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Modelling of complex piezoelectric system by non-classical methods

A. Buchacz, A. Wróbel*
Institute of Engineering Processes Automation and Integrated Manufacturing Systems, Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author: E-mail address: andrzej.wrobel@polsl.pl

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Analysis and modelling

ABSTRACT

Purpose: Present paper is continuation of earlier publications with stack of piezoelectric plates. This work is an author's idea of calculations of complex systems with many elements.
Design/methodology/approach: The base of calculation is matrix method and application of aggregation of graphs to determination characteristic parameters of bimorphic systems, as well as to drawing its characteristics.
Findings: The analysis of complex piezoelectric system was shown to determinate characteristics of it.
Research limitations/implications: In the article problem of mechatronic system analysis on example of longitudinal vibration of piezoelectric plates was presented. In the future analysis of plate with bending vibration will be done.
Practical implications: Presented analysis method of piezoelectric effect in complex systems is well suited for determining the flexibility of bimorphic stack of piezoelectric plates in vibration sensors. This sensors are used to the level detection of materials.
Originality/value: Thanks to the approach, introduced in this paper, analysis of bimorph system was done by means of the graph and structural numbers method.
Keywords: Process systems design; Piezoelectric plate; Bimorph system; Graph

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1. Introduction

In recent years there is growing interest of materials, called smart materials. They have one or more properties that can be significantly changed. Smartness describes abilities of shape, size and state of aggregation changes. The main groups of smart materials are:

- piezoelectric plates,
- magneto-rheostatic materials,
- electro-rheostatic materials,
- shape memory alloys.

Those materials are widely used in technology and their numbers of applications still growing. Piezoelectric effect was discovered by French physicists Peter and Paul Curie in the 1880s. They described generation of electric charge on the surface with various shape during its deformation in different directions.

In their research, first of all, they focused on tourmaline crystal, salt and quartz. In 1881s Gabriel Lippman suggested the existence of the reverse piezoelectric phenomenon, which was confirmed experimentally by the Curie brothers. As a solution of research such two, unique properties of piezoelectric materials were assigned:

- showing of simple piezoelectric effect, which rely on generating of voltage after deformation of material,
- reverse piezoelectric effect, which rely on changing of sizes (by around 4%) after applying a voltage to piezoelectric facing.

Designing of technological systems, which contains piezoelectric elements should not be framed only to mechanical system analysis, but should be taken under consideration also electrical part.

The entity should be considered as complex system, which contains independent subsystem.

Problem with mechanical-physical systems synthesis, first of all electrical and mechanical ones, is well known and frequently published [1, 2, 3, 6, 11, 14, 15]. In articles concerned theory and designing of filters not much space was devoted to mechanical systems with parameters distributed in continuous way. Determination tests of mechatronic systems characteristics, applications of graphs and structural numbers were carried out at Silesian Center repeatedly [7-11, 12, 13, 26, 27, 28]. Those studies gave assumption to analysis of piezoelectric work.

In many publications and papers [9, 10, 15, 27], mechanical systems investigations on example of vibration beams and rods, were introduced. Moreover, rules of modellinig by non-classical method and attempts of analysis by using hypergraph skeletons [4, 7, 8], graphs with signal flow [27] and matrix methods [6, 5], were shown in these works.

Nowadays, numerous piezoelectric advantages caused its multi-application in mechanics and in many replaced field of science [24, 16, 18, 20, 25]. Many times beams configurations, with respect to different boundary conditions and during piezoelectric application in damping of vibrations, were analyzed [19, 24].

In the paper [23] capability of piezoelectric systems modelling using equivalent Manson models were presented.

Analysis of longitudinal vibrations were made taking into consideration dielectric and piezoelement layer. Mason in [21] introduced one-dimensional, equivalent system parameters widely used in modelling systems both free, and loaded [22].

The main disadvantage of such approach is the equivalent of the mechanical system by discreet model. In article [24] author presents 4-port equivalent system of piezoelectric plate, used to identification of system response on mechanical force. A matrix, size 5x5, input-output dependences, with different conditions of support, was also determined.

Another type of piezoelectric transducer, which was based on Masons alternative systems of higher number of piezoelectric layer, was presented in article [24]. Simulation was carried out in frequency domain, furthermore result was compared with values obtained by experimental method. Bellert in his volumes of chosen works [2] many times wrote about modelling of replacing systems, examined as 4-ports. In work [6] Bolkowski provide chain method of connection electric 4-ports. However, both: [6] and [2] concerned primarily electric systems. In research work number N502 071 31/3719 attempts of active, mechanical systems, with damping in scope of graphs and structural numbers methods, were analyzed. In such, rich publications from field of vibration analysis, solution of piezoelectric plate itself with respect to dynamic characteristic was not undertaken, with the exceptions [17, 19, 20, 23, 24].

Previous presented solutions were conducted mainly in field of time and concerned single plate. Present paper is continuation of mentioned publications with stack of piezoelectric plates. This work is an author's idea of calculations of complex systems with many elements. The base of calculation is matrix method and application of aggregation of graphs to determination characteristic parameters of bimorphic systems, as well as to drawing its characteristics.

