A STUDY IN THEORY UNIFICATION: THE CASE OF KALUZA-KLEIN THEORIES

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Abstract

In this paper, I call attention to *the higher dimensional unified field theory program* that has culminated in a class of higher dimensional spacetime theories, called the Kaluza-Klein (KK) theories, aiming to unify gravity with gauge fields in a higher dimensional Riemannian spacetime. I examine theory unification both in the original KK theory, which originated in the works of Theodor Kaluza and Oskar Klein in the twenties, and in the modern KK theories—namely, the higher dimensional superstring and supergravity theories—which date back to the late seventies and which are still considered by the majority of the physics community to be the best hope for a complete unified theory of all fundamental interactions. I use the conclusions of this case-study to assess the merits of the *unificationist* account of scientific explanation advanced by Philip Kitcher. In conclusion, I argue that the conceptions of unity leading to the construction of the KK theories have features that are quite distinct from those asserted by Kitcher's account.

Key-words: Unification; Explanation; Scientific Understanding; Argument Patterns; Higher Dimensions; Gauge Symmetries; Kaluza-Klein Theories; Yang-Mills Theories, Standard Model of Particle Physics.

1. Introduction.

The criticism advanced by Michael Friedman against Carl Hempel's (1962) deductive-nomological (D-N) model of scientific explanation put a new spin not only on the issue of what counts as a successful scientific explanation, but also on the perennial debate concerning the notion of "unity" in science. Friedman (1974) argued that even though the D-N model succeeded in providing an objective account of scientific explanation, it failed in answering the questions of how and why our scientific understanding of the world increases by way of explanations. In this respect, Friedman called attention to the necessity of an *objective* account of scientific understanding and proposed a criterion for distinguishing explanations that yield *genuine* scientific understanding of the world from those that do not. In Friedman's view, as in the D-N model, scientific explanations are deductive derivations of descriptions of natural phenomena under consideration from premises that include laws of nature and statements of initial conditions. However, unlike in the D-N model, they are characterized as *unifications* that maximize the number of explained phenomena while minimizing the number of *independent* laws.

Friedman's account was criticized by Philip Kitcher (1976), who later offered his own account of scientific explanation as unification, namely "explanatory unification" (Kitcher 1981, 1989, 1993). Even though Kitcher's approach to explanation has been criticized on a number of grounds[1], it still stands as one of the notable accounts of scientific explanation in the literature of philosophy of science. In this paper, I will assess the merits of Kitcher's account within the context of the higher dimensional unified field theories, called the Kaluza-Klein (KK) theories, which aim to unify gravity with gauge (nuclear) force-fields in a higher dimensional Riemannian spacetime. By considering both the original five-dimensional KK theory and its modern

[1] See especially Barnes (1992), Humphreys (1993), Halonen and Hintikka (1999) and Morrison (2000).

elaborations, namely the modern KK theories, I will argue that Kitcher's account fails to capture the essential features of the unification agenda implemented in *the higher dimensional unified field theory program*. The present paper has also an aim to explore why KK approach to unification received renewed interest in the seventies and eighties after almost half a century passed since its first emergence in the twenties. But, before embarking on these goals, in the next section, I will take a closer look at Kitcher's account of explanation.

2. Kitcher on explanatory unification

Kitcher's model of explanation shares the basic tenet of Friedman's model, namely that scientific explanation is a deductive process that proceeds through unification. Unlike Friedman, however, Kitcher does not think that unification necessarily results in a reduction in the number of independent laws used in the explanations of natural phenomena. He rather thinks that unification brings about a reduction in the number of *types* of facts that are taken to be fundamental to natural phenomena, and that this is essential to the scientific understanding of nature. In this respect, in Kitcher's view, the essence of scientific explanation, and thus the fundamental goal of scientific theorizing, is unification, and this basically means the derivation of a wealth of conclusions regarding natural phenomena by the *repeated* use of a relatively small number of, what Kitcher calls, "argument patterns".

The notion of "argument pattern" is central to Kitcher's account of explanation. Kitcher uses this notion to express the structural and linguistic *commonalities* among the derivations of the same scientific theory. According to Kitcher's definition, an argument pattern consists of three main elements. The essential element is what is called by Kitcher a "schematic argument" which consists of a set of sentences called "schematic sentences." A schematic sentence is an expression obtained by replacing some, but not necessarily all, non-logical expressions appearing in a given sentence with dummy letters. For instance, the sentence "Organisms homozygous for the sickling allele develop sickle-cell anemia" can be replaced by the following sentence "Organisms homozygous for A develop P." The way how the dummy letters are to be replaced in a given schematic sentence is determined by a set of directions which Kitcher calls "filling instructions." For instance, in the above schematic sentence, specifying that A and B are to be replaced to denote respectively an allele and a phenotypic trait constitutes the filling instructions. And, the last element an argument pattern must possess is a "classification" for a schematic argument. This is a set of sentences containing information as to how inferential relations among the sentences of the same schematic argument work, i.e., which ones of the sentences are premises, and which conclusions are inferred from those premises.

In Kitcher's account, however, not every (deductive) derivation fitting into a general argument pattern counts as *successful* explanation. Only derivations instantiating argument patterns which belong to a particular set qualify as successful (1989, p. 431). Kitcher defines this set to be E(K) and calls it "explanatory store" over K, where K represents the set of sentences or beliefs endorsed by a scientific community at some point in the history of scientific inquiry. Kitcher calls any set of arguments that derives some members of K from other members of K a "systematization" of K. Then, he assumes that for a given set of beliefs K, E(K) is *unique* and represents the set of argument patterns that best systematizes K. For Kitcher, the criterion for best systematization is *unification* that is geared to generating as many conclusions as possible about natural phenomena by using as few and stringent patterns as possible (1989, p. 434). Kitcher defines the degree of stringency of an argument pattern as depending on the restrictions imposed by the filling instructions on the expressions to be used for the substitution of dummy letters in schematic sentences and also on the restrictions placed by the classification of the argument pattern on what kinds of inferences to be made from these schematic sentences (1989, 479-480). Therefore, according to Kitcher's account, "the unifying power [of a scientific theory] depends on paucity of patterns used, size of conclusion set, and stringency of patterns" (1989, p. 478). Note that in Kitcher's account it is not only the number of explanations, but also the ways in which those explanations are derived are decisive in the unifying power of a scientific theory.

