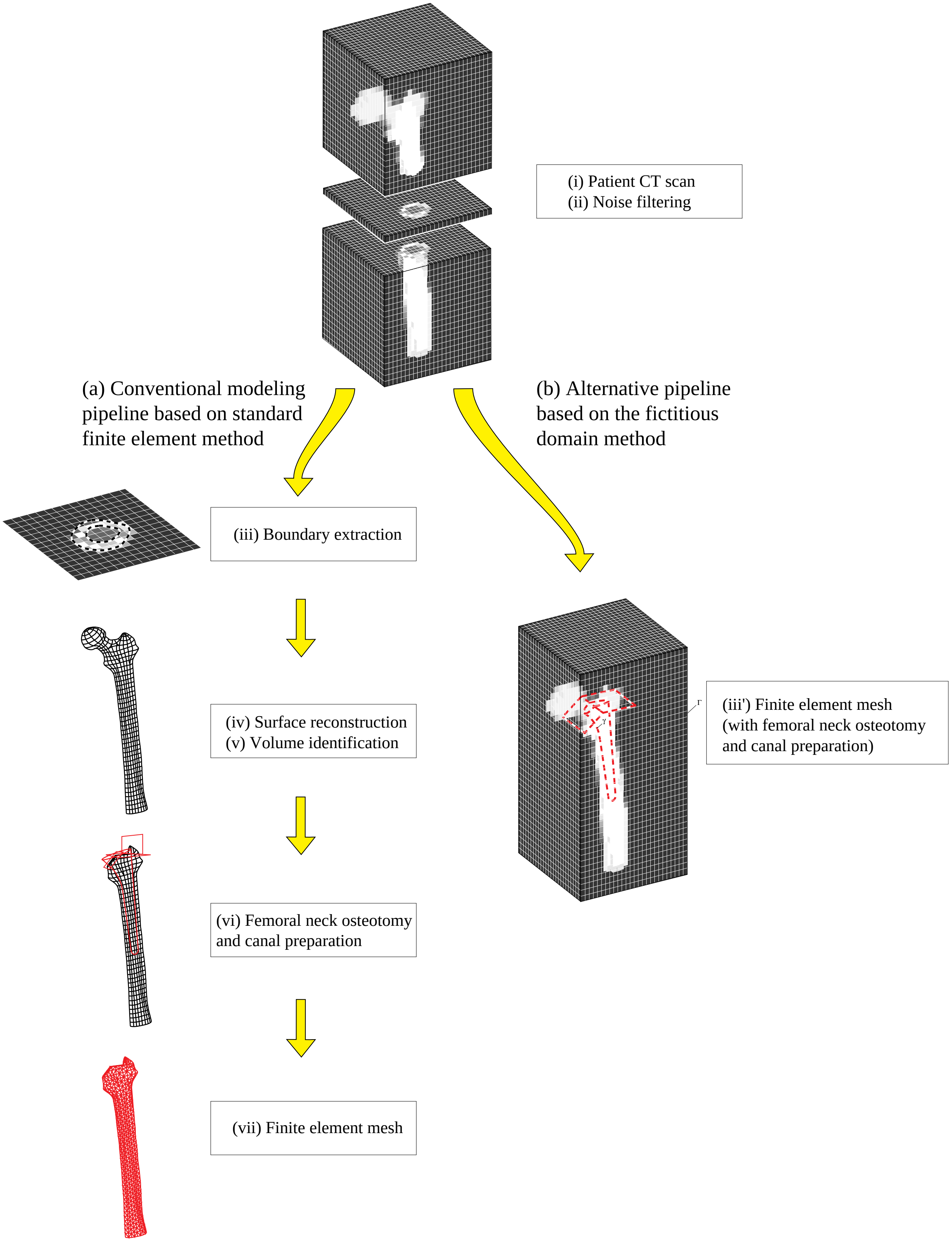


Figure 1: (a) Conventional, and (b) alternative modeling pipeline from CT-scan patient data to finite element model.

The sources of error in the modeling pipeline include CT imaging errors, boundary extraction errors, surface generation errors and meshing errors. Furthermore, the surface reconstruction and the intersection of volumes during the canal preparation phase result in geometrically complex volumes that are difficult to mesh using unstructured techniques, if it is at all possible, given a reasonable set of computational resources - both hardware and software.

