Article ID: 1000-5641(2009)01-0043-05

Boundedness of the solutions of the T-system and its control

SHU Yong-lu, WANG Meng

(*College of Mathematics and Physics, Chongqing University, Chongqing* 400044, *China*)

Abstract: The boundedness of solutions of a chaotic T-system was proved by constructing a new Lyapunov function. Furthermore, the theoretical results of the boundedness of a T-system can be used for chaos control and synchronization. Effective linear feedback controller was proposed for stabilizing chaos to unstable equilibrium$(0, 0, 0)$.

Key words: T-system; Lyapunov function; boundedness; linear feedback control

CLC number: O175.14 **Document code:** A

T系统解的有界性及其控制

舒永录, 王 猛

(重庆大学 数理学院, 重庆 400044)

摘要: T 系统源于著名的 Lorenz 系统, 通过构造一个新的 Lyapunov 函数, 证明了混沌 T 系统的解的有界性。进而将 T 系统有界的理论结果应用在混沌控制和同步上, 设计了有效的线性控制器把混沌控制到不稳定的平衡点 $(0, 0, 0)$.

关键词: T 系统; Lyapunov 函数; 有界性; 线性反馈控制

0 Introduction

Chaos has been found in many engineering systems. Sensitivity to initial conditions is the fundamental characteristic of a chaotic system. In 1963, Lorenz found the first chaotic attractor in a three-dimensional autonomous system when he studied the atmospheric convection[1]. In 1999, Chen and Ueta introduce a 3D polynomial system known as the Chen system[2]. In 2002, Lü and Chen further found a chaotic system, named as Lü system[3]. Gh. Tigan found a new 3D polynomial differential system given by

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a)x - axz, \\ \dot{z} = -bz + xy, \end{cases} \tag{1}$$

收稿日期: 2008-05

基金项目: 国家自然科学青年基金(10601071)

第一作者: 舒永录, 男, 博士, 副教授. E-mail: shuyonglu@163.com.

第二作者: 王猛, 男, 硕士. E-mail: sanxingtown@163.com.

where a, b, c are real parameters, named as T-system[4]. There are some dynamic properties are already studied in [4-6]. T-system allows a larger possibility in choosing the parameters, so it plays a more complex dynamics. A chaotic attractor is ensured by two things, one is that the system has a trapping region which guarantees the existence of an attractor, and the other is that the system displays chaotic behavior on the attractor. It is very important to find a trapping region in chaos synchronization and chaos control. However, it is difficult to find an attracting domain that include the chaotic attractor. In 2003, Pogromsky et al. investigated the globally attractive set of the Lorenz system[7]. In 2006, Qin investaged the boundedness of the solutions of the Chen system with $a > c > 0$ and $b > 2c > 0$[8]. But it is not include the most interesting situation with the chaotic attractor. The chaotic behavior of the T-system has been studied in [4]. However, the problem that the existence of a trapping region is not solved. Considering the similar nature between the T-system and the Chen system, this paper makes some improvements based on the Lyapunov function in [8]. Now the chaotic attractor of the T-system is in it. Based on Lyapunov stability theory, the T-system is stabilized to its equilibrium by using linear feedback control.

1 The boundedness of the solutions of the T-system

The main result is summarized as follows. Let

$$\theta = 2a\tau, \mu = a + 4a^2\tau. \tag{2}$$

$$\omega = 2b\mu - \frac{(2a(2a+b)\tau + a\alpha)^2}{4a\tau}. \tag{3}$$

Lemma1.1 if $c > a > 0$, and $\alpha = \dfrac{b}{2a+b}$, then $\omega > 0$ for a suitable$\tau > 0$.

Proof

$$\begin{aligned} \omega > 0 \quad &\Leftrightarrow \quad 2b\mu > \frac{(2a(2a+b)\tau + a\alpha)^2}{4a\tau} \\ &\Leftrightarrow \quad 8ab\tau(a + 4a^2\tau) > 4a^2(2a+b)^2\tau^2 + 4a^2(2a+b)\alpha\tau + a^2\alpha^2 \\ &\Leftrightarrow \quad 4(2a-b)^2\tau^2 + 4((2a+b)\alpha - 2b)\tau + \alpha^2 < 0. \end{aligned}$$

Because $\alpha = \dfrac{b}{2a+b}$, so $4((2a+b)\alpha - 2b) < 0$. And the discriminant is greater than 0. It means that we will be able to get $\tau > 0$ such that $\omega > 0$.