Kitcher's account is partly a *normative* account about how successful explanations in science ought to be. However, I think, there is also a sense in which Kitcher's account can be taken as having a *descriptive* content with regard to the actual practice of scientific theorizing. By arguing that explanations in science are arrived at by a process of unification that proceeds through derivations of larger number of conclusions by using fewer and more stringent argument patterns than do the previous theories, Kitcher also aims to describe the actual practice of scientific theorizing. In this respect, Kitcher frequently makes appeal to specific cases from the history of scientific practice to vindicate his criterion of theory unification. Kitcher's paradigm examples are Newton's theory of motion and Darwin's theory of evolution. Here are some representative passages from these examples:

> The unifying power of Newton's work consisted in its demonstration that one pattern of argument could be used again and again in the derivation of a wide range of accepted sentences. (1981, p. 514)

> In place of detailed evolutionary stories, Darwin offers explanation-sketches. By showing how a particular characteristic would be advantageous to a particular species, he indicates an explanation of the emergence of that characteristic in the species, suggesting the outline of an argument instantiating the general pattern.
>
> From this perspective, much of Darwin's argumentation in the *Origin* … becomes readily comprehensible. Darwin attempts to show how his pattern can be applied to a host of biological phenomena. (1981, p. 515)

In Kitcher's view, "the moral of the Newtonian and Darwinian examples is that unification is achieved by using similar argument patterns in the derivation of many accepted sentences" (1981, p. 519). Kitcher takes these and other historical cases—such as the classical genetics and the theory of the chemical bond—"to provide *prima facie* support for the view that unification is important to explanation and that *unification works in the way [he] suggested*" (1989, p. 448, emphasis added). So, Kitcher is convinced that scientific practice vindicates his claim that what determines the explanatory power of scientific theories is their unifying power, and this in turn amounts to offering a relatively more stringent and fewer argument patterns from which a relatively larger number of conclusions can be drawn about natural phenomena.

In what follows, I will examine the KK theories and suggest a different conclusion with regard to the issue of unification in relation to the actual practice of scientific theorizing.

3. Kaluza-Klein theory in perspective

What is today known as "Kaluza-Klein theory" was developed in the twenties by the efforts of Theodor Kaluza and Oskar Klein to unify electromagnetism and gravity in a five-dimensional Riemannian spacetime by invoking an extra *spatial* dimension. KK approach to unification in physics has manifested its impact over the decades in many attempts to unify gravity with the other fundamental force-fields of nature.[2] In the later stages of the present paper, I will discuss the issue of how the later developed unified field theories—such as the higher dimensional supergravity and superstring theories—have made use of the idea of higher-dimensional unification à la KK. But, in the first place, I think, a closer look at the original KK theory would be helpful.

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[2] The literature of history and philosophy of science is very sparse with regard to the issue of KK unification, despite the immenseness of the physics literature on the same topic. See for example the accounts by Cao (1988), Vizgin (1994), Huggett and Weingard (1999), and van Dongen (2002). Also, a chronological compilation of important physics papers published on KK unification can be found in Appelquist et al., (1987).

3.1 Kaluza's theory: unification of gravity and electromagnetism

At the outset of his 1921 paper, entitled *On the Unity Problem of Physics*, Kaluza makes explicit that by the *unity* of electromagnetism and gravity he means that "the gravitational and electromagnetic fields stem from a single universal tensor" (1921). Kaluza views such a unity as "a close union of the two forces of the world in principle" (1921). This view manifests itself in Kaluza's account as a *strategy* of writing the Christoffel symbols $\left[{i \atop l}{k}\right] = -\Gamma_{ikl}$ in a five-dimensional Riemannian spacetime (with coordinates x^1, x^2, x^3, x^4, x^5, where x^5 stands for the fifth coordinate, and spacetime metric $g_{l\kappa}$, where $l, k = 1,...,5$), and seeking for a *proportionality* relation between the Christoffel symbols in five dimensions and the components of the four-dimensional electromagnetic field tensor $F_{\mu\nu}$, where $\mu, \nu = 1,...4$. In order for that strategy to work, Kaluza made use of the following conjecture:

> Our previous physical experience contains no hint as to an additional [spacelike dimension], but we are at liberty to view our spacetime as a four-dimensional part of [a five-dimensional Riemannian spacetime]; one has, however, to take account of the fact that we never observe changes of physical quantities other than in spacetime by setting their derivatives with respect to the [fifth dimension] to zero, or treats them as small of higher order ("cylinder condition"). (1921)