We remark that in this process the CT-scan data are perforce considered as the best anatomical information that is available for modeling - a form of “ground truth”. The approximation errors that are introduced along the modeling pipeline, and prior to the analysis module, greatly distort that ground truth. Therefore, if one were to avoid the accumulation of the errors, while simultaneously resolving or sidestepping the meshing difficulties without sacrificing computational speed or accuracy, one would have a strongly promising combination for a fast and robust analysis tool. We turn to the fictitious domain method in search of such a tool.

2 Fictitious domain method

The important promise of the fictitious domain method lies in the fact that one need not respect the geometric constraints in the way that one need do with the standard finite element method [2]-[5]. Instead, one can use structured meshes that do not necessarily follow the geometric boundaries of the solid under analysis. Accordingly, one could bypass most of the steps in the modeling pipeline, and hence eliminate the approximation errors associated with them (Fig. 1b). For example, in our prototype problem there would be no need for boundary extraction, surface reconstruction and canal preparation prior to meshing. The method is especially well suited for problems with input data given already on a structured mesh; our prototype problem conforms precisely to that description since the primary input stems from CT-scan data that are typically offered on rectangular grids.

To illustrate the concept of the fictitious domain method we first consider a simple two-dimensional case; we return later to our three-dimensional prototype problem. Thus, let γ denote a closed curve on the plane with interior domain Ω (Fig. 2a). A typical boundary value problem in Ω involves the solution of a partial differential equation governing the physical problem within Ω, subject to appropriate boundary conditions on γ. In standard finite element methods, solutions to the problem are found by seeking to satisfy a weak form of the governing partial differential equation in the interior domain Ω subject to the same conditions on γ. A typical unstructured domain discretization, as needed for carrying out such a solution,

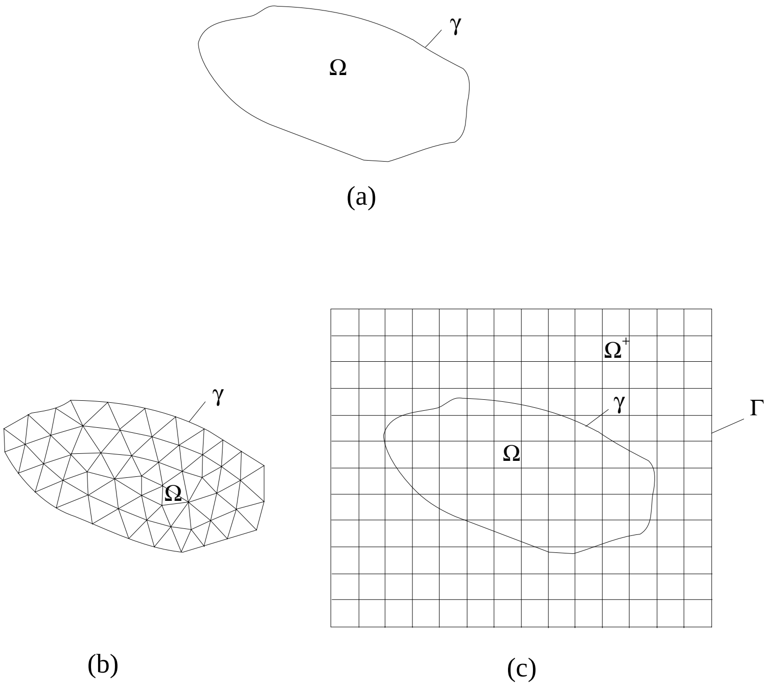
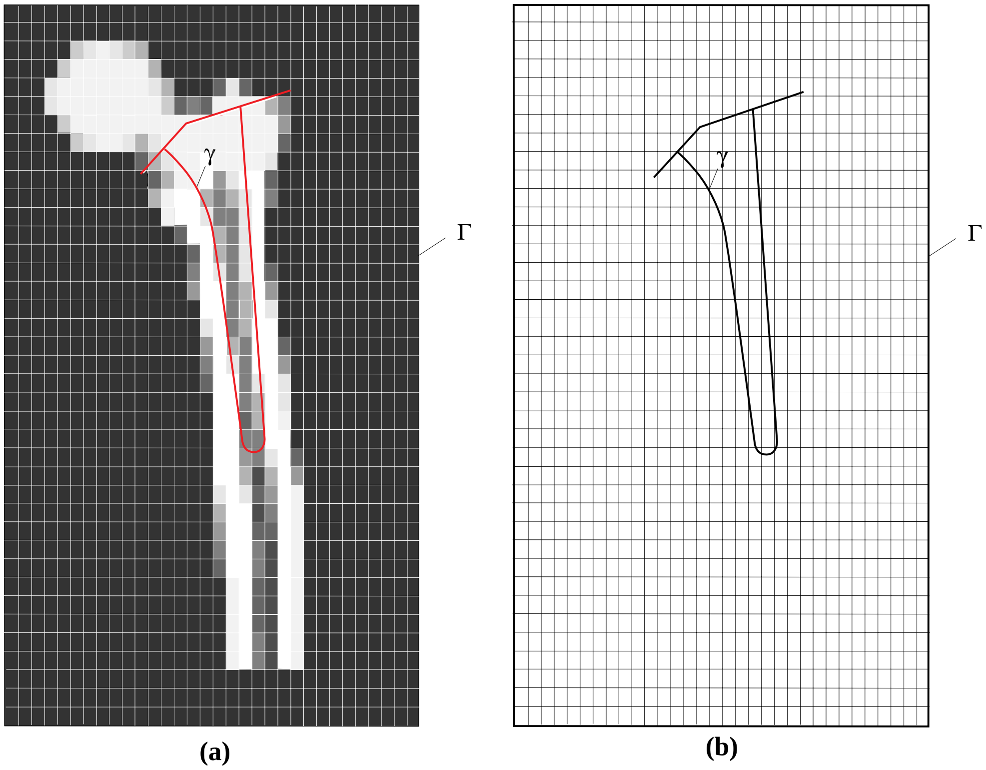