2. Modelling mechatronic systems

Under consideration is vibrating piezoelectric plate with parameters distributed in a continuous way. The model has the section A, thickness d and is made of a uniform material with density ρ. It was assumed, that one of the size - the thickness is much smaller than the other two. The model is shown in Figure 1.

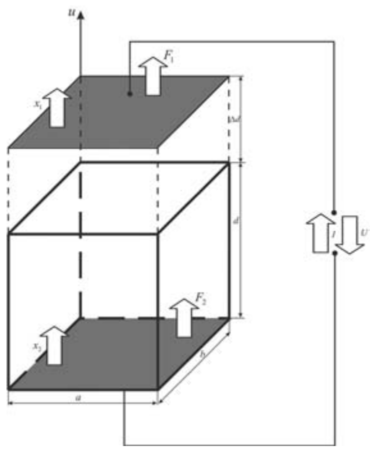

Fig. 1. Continuous and limited piezoelectric model

In the case, a matrix characteristics of piezoelectric transducer was derived from the wave equation, with the assumed boundary conditions. In the analysis of complex systems, it is not necessary solving sets of equation successive layers, a matrix of single piezoelectric plate characteristics already designated can be used. The paper also accepted, that the investigated object will be vibrating, piezoelectric plate, changing only its thickness - one-dimensional system. A piezoelectric plate constitutive equation is as follows [25]:

$$\begin{cases} T = c^E \dfrac{\partial x}{\partial u} - eE, \\ D = \varepsilon^S E + e\dfrac{\partial x}{\partial u}, \end{cases} \tag{1}$$

where:

- c^E - modulus of elasticity,
- E - the value of electric field intensity,
- e - strain,
- ε^S - electric permeability,
- D - electric induction.

Applying boundary conditions to equation (1):

$$F_1 = Ack\left[\frac{x_1}{tg(kd)} - \frac{x_2}{\sin(kd)}\right] + \frac{e}{\varepsilon^S} D, \tag{2}$$

$$-F_2 = -Ack\left[\frac{x_1}{\sin(kd)} - \frac{x_2\cos(kd)}{\sin(kd)}\right] - \frac{e}{\varepsilon^S}D. \tag{3}$$

$$U = \frac{h}{\omega}(x_1 - x_2) + \frac{1}{\omega C_0}i, \tag{4}$$

The mechanical and electrical values in equations (2)-(4) are presented in the form of 3-port system [6]:

$$\begin{bmatrix} F_1 \\ F_2 \\ U \end{bmatrix} = \begin{bmatrix} \frac{\rho VA}{tgkd} & -\frac{\rho VA}{\sin kd} & \frac{h}{\omega} \\ \frac{\rho VA}{\sin kd} & -\frac{\rho VA}{tgkd} & \frac{h}{\omega} \\ \frac{h}{\omega} & -\frac{h}{\omega} & \frac{1}{\omega C_0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ i \end{bmatrix}. \tag{5}$$

System analysis as a mechanical one are correct according to presented in the 2x2 matrix, as on equation (6):

$$\begin{bmatrix} F_1 \\ F_2 \\ U \end{bmatrix} = \begin{bmatrix} \frac{\rho VA}{tgkd} & -\frac{\rho VA}{\sin kd} & \frac{h}{\omega} \\ \frac{\rho VA}{\sin kd} & -\frac{\rho VA}{tgkd} & \frac{h}{\omega} \\ \frac{h}{\omega} & -\frac{h}{\omega} & \frac{1}{\omega C_0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ i \end{bmatrix} \tag{6}$$

In the case of the analysis of piezoelectric plates as electrical system, the following elements of matrix must be consider.

$$\begin{bmatrix} F_1 \\ F_2 \\ U \end{bmatrix} = \begin{bmatrix} \frac{Z}{tgkd} & -\frac{Z}{\sin kd} & \frac{h}{\omega} \\ \frac{Z}{\sin kd} & -\frac{Z}{tgkd} & \frac{h}{\omega} \\ \frac{h}{\omega} & -\frac{h}{\omega} & \frac{1}{\omega C_0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ i \end{bmatrix} \tag{7}$$

3. Mapping of piezoelectric plate into graph

In order to determine graph, representing modeled system of piezoelectric plates, as the symbols in the matrix were used:

$$Z = \begin{bmatrix} \frac{\rho VA}{tgkd} & \frac{h}{\omega} & -\frac{\rho VA}{\sin kd} \\ \frac{h}{\omega} & \frac{1}{\omega C_0} & -\frac{h}{\omega} \\ \frac{\rho VA}{\sin kd} & \frac{h}{\omega} & -\frac{\rho VA}{tgkd} \end{bmatrix} \tag{8}$$