In the above passage, Kaluza makes explicit how he conceives of enlarging the dimensionality of spacetime. For him, the introduction of the fifth dimension is *legitimate* as long as physical quantities do not depend appreciably on the fifth dimension. This, according to Kaluza, follows from the fact that neither the fifth dimension nor its effect whatsoever is *physically* perceivable. This consideration led Kaluza to set forth the condition which he called the "cylinder condition", according to which all partial derivatives of the components of the spacetime metric and those of other matter fields—such as the electromagnetic field—with respect to the fifth dimension vanish. Kaluza's treatment of the fifth dimension via the cylinder condition is central to his theory in that it enabled him to construct the proportionality relation that he was seeking for. By means of the cylinder condition, and with the aid of the identifications: $g_{5\mu} = 2\alpha A_\mu$ and $g_{55} = 2\phi$, where ϕ is a scalar function, and α is a constant of proportionality, Kaluza was able to write:

$$\Gamma_{5\mu\nu} = \alpha(A_{\mu,\nu} - A_{\nu,\mu}) = \alpha F_{\mu\nu}, \tag{1a}$$

$$\Gamma_{\mu\nu 5} = -\alpha(A_{\mu,\nu} + A_{\nu,\mu}) = -\alpha\Sigma_{\mu\nu}, \tag{1b}$$

$$\Gamma_{55\mu} = -\Gamma_{5\mu 5} = \phi_{,\mu}, \tag{1c}$$

where A_μ and $F_{\mu\nu}$ represent respectively the electromagnetic vector potential and the electromagnetic field strength tensor in four dimensions, and $\Sigma_{\mu\nu} = A_{\mu,\nu} + A_{\nu,\mu}$ was called the "associated" field (*Nebenfeld*) by Kaluza.[3] The first relation was exactly the kind of proportionality relation that Kaluza was seeking for between the Christoffel symbols in five dimensions and the components of the four-dimensional electromagnetic field tensor.

Upon these results, Kaluza assumed that the gravitational field is *weak* and that the five-dimensional Riemannian spacetime deviates slightly from the Minkowski spacetime. In this limit, called the *weak-field* limit, Kaluza took the metric tensor in five-dimensional Riemannian spacetime to be $g_{lk} = \delta_{lk} + \gamma_{lk}$, where δ_{lk} is the flat Minkowski metric in the five-dimensional spacetime, and γ_{lk} stands for a symmetric second-rank tensor representing small perturbation whose contributions of order higher than one are ignored.[4] In the light of this approximation,

[3] Kaluza did not account for ϕ and the role of this term remained unsettled in the original formulation of the KK theory.
[4] The weak-field limit is used as part of a standard procedure to *linearize* the non-linear gravitational field equations.

Kaluza calculated the components of the five-dimensional analog of the Ricci tensor as in the following:

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda}, \quad (2a)$$

$$R_{5\nu} = -\alpha\partial^{\mu}F_{\mu\nu}, \quad (2b)$$

$$R_{55} = -\Box\varnothing. \quad (2c)$$

Note that the associated field $\Sigma_{\mu\nu}$ does not appear in the above set of equations. Then, Kaluza assumed that the five-dimensional word is *devoid* of both matter and electromagnetic fields, that is, its total energy-momentum is zero, meaning that the components of the five-dimensional Ricci tensor are all equal to zero. Under this assumption and the weak-field limit, note that the first equation in the above set can be recognized as the four-dimensional Einstein field equations in *vacuum*. In addition, by making use of the identity: $(\Gamma_{ikl} + \Gamma_{kli} + \Gamma_{lik})_{,n} = \Gamma_{nik,l} + \Gamma_{nkl,i} + \Gamma_{nli,k}$, Kaluza also obtained the expression: $F_{\mu\nu,\lambda} + F_{\nu\lambda,\mu} + F_{\lambda\mu,\nu} = 0$, which, together with the equation in (2b), gives the full set of Maxwell's equations in *vacuum*. And, the third equation in the above set may be taken as a *Poisson* type of equation for the *uninterpreted* function $\varnothing$. These results were striking in the sense that the transition from the five-dimensional spacetime to ordinary four-dimensional spacetime via the cylinder condition had led Kaluza to the correct mathematical forms of the field equations of both Einstein's general theory of relativity (GTR) and Maxwell's electrodynamics (EMT) in *vacuum*. In Kaluza's view, "[t]herein lies the first justification … for the hope to recognize gravitation and electricity as manifestations of a universal field" (1921).

It is to be noted that in Kaluza's theory the *recovery* of the four-dimensional gravitational and electromagnetic field equations, as given in GTR and EMT respectively, became possible only in the weak-field limit, where the *non-linear* contributions from the spacetime metric are taken to be *negligibly* small. Now, I shall turn to Klein's theory where the *exact* splitting of the five-dimensional field equations into the Einstein-Maxwell equations was obtained.

3.2 Klein's theory: an elucidation and elaboration of Kaluza's theory

In two papers written a few years after Kaluza's 1921 paper, Klein (1926a, 1926b) elaborated on Kaluza's theory and established its connection with the wave-mechanics formulation of the quantum theory (QT)—which had been recently developed by Erwin Schrodinger. At the outset of his 1926a paper, entitled *Quantum Theory and Five-dimensional Theory of Relativity*, Klein adopts the following five-dimensional Riemannian metric: $d\sigma = \sqrt{\sum \gamma_{ik} dx_i dx_k}$, where γ_{ik} stands for the covariant components of the metric tensor; x_1 and x_2, x_3, x_4, x_5 represent respectively the time and the space coordinates; and Σ indicates a summation over $i, k = 1, 2, 3, 4, 5$. One remarkable novelty brought out by Klein in the first of these papers, is the specification of the group of coordinate transformations under which the cylinder condition holds. Klein specifies this group as follows:

$$x^5 = x^{5'} + \Psi_0(x^{1'}, x^{2'}, x^{3'}, x^{4'}), \tag{3a}$$

$$x^i = \Psi_i(x^{1'}, x^{2'}, x^{3'}, x^{4'}) \qquad (i = 1, 2, 3, 4) \tag{3b}$$

At this point, it is to be noted that one remarkable feature of GTR is its being a *generally covariant* theory, meaning that the field equations of GTR retain their mathematical forms under arbitrary differentiable spacetime coordinate transformations. This is also true for any higher dimensional formulation of GTR like the five-dimensional one considered by both Kaluza and Klein. The general covariance feature of GTR provides a freedom, called "gauge freedom", in the choice of spacetime coordinate transformations. And, the choice of a particular gauge singles out the mathematically admissible coordinate transformations that constitute the general covariance group of GTR. In this sense, the postulation of the cylinder condition by Kaluza in his 1921a paper can be interpreted as a choice of gauge, and the group of coordinate transformations specified by Klein, as given in (3a) and (3b), under which the cylinder condition holds, can be seen as the general covariance group of coordinate transformations admitted by the five-dimensional KK theory.