Figure 2: Simple two-dimensional case: (a) Domain of interest Ω bounded by external curve γ; b) typical unstructured discretization of Ω (standard finite element method); (c) structured discretization of Ω embedded in Ω^+ (fictitious domain method).

is shown in Fig. 2b. In the fictitious domain approach, the interior domain is embedded within a larger domain Ω^+ that is bounded (here Γ bounds Ω^+ as per Fig. 2c) and it is usually of canonical shape in order to allow for such structured discretizations as the one shown in Fig. 2c. Solutions are then sought to the same partial differential equation over the enlarged domain Ω^+, while boundary conditions are weakly enforced on the boundary γ; in this case γ need be discretized independently of the discretization of the background grid (Fig. 2c). Appropriate boundary conditions need also be imposed on the external boundary Γ. Naturally, the interior domain Ω is still the region of interest; it can be shown that the solution to the discrete problem over the enlarged domain Ω^+ coincides with the solution within Ω -in the limit as the boundary and domain mesh sizes tend to zero.

There are substantial benefits to this approach; for one, the presence of a structured mesh allows for the efficient use of computational resources. Conceivably, there is no need for repeated numerical integration in order to evaluate the element matrices associated with each grid cell (finite element) of the background grid, as the geometry of the elements is the same across the entire domain. In addition, the structured mesh is well suited for taking advantage of fast iterative solvers and for exploiting the computational speed of parallel architecture computers. But, by far, the advantages of the fictitious domain method can be seen in the prototype problem. Consider the two-dimensional cross-section shown in Fig. 3a: the elements of the background grid correspond precisely to the voxels of the CT-scan; the shades of gray represent different CT-scan densities which are correlated to material properties (either of the cortical or cancellous bone, or of void). Therefore, there is no need to revisit the discretization, as one would do with the standard finite element method, in order to assign material properties. The closed curve denoted by γ in Fig. 3a represents the implant, after the femoral neck osteotomy. In the cementless press-fit case, traction-free conditions are imposed on the upper portion (open part) of γ, while displacements are prescribed on the remainder of γ to represent the amount of press-fit. The geometry of the resulting boundary value problem is shown in Fig. 3b.

The application of the method is not without difficulties. It is unclear, for example, what the optimal ratio of element sizes of the background grid to the boundary discretization should be for achieving a desired rate of convergence. What is the role of the traction- vs. displacement-like conditions on the rate of convergence? What is the appropriate combination of polynomial approximants for the background grid versus the boundary discretization?