The inverse matrix with respect to Z becomes:

$$Z^{-1} = \frac{1}{\det Z}\begin{bmatrix} -\frac{\rho VA}{\omega C_0 tgkd} - \frac{h^2}{\omega^2} & -\frac{\rho VA}{\sin kd}\frac{h}{\omega} - \frac{h}{\omega}\frac{\rho VA}{\sin kd} & \frac{h^2}{\omega^2} - \frac{1}{\omega C_0}\frac{\rho VA}{\sin kd} \\ \frac{h}{\omega}\frac{\rho VA}{tgkd} + \frac{\rho VA}{\sin kd}\frac{h}{\omega} & -\left(\frac{\rho VA}{tgkd}\right)^2 + \left(\frac{\rho VA}{\sin kd}\right)^2 & -\frac{\rho VA}{\omega C_0 tgkd} - \frac{h^2}{\omega^2} \\ \frac{h^2}{\omega^2} + \frac{\rho VA}{\omega C_0 \sin kd} & \frac{h}{\omega}\frac{\rho VA}{tgkd} - \frac{\rho VA}{\sin kd}\frac{h}{\omega} & \frac{h^2}{\omega^2}\frac{\rho VA}{tgkd} + -\frac{\rho VA}{\omega C_0 tgkd} \end{bmatrix} \tag{9}$$

The elements of matrix (9) assigned to the edges of graph are presented as:

$$Y_{11} = (x_1, F_1) = \frac{1}{\det Z}\left\{-\frac{\rho VA}{\omega C_0 tgkd} - \frac{h^2}{\omega^2}\right\} \tag{10}$$

$$Y_{12} = (x_1, U) = \frac{1}{\det Z}\left\{-\frac{\rho VA}{\sin kd}\frac{h}{\omega} - \frac{h}{\omega}\frac{\rho VA}{\sin kd}\right\} \tag{11}$$

$$Y_{13} = (x_1, F_2) = \frac{1}{\det Z}\left\{\frac{h^2}{\omega^2} - \frac{1}{\omega C_0}\frac{\rho VA}{\sin kd}\right\} \tag{12}$$

$$Y_{21} = (i, F_1) = \frac{1}{\det Z}\left\{\frac{h}{\omega}\frac{\rho VA}{tgkd} + \frac{\rho VA}{\sin kd}\frac{h}{\omega}\right\} \tag{13}$$

$$Y_{22} = (i, U) = \frac{1}{\det Z}\left\{-\left(\frac{\rho VA}{tgkd}\right)^2 + \left(\frac{\rho VA}{\sin kd}\right)^2\right\} \tag{14}$$

$$Y_{23} = (i, F_2) = \frac{1}{\det Z}\left\{-\frac{\rho VA}{\omega C_0 tgkd} - \frac{h^2}{\omega^2}\right\} \tag{15}$$

$$Y_{31} = (x_2, F_1) = \frac{1}{\det Z}\left\{\frac{h^2}{\omega^2} + \frac{\rho VA}{\omega C_0 \sin kd}\right\} \tag{16}$$

$$Y_{32} = (x_2, U) = \frac{1}{\det Z}\left\{\frac{h}{\omega}\frac{\rho VA}{tgkd} - \frac{\rho VA}{\sin kd}\frac{h}{\omega}\right\} \tag{17}$$

$$Y_{33} = (x_2, F_2) = \frac{1}{\det Z}\left\{\frac{h^2}{\omega^2}\frac{\rho VA}{tgkd} - \frac{\rho VA}{\omega C_0 tgkd}\right\} \tag{18}$$

The graphical representation of mapping is shown in Fig. 2:

Fig. 2. Mapping Y_{ij}

The symbol Y_{ij} is the mechanical flexibility, electrical admittance or characteristics of the system. In mapping of the parameters into the graph, mark Y_{ij} means the relationship between the vertex of graph, directed from the apex i to apex j, with the symbol $i = j$, then the following relationship were true:

$$Y_{11} = Y_{10}, \quad Y_{22} = Y_{20}, \quad Y_{33} = Y_{30}. \tag{19}$$

Dependences according to the index $j = 0$ maps a connection of the vertex with the base vertex. Following this systematic, assignment by an edge of following relations was made:

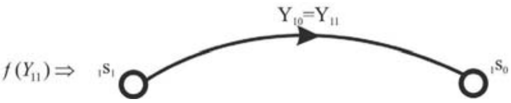

Fig. 3. Mapping Y_{10}

where $f(Y_{11})$ is the mechanical flexibility;

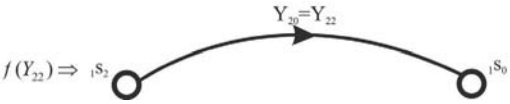

Fig. 4. Mapping Y_{20}

where $f(Y_{22})$ is admittance of electrical system;

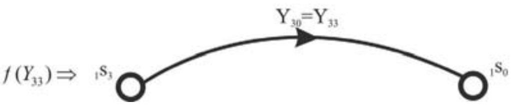

Fig. 5. Mapping Y_{30}

where $f(Y_{33})$ is mechanical flexibility;

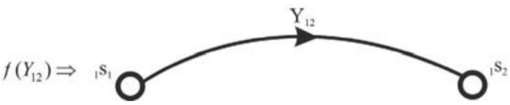

Fig. 6. Mapping Y_{12}

where $f(Y_{12})$ is system characteristic;

Fig. 7. Mapping Y_{21}

where $f(Y_{21})$ is system characteristic;