Under this group of coordinate transformations, Klein observes that γ_{55} remains invariant. Upon this, he takes $\gamma_{55} = \alpha = constant$, and then writes the five-dimensional metric as $d\sigma^2 = \alpha d\theta^2 + ds^2$, where

$$d\theta = dx^5 + \frac{\gamma_{5i}}{\gamma_{55}} dx^i, \quad (4a)$$

$$ds^2 = (\gamma_{ik} - \frac{\gamma_{5i}\gamma_{5k}}{\gamma_{55}}) dx^i dx^k. \quad (4b)$$

Klein notes that the differential quantities in (4a) and (4b) remain invariant under the set of coordinate transformations given in (3a) and (3b), and then he shows that the invariance of $d\theta$ entails that γ_{5i} transforms under the same coordinate transformations as $\gamma_{5i} \rightarrow \gamma_{5i} + \alpha\nabla\Psi_0$, where ∇ is a four-dimensional gradient operator. Note that the way γ_{5i} transforms under the coordinate transformations given in (3a) and (3b) is indeed the same as the way the electromagnetic vector potential A_μ transforms under U(1) electromagnetic gauge transformation in EMT.[5] Upon this observation, Klein identifies γ_{5i} as the components of A_μ and takes $\gamma_{5i} = \alpha\beta\phi_i$, where β is a constant and ϕ_i represents the components of the electromagnetic four potential. An important consequence of this identification can be noted as

[5] Under electromagnetic U(1) gauge transformation, $A_\mu \rightarrow A_\mu^{'} = A_\mu - \partial_\mu \chi$, where χ is an arbitrary scalar function.

the interpretation of the electromagnetic gauge transformation as a coordinate transformation associated with the fifth dimension.

After having specified the spacetime metric in the way mentioned above, Klein constructed the five-dimensional field equations from the *variational principle*: $\delta\int P\sqrt{-\gamma}dx^1dx^2dx^3dx^4dx^5 = 0$, where P is the five-dimensional scalar curvature, and γ is the determinant of γ_{ik}. This procedure enabled Klein through the cylinder condition to derive the following set of field equations:

$$R^{ik} - \frac{1}{2}g^{ik}R + \frac{\alpha\beta^2}{2}S^{ik} = 0, \qquad (i,k = 1, 2, 3, 4) \tag{5a}$$

and

$$\frac{\partial\sqrt{-g}F^{i\mu}}{\partial x^{\mu}} = 0, \qquad (i = 1, 2, 3, 4) \tag{5b}$$

where S^{ik} represents the contravariant components of the electromagnetic energy-momentum tensor. If one makes the identification $\alpha\beta^2/2 = \kappa$, where κ denotes the gravitational constant, the equation in (5a) can be recognized as the four-dimensional gravitational field equations of GTR, for which the energy-momentum tensor $T_{\mu\nu}$ is that of a electromagnetic field in free space (i.e., in vacuum). And, Eq. (5b) can be recognized as the Maxwell equations in vacuum.[6]

It is to be noted that the cylinder condition plays an essential part in the theoretical framework of KK theory. By way of the cylinder condition, the five-dimensional gravitational field equations of KK theory get reduced into the four-dimensional Einstein-Maxwell equations for a source-free electromagnetic field. This reduction, later called "dimensional reduction", can

[6] Klein's derivation of the *exact* mathematical forms of the four-dimensional Einstein-Maxwell equations for a source-free electromagnetic field has been possible by virtue of his treatment of g_{55} as a constant. By this move, Klein indeed ruled out ϕ as a new scalar field. On the other hand, Kaluza's treatment of g_{55} as a new scalar field has been acknowledged by the later developed modern KK theories, and this new scalar field was given the name the "dilaton" field.

be viewed as a *projection* of the five-dimensional field equations of KK theory onto the usual four-dimensional spacetime structure in the limit where the effects of the fifth dimension on physical quantities becomes negligibly small. At first glance, Kaluza's use of the cylinder condition might look like a simple mathematical *manoeuvre* aiming to evade the burden of explaining the effects of the fifth dimension, and his subsequent *recovery* of the field equations of both EMT and GTR might similarly be seen as the product of a simple mathematical *coincidence*. However, Klein's identification of the group of coordinate transformations—under which the cylinder condition holds—shows that the cylinder condition indeed represents a particular choice of gauge—namely, the electromagnetic U(1) gauge—which puts a certain restriction on the general covariance group of the five-dimensional KK theory, in the sense that spacetime coordinate transformations along the coordinate associated with the fifth dimension are taken to be (electromagnetic) U(1) gauge transformation. Within the theoretical framework of KK theory, this means that the general covariance group associated with the fifth dimension is taken to be consisting of coordinate transformations that leave the electromagnetic field tensor invariant. And, the rest of the general covariance group represents the usual covariance group of GTR in four-dimensional spacetime. Therefore, by way of the cylinder condition, the general covariance group of KK splits into two distinct parts[7], and this manifests itself as a split of the five-dimensional field equations of KK into two parts as representing separately the field equations of GTR and EMT in vacuum. This explains why it was not surprising at all that Kaluza was able to recover the vacuum field equations of EMT and GTR from the five-dimensional field equations of KK theory. The above discussion also shows how within the formalism of the five-dimensional KK theory, both the general covariance group of GTR and the gauge invariance

[7] In modern parlance, this split, and thus the process of dimensional reduction, may be taken to amount to a process of "symmetry breaking" in the sense that the original symmetry group of KK theory is broken via the cylinder condition into two sub-groups, namely, the general covariance group of the four-dimensional GTR and the gauge invariance group of EMT.

group of EMT are combined and represented as parts of a more general covariance group. This further indicates that within the theoretical framework of KK theory electromagnetic gauge invariance turns out to be a geometric invariance under spacetime coordinate transformations along the coordinate associated with the fifth dimension. In this sense, one might also view KK theory as offering an interpretation of gauge symmetry as a *geometric* symmetry of spacetime.