(a) (b)

Figure 3: (a) Cross-section of CT-scan data and (b) corresponding fictitious domain model.

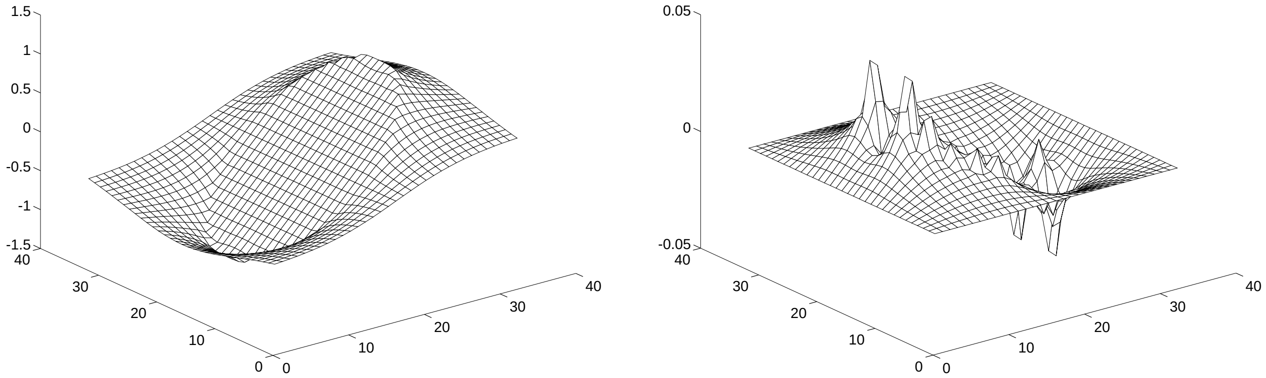


Figure 4: (a) Fictitious domain solution and (b) relative error.

How does the computational speed of the fictitious domain method compare with the standard finite element method for a given accuracy? (see also [3]). We attempt to answer some of the above questions with our numerical experiments.

3 Numerical experiments

Our preliminary investigations focused on two-dimensional problems involving regions of arbitrary geometry with Poisson's equation as the governing field equation. We later extended our studies to the all important elasticity case. Excerpts from our initial results are shown in Fig. 4 for a circular domain Ω embedded in a domain Ω^+. Dirichlet conditions (displacement-like) were applied on both the inner boundary γ and the outer one Γ. Figure 4a depicts the numerical solution obtained by the fictitious domain method, whereas Fig. 4b depicts the relative error across the entire computational domain Ω^+. Notice that the concentration and the error amplitudes are highest along the boundary γ. Our present efforts focus on reducing the error on γ, since in our prototype problem (simulation of the femur) γ represents the critical interface between the implant and the bone. We will report on the theoretical foundation of the method, on the results of our convergence studies, on comparisons with the standard finite element method and on the remaining open questions. Given the advantages of the method we are confident that the suggested approach represents a solid path to automating patient-specific modeling for use in pre-operative planners.

4 Acknowledgments

This work was supported in part by a National Challenge grant from the National Science Foundation (award ECS-9422734) and by a grant from Arthritis Foundation.

5 References

[1] Taylor R.H. et al., An image-directed robotic system for precise orthopaedic surgery, *IEEE Transactions on Robotics and Automation*, Vol. 10, No. 3, 261-270, June 1994.

[2] Astrakhantsev G. P., Method of fictitious domains for a second-order elliptic equation with natural boundary conditions, *USSR Computational Mathematics and Mathematical Physics*, 18, 114-121, 1978.

[3] Bespalov A., Kuznetsov Y., Pironneau O. and Vallet M-G., Fictitious domains with separable preconditioners versus unstructured adapted meshes, *Impact of Computing in Science and Engineering*, 4, 217-249, 1992.

[4] Glowinski R., Pan T-W and Periaux J., A fictitious domain method for Dirichlet problem and applications, *Computer Methods in Applied Mechanics and Engineering*, 111, 283-303, 1994.

[5] Glowinski R., Pan T-W and Periaux J., A fictitious domain method for external incompressible viscous flow modeled by Navier-Stokes equations, *Computer Methods in Applied Mechanics and Engineering*, 112, 133-148, 1994.