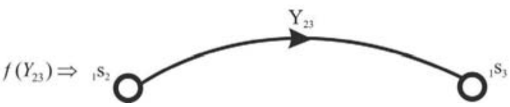

Fig. 8. Mapping Y_{23}

where $f(Y_{23})$ is system characteristic;

Fig. 9. Mapping Y_{32}

where $f(Y_{32})$ is mechanical flexibility;

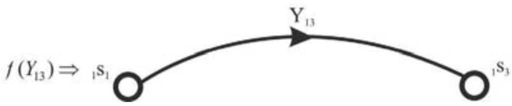

Fig. 10. Mapping Y_{13}

where $f(Y_{13})$ is mechanical flexibility;

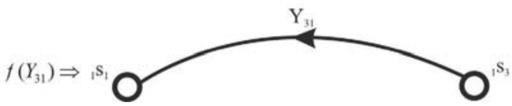

Fig. 11. Mapping Y_{31}

where $f(Y_{31})$ is mechanical flexibility.

A set of drawings of the relation (Figs. 3-11), represents 4-vertex graph (20), were created and presented in Figure 12:

$$_2X = \{Y_{11}, Y_{22}, Y_{33}, Y_{12}, Y_{21}, Y_{31}, Y_{13}, Y_{23}, Y_{32}\} \tag{20}$$

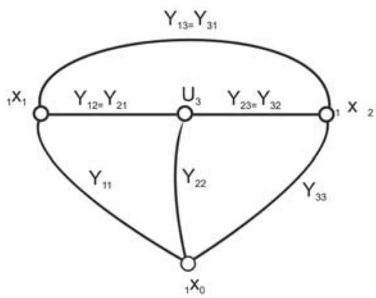

Fig. 12. Geometric representation of mapping in the graph

In the rest of the work earlier created 4-vertex graph was replaced by structural number method by the 3-vertex graph.

4. Construction of the replacement graph

Furthermore, the use of an extended 4-vertex graph may prove to complicated calculations. In such case, a modelling of system using the replaced graph was performed. In order to maintain clearness of mapping, characteristics determined in paragraph 3 are indicated by Arabic numerals in parentheses, in accordance with [2]. As a consequence of introduction of the replaced graph, a graph presented in Figure 13 was obtained. It is the basis for further network analysis methods.

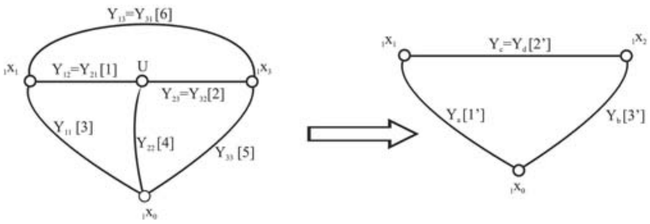

Fig. 13. Construction of the replacement graph

As a result of insertion of replaced graph, replaced flexibility of the system was calculated by structural number method:

$$Y_b = Y_{3'} = \frac{\det\limits_Z (A_{1'} \cap A_{2'})}{\det\limits_Z A_{1'2'}} = \frac{Y_4(Y_2+Y_5)+Y_5(Y_1+Y_3)}{Y_1+Y_2+Y_4}. \qquad (22)$$

$$Y_c = -Y_d = Y_{2'} = \frac{\det\limits_Z (A_{1'} \cap A_{3'})}{\det\limits_Z A_{1'3'}} = \frac{Y_1(Y_2+Y_6)+Y_6(Y_2+Y_4)}{Y_1+Y_2+Y_4}. \qquad (23)$$

$$Y_a = Y_{1'} = \frac{\det\limits_Z (A_{2'} \cap A_{3'})}{\det\limits_Z A_{2'3'}} = \frac{Y_1(Y_3+Y_4)+Y_3(Y_2+Y_4)}{Y_1+Y_2+Y_4}. \qquad (24)$$

5. Chain equation of simple plate

On the Figure 14 a piezoelectric plate with parameters distributed in the continuous way, the left and right end is free, was presented. The model of a single plate was marked by (i). Currently considered a model system is reduced system in the previous graph from 4-vertex to 3-vertex graph, as shown in Figure 14.

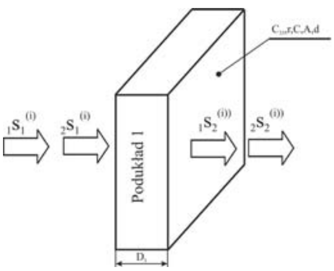

Fig. 14. Model of single piezoelectric plate after reduction

Longitudinal vibrations of piezoelectric plate were considered, in the literature described also as thickness. The parameters speciyfying the system, in accordance with the previously accepted assumptions, were the sizes of input ${}_1s_1, {}_2s_1$ and output ${}_1s_2, {}_2s_2$ values, which were presented as:

$${}_1S^{(i)} = Y\,{}_2S^{(i)} \qquad (25)$$

where:

Y is a value characterized input-output dependences.