3.3 Klein's compactification of the fifth dimension: explaining the *unobserved*

In the second part, entitled *The Wave Equation of Quantum Theory*, of his 1926a paper, Klein expresses his conviction that it is unlikely that QT could be formulated under a unified spacetime representation in four dimensions. He rather thinks that a five-dimensional formulation of the laws of quantum phenomena might yield the desired representation of QT, even though the postulation of the fifth dimension does not seem to be compatible with our physical experience of the nature. In Klein's view, the reason why quantum phenomena cannot be represented under a unified spacetime description in four-dimensions lies in an *inadequate* conception of the duality that is taken to exist between particle motion and wave motion—as asserted in de Broglie's hypothesis (de Broglie 1923)—which grounded Schrodinger's wave-mechanics formulation of QT (Schrodinger 1926). Klein views this duality as a direct result of seeing the wave-mechanics as governing wave motion in an analogous way the classical mechanics describes particle motion. Klein's claim is that the analogy between the wave-mechanics and the classical mechanics is "incomplete as long as one considers wave-propagation in a space of only four dimensions" (1926a). This shortcoming, argues Klein, can however be remedied if "one views the observed motion as a kind of projection onto spacetime of a wave propagation taking place in a space of five dimensions" (1926a). More importantly, such an analogy, in Klein's view, might provide the necessary unitary mathematical representation that QT lacks.

Unlike Kaluza and others[8] who sought the unification of electromagnetism and gravity under a classical field theory, Klein considered the possibility of unifying these two force-fields in the context of the wave-mechanics formulation of QT. Klein did not think of the duality between particle motion and wave propagation as existing only in a space of three dimensions. Instead, by recognizing the possibility that our spacetime possesses one additional dimension, he conceived of the duality that had been previously postulated by de Broglie as a more general feature of nature that also extends to a space of four dimensions. This led him to the generalization of the wave-particle duality to a higher dimensional spacetime where the five-dimensional formulation of GTR in *vacuum* generates the four-dimensional Einstein-Maxwell equations for a source-free electromagnetic field.

For Klein, the generalization of de Broglie's hypothesis is mathematically meant to be that "the Hamilton-Jacobi equation [of the classical mechanics] can be regarded as an equation of characteristics not for a four-dimensional but for a five-dimensional wave equation" (1926a). This idea enabled Klein to derive the expression: $\beta p_5 = \pm e / c$, for the fifth component of the linear momentum of an electron moving under electric and magnetic fields in a five-dimensional spacetime. In a short follow up note published in *Nature* (1926b), Klein reconsidered the above proportionality relation for any electrically charged particle with charge $N\varepsilon$, and obtained $p_5 = N\varepsilon / \beta c$, where ε and N stand for the charge of the electron and an integer number, respectively. And, supposing that the five-dimensional spacetime is *closed* in the direction of x^5 with a period of l, Klein quantized p_5 by making use of the Sommerfeld-Wilson quantization

[8] Namely, Gustav Mie, Gunnar Nordstrom and Hermann Weyl, who developed classical unified field theories of electromagnetism and gravitation before Kaluza's theory in 1921. See Vizgin (1994) for details.

rule[9] and obtained: $p_5 = Nh/l,$ where h is the Planck's constant, and l represents the length of the closed fifth coordinate. Upon comparing the foregoing two relations obtained for p_5, he obtained the length of the period as $l = hc\sqrt{2\kappa}/\varepsilon = 0.8\times10^{-30}\,cm$, where $\beta = \sqrt{2\kappa}$ is used. Thus, Klein was able to show that the conjecture of the *periodicity* of spacetime along the fifth coordinate brings about the *compactification* of the fifth coordinate to a size comparable with the Planck length ($\approx 10^{-33}\,cm$). This result was interpreted by Klein as a possible reason of the *unobservability* of the fifth dimension. In his words:

> The small value of this length together with the periodicity in the fifth dimension may perhaps be taken as a support of the theory of Kaluza in the sense that they may explain the non-appearance of the fifth dimension in ordinary experiments as the result of averaging over the fifth dimension." (1926b)

3.4 Unity in Kaluza-Klein theory

In the theoretical framework of KK theory, the dynamics of electromagnetism and gravity, which were previously represented in EMT and GTR respectively by *separate* mathematical structures, are combined and represented under a *unified* and more comprehensive mathematical structure that consists of the field equations of the five-dimensional formulation of GTR in vacuum. In this *unifying* mathematical structure, the five-dimensional Ricci (or Riemann) curvature tensor—which Kaluza calls the "universal tensor"—can be identified as the key *structural element* in the sense that it incorporates the components of *both* the electromagnetic field tensor and the Ricci tensor in four dimensions. This indicates that within the mathematical structure of KK theory the foregoing four-dimensional tensor quantities cease to exist as separate and independent structural elements. Instead, their components are *coupled* to form the components of the five-dimensional

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[9] Sommerfeld-Wilson quantization rule, after the works of Arnold Sommerfeld (1916) and William Wilson (1915), states that the action integral in phase space around a closed path is quantized according to $\oint p_k dq_k = n_k h$, where p_k is the generalized momentum corresponding to the generalized coordinate q_k, and n_k is a positive integer number.

Ricci curvature tensor of KK theory, thereby the distinction that formerly existed between the mathematical structures of EMT and GTR used to represent respectively the dynamics of electromagnetism and gravity is completely eliminated. As a result, in KK theory, the dynamics of both of those phenomena are represented by the same set of field equations—namely, the five-dimensional vacuum gravitational field equations of KK theory—, as opposed to by different sets of field equations—namely, the Maxwell and Einstein field equations. Viewed in this way, the theoretical framework of KK theory can be said to provide a *structural* unity insofar as it enjoys a more comprehensive mathematical structure that accommodates and combines the mathematical structures formerly used in EMT and GTR to represent the dynamics of electromagnetism and gravity.

The structural unity achieved in KK theory also produces an *ontological* unity between electromagnetism and gravity insofar as one regards those phenomena as force-fields and thinks of the five-dimensional Ricci curvature tensor—as does Kaluza—as unifying the four-dimensional electromagnetic and gravitational fields in a *single* force-field, namely, in the five-dimensional Ricci curvature tensor field. Interpreted in this way, the five-dimensional Ricci curvature tensor, which I have identified above as the key unifying structural element in the mathematical structure of KK theory, brings about an *ontological* commitment in the sense that electromagnetism is treated as an aspect of *vacuum* gravity in the five-dimensional world, as opposed to as a *separate* force as previously understood within the four-dimensional theoretical framework of EMT. Bearing in mind that—as Albert Einstein taught us with his GTR—gravity is in fact an aspect of the *geometrical* structure of spacetime, one can thus regard KK theory as offering a purely *geometrical* interpretation of electromagnetism.[10] This feature is also manifest

[10] In passing, let me note that this feature of KK theory complies well with Einstein's program of *geometrization of physics*; for the latter, see Ryckman (2005).

in the identification of the U(1) gauge invariance by KK theory as a geometric invariance of spacetime. Therefore, within the theoretical framework of KK theory, electromagnetism can be said to be *reduced* to an aspect of the geometrical structure of the five-dimensional spacetime, and thus the fundamental *ontological* distinction that formerly existed between electromagnetism and gravity, as described in EMT and GTR respectively, can be said to be eliminated. This conclusion suggests that, besides being structurally unified, KK theory can also be viewed as an *ontologically* unified field theory of electromagnetism and gravity.

4. A general outlook on the emergence of modern Kaluza-Klein theories

In this section I shall provide a general outlook on how the mathematical formalism of the original KK theory was used by the later developed higher dimensional unified field theories. I will basically argue that the revival of KK approach to unification in the late seventies and early eighties was largely fueled by the *confluence* of two factors: first, the desire to incorporate gravity into the Standard Model (SM) of particle physics, and second, the realization that with the addition of *extra* spatial dimensions the formalism of the original KK theory can be extended as to include also the mathematical representations of gauge fields with non-Abelian symmetry groups.

To this end, for future reference, at the outset, I want to briefly touch upon the emergence of Yang-Mills (YM) theory[11] and the SM. In 1954, Chen Ning Yang and Robert Mills jointly published a paper (1954), where they considered *isotopic spin* symmetry as a gauge symmetry, and based on that they derived a set of field equations which were invariant under SU(2) group transformations. YM theory has been revolutionary for the later development of particle physics in the sense that it offered a theoretical framework that allowed to express conserved quantities

[11] For details see for instance Moriyasu (1983), and Aitchison and Hey (1989).

in nuclear interactions in terms of internal gauge symmetries, thereby revealing that the dynamics of nuclear interactions could be characterized by using gauge fields.

Even though YM theory was originally proposed for the description of strong interactions, its gauge symmetry formalism was later used in the construction of the electroweak theory (EWT) of weak and electromagnetic interactions[12]. EWT can be regarded as a special type of YM theory possessing a *double* invariance, namely, SU(2)xU(1) gauge invariance representing "weak isotopic spin" (or "flavor") invariance and "charge conservation", respectively. The discovery of "color" as a new quantum number brought out the possibility that strong interactions could also be described by a YM-type gauge theory in the same manner as were weak interactions described by EWT. The expectations turned out to be true and the theory of quantum chromodynamics (QCD) emerged in the early seventies as a gauge theory of strong interactions exhibiting color invariance with symmetry-group SU(3).[13] Finally, this was followed by the formation of the SM from the *grouping* of QCD and EWT as a unified framework of all the fundamental interactions excluding gravity. [14]

At this point, it is to be recalled that the basic insight the original KK theory gave with regard to the unification of force-fields consisted in showing how the U(1) (gauge invariance) symmetry-group of EMT and the four-dimensional general covariance group of GTR could be accommodated within a larger (five-dimensional) general covariance group. Taking into account the fact that electromagnetic field is described by a YM–type field theory, and that it can be accommodated within the formalism of the KK theory, the following question naturally arises: is it possible to extend the formalism of the original KK theory in such a way as to accommodate gauge fields with non-Abelian symmetry groups? This question was addressed by different

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[12] EWT was formulated in the sixties by Glashow (1961), Weinberg (1967) and Salam (1968).
[13] The formulation of QCD is due to Gross and Wilczek (1973) and Politzer (1973).
[14] For the emergence of SM see Hoddeson et al. (1997), and for the gauge field program see Cao (1997).

physicists in the sixties—namely, DeWitt (1964), Kerner (1968) and Trautman (1970)—, and as a result, it was understood that the mathematical formalism of the original KK theory could be generalized for the description of gauge fields with non-Abelian gauge groups. The non-Abelian extension of KK unification consists in accommodating the non-Abelian gauge-symmetry groups representing fundamental nuclear interactions within the same general covariance group of a higher dimensional spacetime theory, i.e., of a higher dimensional formulation of GTR. The number of extra dimensions necessary for the extension procedure is determined by the group structures of the symmetry groups associated with the gauge fields to be incorporated. Just like in the original KK theory, where the general covariance group is restricted to the set of coordinate transformations—via the cylinder condition—that satisfy U(1) gauge invariance and that leave the five-dimensional metric invariant, in the generalized KK theory general covariance group is restricted to the set of coordinate transformations that satisfy the non-Abelian gauge invariances and that leave the (N+1)-dimensional metric invariant.