The relations between displacements of plate, and the forces acting on them, written in matrix form:

$$\begin{bmatrix} {}_1s_1^{(i)} \\ {}_1s_2^{(i)} \end{bmatrix} = \begin{bmatrix} Y_a^{(i)} & Y_c^{(i)} \\ Y_d^{(i)} & Y_b^{(i)} \end{bmatrix} \begin{bmatrix} {}_2s_2^{(i)} \\ {}_2s_2^{(i)} \end{bmatrix} \qquad (26)$$

Transforming the matrix (26) to the chain form expects to receive in the form of matrices:

$$\begin{bmatrix} {}_2s_1^{(i)} \\ {}_1s_1^{(i)} \end{bmatrix} = \begin{bmatrix} A_{11}^{(i)} & A_{12}^{(i)} \\ A_{21}^{(i)} & A_{22}^{(i)} \end{bmatrix} \begin{bmatrix} {}_2s_2^{(i)} \\ {}_1s_2^{(i)} \end{bmatrix} \qquad (27)$$

where:

$$\begin{cases} A_{11}^{(i)} = \dfrac{Y_b^{(i)}}{Y_c^{(i)}}, \\ A_{12}^{(i)} = \dfrac{1}{Y_c^{(i)}}, \\ A_{21}^{(i)} = \dfrac{Y_c^{(i)}Y_d^{(i)} - Y_a^{(i)}Y_b^{(i)}}{Y_d^{(i)}}, \\ A_{22}^{(i)} = \dfrac{Y_a^{(i)}}{Y_d^{(i)}}. \end{cases} \qquad (28)$$

Figure 15 presents the free system, consisting of two plates. Superscript indicates the subsequent number of subsystem.

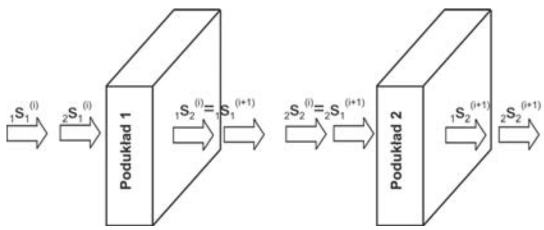

Fig. 15. Diagram of a connection between the two cells and the relation between them

$$B = \begin{bmatrix} {}_2s_1^{(i)} \\ {}_1s_1^{(i)} \end{bmatrix} = \begin{bmatrix} A_{11}^{(i)} & A_{12}^{(i)} \\ A_{21}^{(i)} & A_{22}^{(i)} \end{bmatrix} \begin{bmatrix} A_{11}^{(i+1)} & A_{12}^{(i+1)} \\ A_{21}^{(i+1)} & A_{22}^{(i+1)} \end{bmatrix} \begin{bmatrix} {}_2s_2^{(i+1)} \\ {}_1s_2^{(i+1)} \end{bmatrix} \qquad (29)$$

Finally, chain equation was written in general form:

$$A^{(k)} = A^{(i)} A^{(i+1)}. \qquad (30)$$

After the operations carried out according to (30) it was found, that the chain matrix with cascade structure is the ratio of chain matrix of individual cells of the complex system. Obtained transition matrix is presented as:

$$B = \begin{bmatrix} {}_2s_1^{(i)} \\ {}_1s_1^{(i)} \end{bmatrix} = \begin{bmatrix} A_{11}^{(i+1)} A_{11}^{(i)} + A_{12}^{(i)} A_{21}^{(i+1)} & A_{11}^{(i)} A_{12}^{(i+1)} + A_{12}^{(i)} A_{22}^{(i+1)} \\ A_{21}^{(i)} A_{11}^{(i+1)} + A_{22}^{(i)} A_{21}^{(i+1)} & A_{21}^{(i)} A_{12}^{(i)} + A_{22}^{(i)} A_{22}^{(i+1)} \end{bmatrix} \begin{bmatrix} {}_2s_2^{(i+1)} \\ {}_1s_2^{(i+1)} \end{bmatrix} \tag{31}$$

Calculated coefficients (31) were substituted and the final form of the transition matrix was received:

$$B = \begin{bmatrix} B_{11}^{(k)} & B_{12}^{(k)} \\ B_{21}^{(k)} & B_{22}^{(k)} \end{bmatrix} \tag{32}$$

where:

$$\begin{cases} B_{11}^{(k)} = \dfrac{Y_b^{(i)} Y_b^{(i+1)}}{Y_d^{(i)} Y_d^{(i+1)}} + \dfrac{-Y_a^{(i+1)} Y_b^{(i+1)} + Y_c^{(i+1)} Y_d^{(i+1)}}{Y_d^{(i)} Y_d^{(i+1)}}, \\ B_{12}^{(k)} = \dfrac{Y_b^{(i)}}{Y_d^{(i)} Y_d^{(i+1)}} + \dfrac{Y_a^{(i+1)}}{Y_c^{(i)} Y_d^{(i+1)}}, \\ B_{21}^{(k)} = \dfrac{(-Y_a^{(i)} Y_b^{(i)} + Y_c^{(i)} Y_d^{(i)}) Y_b^{(i+1)}}{Y_d^{(i)} Y_d^{(i+1)}} + \dfrac{Y_a^{(i)} (-Y_a^{(i+1)} Y_b^{(i+1)} + Y_c^{(i+1)} Y_d^{(i+1)})}{Y_d^{(i)} Y_d^{(i+1)}}, \\ B_{22}^{(k)} = \dfrac{-Y_a^{(i)} Y_b^{(i)} + Y_c^{(i)} Y_d^{(i)}}{Y_d^{(i)} Y_d^{(i+1)}} + \dfrac{Y_a^{(i)} Y_a^{(i+1)}}{Y_d^{(i)} Y_d^{(i+1)}}, \end{cases} \tag{33}$$