In the seventies, the realization that the mathematical formalism of the original KK theory also admits generalizations to non-Abelian gauge fields opened the door to the possibility of unifying gravity with nuclear force-fields. This led physicists to consider the formalism of KK unification as a general unification *scheme* that would fulfill the ideal of unifying gravity with the other fundamental interactions of nature. In this regard, several attempts were made, and as a result, the higher dimensional spacetime theories which are today known as the eleven dimensional supergravity and the ten dimensional superstring theories came out.[15] Notwithstanding the existing differences between them, these theories have been given the generic name "modern KK theories" in the literature of contemporary physics.

[15] For an analysis of the superstring and supergravity theories see, for instance, Kaku (1988) and West (1986) respectively.

The mechanism of "spontaneous compactification" of extra dimensions[16] constitutes the most characteristic aspect of the modern KK theories. According to this mechanism, the extra dimensions are *real* like the observed ones, but differ from them in that they are deemed to be *confined* to a region of space whose size (or *radius*) lies at the order of the Planck length.[17] Moreover, it is also conjectured that spontaneous compactification of extra space dimensions, which constitutes a mechanism of *dimensional reduction* down to ordinary space dimensions, takes place in the form of a process of "spontaneous symmetry breaking", according to which the original symmetry of spacetime, which is *deemed* to be "supersymmetry"[18], breaks down around the Planck energy ($\approx 10^{19}\,GeV$), and generates the individual gauge symmetries—namely, U(1), SU(2) and SU(3) symmetries—observed in nuclear phenomena.[19] During this process, it is also taken that the original space is split into two subspaces, and the unobserved (also called *compact*) dimensions continue to exist in the subspace whose size remains close to the Planck length. In order for this split to occur, the group of admissible coordinate transformations associated with the extra dimensions should be identified to be gauge transformations under which the internal gauge symmetries hold.[20] This means that the theoretical framework in the modern KK theories allows one to interpret different internal gauge symmetries observed in nuclear interactions as different manifestations of space-symmetries associated with extra dimensions. In this sense, the

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[16] This mechanism, first proposed by Cremmer and Scherk (1976, 1977), is also known as "dynamical compactification" in the literature.

[17] Note that the way the mechanism of "spontaneous compactification" was conceived in modern KK theories closely echoes Klein's compactification of the fifth dimension in the original KK theory.

[18] "Supersymmetry" is the idea that the laws of nature are invariant under the interchange of fermions and bosons of the same mass.

[19] For a philosophical discussion of the notion of "symmetry breaking" in physics, see, for instance, Brading and Castellani (2003).

[20] Note the interesting analogy here between the mathematical formalisms of the original KK theory and the modern KK theories. In the former case, recall that, in order for the "cylinder condition" to hold the group of admissible coordinate transformations associated with the fifth dimension should be identified with the group of coordinate transformations under which U(1) gauge invariance holds.

conception of unity underlying the modern KK theories can be said to consist of a *synthesis* of higher dimensional unification with gauge symmetry formalism.

The foregoing analysis suggests the conclusion that the formation process of the modern KK theories proceeded through a search for a mathematical formalism that could represent the individual gauge symmetries under the same general covariance group. By treating all kinds of gauge symmetries as space-symmetries and accommodating them under the general covariance group of a higher dimensional spacetime theory, the mathematical formalism of the modern KK theories provides a unified and more comprehensive group structure for the mathematical representations of the dynamics of all types of interactions, including that of gravity, as opposed to different group structures exhibited by the gauge theories of particle physics for different types of interactions. This, in my view, points to a *structural* unity not only on the level of gauge-symmetry group structures of gauge theories of particle physics, but also, more comprehensively, on the level of symmetry group structures of GTR and the gauge theories of particle physics.[21]

5. A critical assessment of Kitcher's model of explanatory unification in the light of Kaluza-Klein unification

In this penultimate section, my goal is to assess the merits of Kitcher's model of explanation in relation to the unified field theory program exemplified by the KK theories. According to Kitcher's unificationist approach to explanation, one would interpret the KK theories as offering a small number of argument patterns from which a larger number of conclusions concerning natural phenomena can be drawn. In what follows, I shall seek to argue that neither the five-dimensional KK theory and nor the modern KK theories can be interpreted in this way. In what follows, I will consider both cases in turn.

[21] Whether, or to what extent, the structural unity achieved in modern KK theories also produces an *ontological* unity of force fields as in the case of the five-dimensional KK theory is a topic that I do not want to broach here, as its discussion would take us much beyond the scope of the present paper. Rather, I here want to indicate the fundamental constitutive role that *structural* unification played in the construction of the modern KK theories.

First, it is to be noted that the five-dimensional KK theory is unable to account for the electromagnetic and gravitational phenomena occurring in (1+3)-dimensional spacetime *directly* by means of its five-dimensional field equations. It is only after the cylinder condition is imposed and the process of dimensional reduction down to the ordinary space dimensions is carried out that the five-dimensional KK theory becomes able to account *only* for the *vacuum* electromagnetic and gravitational phenomena occurring in (1+3)-dimensional spacetime that we live in. The process of dimensional reduction consists of the derivations of the (1+3)-dimensional field equations from the (1+4)-dimensional field equations of the KK theory. In Klein's version of KK theory, where the exact mathematical forms of the Einstein-Maxwell equations for a source-free electromagnetic field were obtained, the reduction process consisted of the derivational steps resulting in the set of field equations earlier given in (5a) and (5b) in this paper. This in turn indicates that with respect to the explanation of both electromagnetic and gravitational phenomena occurring in vacuum in (1+3)-dimensional spacetime the set of patterns of derivation employed by the five-dimensional KK theory is larger than those used by EMT and GTR.

Moreover, as has been stated above, both Kaluza's and Klein's versions of KK theory are able to furnish *only* the source-free Maxwell field equations and the Einstein field equations for which the energy-momentum tensor $T_{\mu\nu}$ is that of an electromagnetic field in free space (i.e., in vacuum). The former set of field equations is not applicable to the electromagnetic phenomena for which the electromagnetic field tensor contains source terms, i.e., ρ and J. And likewise, the latter set of field equations is not applicable to gravitational phenomena for which the energy-momentum tensor contains matter terms. However, we know that these foregoing phenomena constitute the great bulk of the known electromagnetic and gravitational phenomena and are successfully accounted for by EMT and GTR respectively. This in turn leads to the conclusion that about the descriptions of electromagnetic and gravitational phenomena occurring in (1+3)-dimensional spacetime the conclusion sets[22] offered by EMT and GTR are of much greater size than the one offered by the five-dimensional KK theory.

Therefore, the above discussion indicates that with respect to the breadth of the conclusions generated about gravitational and electromagnetic phenomena, as well as to the paucity and stringency of argument patterns employed in reaching those conclusions, both EMT and GTR outperform the five dimensional KK theory. This conclusion clearly stands in conflict with the basic tenet of Kitcher's account, according to which unification in scientific practice operates towards the generation of as many conclusions as possible about natural phenomena from the paucity and stringency of the argument patterns employed. This in turn suggests that KK unification of electromagnetism and gravity illustrates an instance in the practice of science in which unification does not work in the way it is described in Kitcher's account.

Like the five-dimensional KK theory, the modern KK theories also pose a challenge for Kitcher's model of explanation. As mentioned earlier, the commonly-held view among physicists is that those theories should accommodate a successful mechanism of dimensional reduction from higher dimensions down to our ordinary space dimensions, if they are to account for gauge phenomena occurring in (1+3)-dimensional spacetime. However, the lack of such a mechanism of dimensional reduction constitutes the most important shortcoming of the modern KK theories. [23] In the case of the eleven-dimensional supergravity theory, even though dimensional reduction was obtained, it led to an *incomplete* description of known gauge phenomena in the sense that the theory's symmetry group—namely, O(8)—was not large enough to accommodate *simultaneously* the symmetry groups of all quarks and leptons and thus unable

[22] Here, by "conclusion set" I mean the set of explanations offered by the theory under consideration about its target phenomena.
[23] For a notable account of this shortcoming, see for example Michio Kaku (1988, p. 12-18).

to accommodate the minimal symmetry group of particle physics, namely, U(1)xSU(2)xSU(3) symmetry group, as given by the SM. And in the case of the superstring theories, no mechanism of dimensional reduction has been developed thus far; so those theories lack any *real* contact with the (1+3)-dimensional spacetime that we live in.

These above considerations suggest that the number of conclusions advanced thus far by the modern KK theories about gauge (nuclear) phenomena is by far smaller than those offered by the gauge theories of particle physics. The progress of theoretical physics would show us whether or not it would be possible to develop a mechanism of dimensional reduction also for the superstring theories. However, the case of the eleven-dimensional supergravity theory illustrates that the process of dimensional reduction does not necessarily result in an explanatory gain over gauge theories of particle physics; whereas it requires certain additional dynamical derivations to be carried out. [24] All these considerations suggest the conclusion that the current state of the art in the modern KK theories shows no indication that unification process in these theories proceeds by the derivation of a larger number of conclusions regarding natural phenomena through a derivational economy as specified by Kitcher's criterion of unification.

6. Concluding remarks

By examining a series of case-studies from the history of physical and biological sciences, Kitcher argues for "explanatory unification" and suggests that it is the kind of unification that scientific theories ought to exhibit, if they are to provide a unified and systematic description of natural phenomena. His main methodological thesis is that explanatory unification is an important *desideratum* in theory formation and in this sense constitutes the true motive of scientific inquiry. By contrast, my account of the emergence and the historical development of

[24] For the derivational details concerning the dynamics of dimensional reduction in the eleven-dimensional supergravity theories, see, Freund and Rubin (1980).

the higher dimensional unified field theory program à la KK reveals a different type of unification agenda, one which does not fit with Kitcher's account. My analysis has shown that the construction process of both the original KK theory and its modern elaborations, namely modern KK theories, proceeded from a process of unification that was mainly intended to provide the dynamics of the known force-fields with a unified mathematical representation. I have called this process of theoretical unification "structural unification" and argued that it does not operate to explain natural phenomena in the way suggested by Kitcher's account.

The foregoing suggests that within the context of the higher dimensional unified field theory program "structural unification" closely bears on the process of theory formation, and more importantly proves to be an essential part of it. I take this to be an important *methodological* lesson with regard to the practice of contemporary physics. On the other hand, taking into account the fact that KK unification currently constitutes the mainstream approach to the "unity" problem in physics and has produced a great impact in the practice of modern physics, this case-study also suggests an important *descriptive* failure on behalf of Kitcher's model of explanation.

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