In order to obtain the flexibility of the complex system, calculated coefficients of chain equation (29), was transformed to the basic form:

$$\begin{cases} Y_a^{(k)} = \dfrac{B_{22}{}^{(k)}}{B_{12}{}^{(k)}}, \\ Y_c^{(k)} = -\dfrac{B_{11}{}^{(k)} B_{22}{}^{(k)}}{B_{12}{}^{(k)}} - B_{21}{}^{(k)}, \\ Y_d^{(k)} = \dfrac{1}{B_{12}{}^{(k)}}, \\ Y_b^{(k)} = \dfrac{B_{11}{}^{(k)}}{B_{12}{}^{(k)}}. \end{cases} \tag{34}$$

Equation (34) is the components of the complex characteristics of the matrix taking into account obtained chain parameters of complex system.

6. Charts of simple and bimorph system

In this paragraph, graphical charts of characteristics of piezoelectric plates were shown. Figures 16-19 contain the influence of different parameters, such as: the thickness, the surface area, the density of the material.

In Figures 20 and 21 characteristics of the complex system with respectively the same/different thickness and in domain of the frequency are presented.

Furthermore, in Figure 22 three dimensional characteristics of the complex system with different thickness and in domain of the frequency are presented.

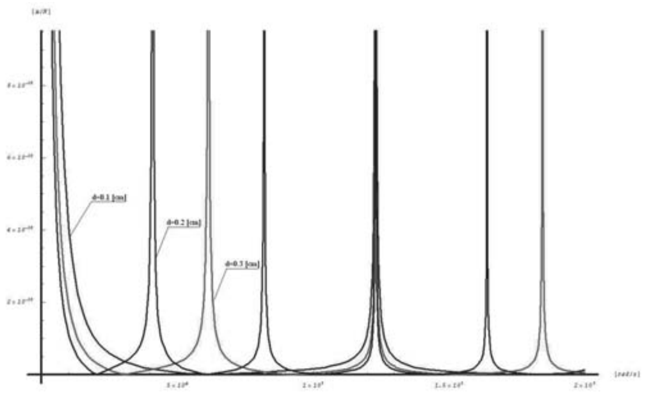

Fig. 16. Characteristics of a single piezoelectric plate in domain of the frequency depending on the thickness of the plate

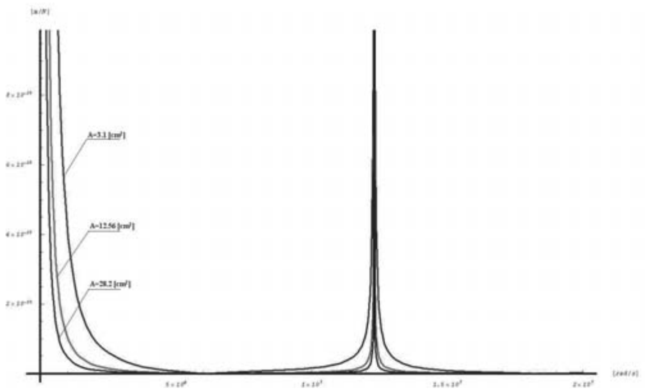

Fig. 17. Characteristics of a single piezoelectric plate in domain of the frequency depending on the plate surface area

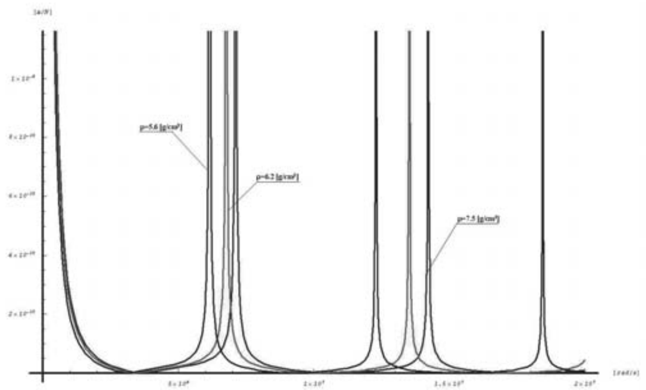

Fig. 18. Characteristics of a single piezoelectric plate in domain of the frequency depending on the piezoelectric density

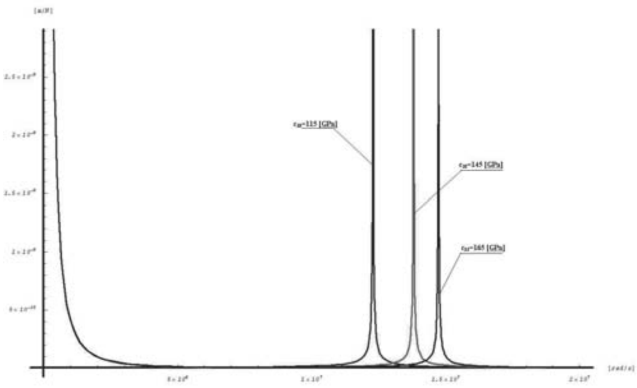

Fig. 19. Characteristics of a single piezoelectric plate in domain of the frequency depending on the module of piezoelectric elasticity

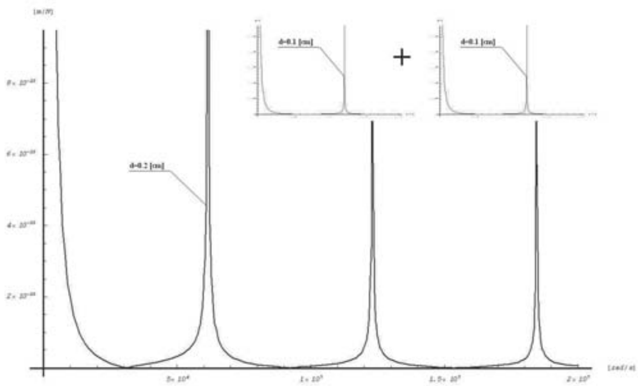

Fig. 20. Characteristics of piezoelectric stack containing two plates with the same thickness and in domain of the frequency

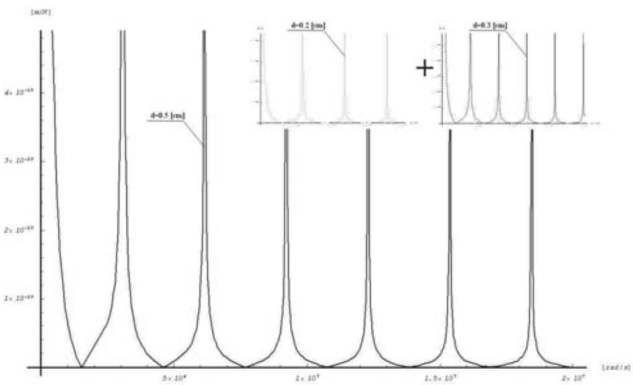

Fig. 21. Characteristics of piezoelectric stack containing two plates with different thickness and in domain of the frequency

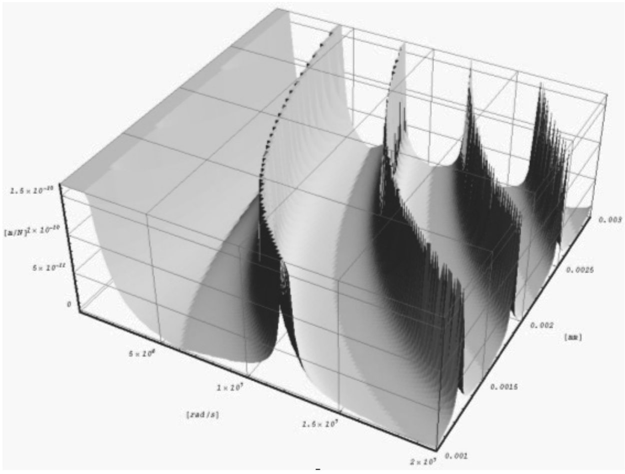

Fig. 22. 3D characteristics of the complex system with different thickness and in domain of the frequency

7. The practical application of the complex systems analysis

Presented analysis method of piezoelectric complex systems is well suited for determining the flexibility of piezoelectric stack in vibration sensors (Fig. 23). Those sensors are used to the level detection of materials in open or pressurized tanks. Output signal is transmitted to the automation systems via relay.

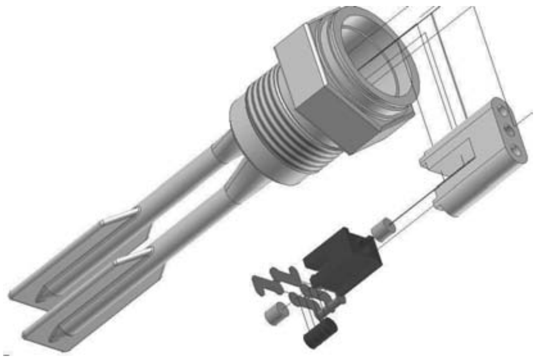

Fig. 23. Construction of level switch

The sensors consist of two pairs of receiving plates and two or three supplying plates, connected in a bimorphic system. Variable voltage, which feeds the supply to the plate thickness results in a change in their proportion to the value of applied voltage. Changes in the thickness of the plate causing mechanical vibrations of the element, called "fork". When the "forks" are not covered, full vibrations of supplying plates are transferred to the receiving ones.

As a result of elongation of receiving plates, on its facing, there is a difference potential proportional to the force. The value of this voltage is transformed by an electronic system. In the case of "forks" not covered by material, the receiving plates are generating the same potential as in supplied plates. Presented process and application are used in vibrating sensors produced by "Nivomer" company.

8. Conclusions

In the article problem of mechatronic system analysis on example of vibrational longitudinal piezoelectric plates was presented.

Characteristic dependences of single piezoelectric plate was derived, taking into account electric properties such as current and voltage. An image representation of the matrix, and furthermore replacing graph by structural numbers method was determined.

Finally, the charts of obtained flexibility of complex system, containing several piezoelectric plates and graphs, with impact of system parameters on the characteristics, were presented.

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References

[1] K. Arczewski, Structural Methods of the Complex Mechanical Systems Analysis, WPW, Warsaw, 1988.
[2] S. Bellert, Chosen works, PWN, Warsaw, 1981.
[3] M. Białko, Active filters RC, WNT, Warsaw, 1979.
[4] C. Berg, Graphs and hypergraphs. Amsterdam-London:North Holland Publishing Co, American Elsevier Publishing Co, Inc., New York, 1973.
[5] R.E.D. Bishop, G.M.L. Gladwell, S. Michaelson, Matrix analysis of vibration, WNT, Warsaw, 1972.
[6] S. Bolkowski, Theoretical electrical engineering, WNT, Warsaw, 1986.
[7] A. Buchacz, J. Świder, Skeletons hypergraph in modeling, examination and position robot's manipulator and subassembly of machines, Silesian University of Technology Press, Gliwice, 2000.
[8] A. Buchacz, Hypergrphs and their subgraphs in modelling and investigation of robots, Journal of Materials Processing Technology 157-158 (2004) 37-44.
[9] S. Żółkiewski, Dynamical flexibilities of mechanical rotational systems, Journal of Achievements in Materials and Manufacturing Engineering 31/2 (2008) 602-609.
[10] A. Buchacz, S. Żółkiewski, Longitudinal vibrations of the flexible n-bar manipulator in terms of plane motion and taking into consideration the transportation effect, Proceedings of the 7th International Conference on Computer Integrated Manufacturing - Intelligent Manufacturing Systems, Gliwice – Wisła, 2005, 26-29.
[11] A. Buchacz, Dynamical flexibility of discretecontinuous vibrating mechatronic system, Journal of Achievements in Materials and Manufacturing Engineering 28/2 (2008) 159-166.
[12] A. Buchacz, A. Wróbel, The investigation of the piezoelectric effect of bimorph system and his practical application in level sensors, XII The Scientific conference, of vibroacoustics and vibrotechnique, Jachranka, 2007, 63-66.
[13] A. Buchacz, A. Wróbel, The analysis of simple and complex piezoelectric systems, XVII Sympozjon "The modelling in the mechanics", project N 502 071 31/3719, Wisla, 2008, 25-29.
[14] K. Białas, Graphs and structural numbers in analysis and synthesis of mechanical systems, Journal of Achievements in Materials and Manufacturing Engineering 29/2 (2008) 151-154
[15] Z. Bychawski, The mechanics of continuous centres. University of Rzeszów, Press, Rzeszów, 1979.
[16] L.-W. Chen, C.-Y. Lin, C.-C Wang, Dynamic stability analysis and control of a composite beam with piezoelectric layers, Composite Structures 56/1 (2002) 97-109.
[17] S.K. Ha, Analysis of a piezoelectric multimorph in extensional and flexular motions, Journal of Sound and Vibration 253/3 (2002) 1001-1014,.
[18] B. Heimann, W. Gerth, K. Popp, Mechatronic - components, methods, examples, PWN, Warsaw, 2001.
[19] R. Kacprzyk, E. Motyl, J.B. Gajewski, A. Pasternak, Piezoelectric properties of nouniform electrets, Journal of Electrostatics 35 (1995) 161-166.
[20] G.Q Li, Chen Chuan-Yao, Hu Yuan-Tai, Equivalent electric circuits of thin plates with two-dimensional piezoelectric actuators, Journal of Sound and Vibration 286 (2005) 145-165.
[21] W.P. Mason, Electromechanical Transducers and Wale Filters, Van Nostrand, 1948.
[22] W.P. Mason, Physical acoustics and the properties of solids, Princeton NJ, Van Nostrand, 1958.
[23] S. Sherrit, S.P. Leary, B.P. Dolgin, Comparison of the Mason and KLM equivalent circuits for piezoelectric resonators in thickness mode, Ultrasonics Symposium, 1999.
[24] H. Shin, H. Ahn, D.Y. Han, Modeling and analysis of multilayer piezoelectric transformer, Materials Chemistry and Physics 92 (2005) 616-620.
[25] W. Soluch, The introduction to piezoelectronics, WKiŁ, Warsaw, 1980,
[26] A. Sękala, J. Świder, Hybrid Graphs in Modelling and Analysis of Discrete–Continuous Mechanical Systems, Journal of Materials Processing Technology 164-165 (2005) 1436-1443.
[27] A. Sękala, J. Świder, Analysis of continuous mechanical systems by means of signal flow graphs, Proceedings of the 7th International Conference on Computer Integrated Manufacturing - Intelligent Manufacturing Systems, Gliwice – Wisła, 2005, 208-211.
[28] G. Wszołek, Modelling of mechanical systems vibrations by utilisation of GRAFSIM software, Journal of Materials Processing Technology 164-165 (2005) 1466-1471.