Theorem 1.1 The solutions of system (1) with $c > a > 0$ are globally bounded for $t \in [0, +\infty)$.

Proof constructing a Lyapunov function as follow

$$\begin{aligned} V &= (\alpha x - y)^2 + \mu z^2 - 2\theta x^2 z + \tau x^4 - 2rz + \tfrac{r^2}{a} \\ &= (\alpha x - y)^2 + a(z - \tfrac{r}{a})^2 + \tau(2az - x^2)^2. \end{aligned} \tag{4}$$

The values of θ and μ follows from (2).

For the derivative of Lyapunov function (4), along the trajectories of the T-system (1), we have

$$\begin{aligned}\dot{V} &= 2(\alpha x - y)(\alpha\dot{x} - \dot{y}) + 2\mu z\dot{z} - 4\theta xz\dot{x} - 2\theta x^2\dot{z} + 4\tau x^3\dot{x} - 2r\dot{z} \\ &= 2a\alpha^2 xy - 2a\alpha y^2 - 4a\theta xyz + 4a\tau x^3 y - 2a\alpha^2 x^2 + 2a\alpha xy - 4a\tau x^4 \\ &\quad +4a\theta x^2 z + 2(c-a)xy - 2(c-a)\alpha x^2 - 2axyz + 2a\alpha x^2 z \\ &\quad -2b\mu z^2 + 2b\theta x^2 z + 2brz + 2\mu xyz - 2\theta x^3 y - 2rxy \\ &= -2(a\alpha^2 + (c-a)\alpha)x^2 - 2a\alpha y^2 - 2b\mu z^2 - 4a\tau x^4 + 2brz + 2(\mu - 2a\theta - a)xyz \\ &\quad +2[a\alpha^2 + (c-a) + a\alpha - r]xy + 2(2a\theta + b\theta + a\alpha)x^2 z + 2(2a\tau - \theta)x^3 y \\ &= -2(a\alpha^2 + (c-a)\alpha)x^2 - 2a\alpha y^2 - \omega z^2 - (2\sqrt{a\tau}x^2 - \frac{2a(2a+b)\tau + a\alpha}{2\sqrt{a\tau}}z)^2 + 2brz. \quad (5)\end{aligned}$$

In the above, we suppose that $r = a\alpha^2 + (c-a) + a\alpha$, and the values of θ , μ , and ω follow from (2) and (3).

According to Lemma 1.1, there is a $\tau > 0$ such that $\omega > 0$, and

$$\dot{V}(x,y,z) \leqslant -2(a\alpha^2 + (c-a)\alpha)x^2 - 2a\alpha y^2 - \omega(z - \frac{br}{\omega})^2 + \frac{b^2 r^2}{\omega}.$$

Therefore, we can find large enough S_0 such that

$$2(a\alpha^2 + (c-a)\alpha)x^2 + 2a\alpha y^2 + \omega(z - \frac{br}{\omega})^2 \geqslant \frac{b^2 r^2}{\omega}.$$

Here (x,y,z)satisfies $V(x,y,z) = S$ with $S > S_0$. It shows that $\dot{V}(x,y,z) < 0$ on the surface $\{(x,y,z)|V(x,y,z) = S\}$. Hence, we can say that the solutions of the T-system are globally bounded.

2 Controlling chaos via linear feedback control

In this section, simple but effective linear feedback controller is designed to drive the chaotic trajectories to the unstable equilibrium.

The T-system exhibits a horseshoe chaos at the parameter values $a = 2.1, b = 0.6, c = 30$(see Fig. 1). There is only one equilibrium$E_0 = (0,0,0)$ with $a \geqslant c$. There are three equilibria

$$E_0 = (0,0,0), \quad E_1 = (\sqrt{\frac{(c-a)b}{a}}, \sqrt{\frac{(c-a)b}{a}}, \frac{(c-a)b}{a}),$$

$$E_2 = (-\sqrt{\frac{(c-a)b}{a}}, -\sqrt{\frac{(c-a)b}{a}}, \frac{(c-a)b}{a}) \text{ with } c > a.$$

The equilibrium $E_0 = (0,0,0)$ of the controlled system

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = (c-a)x - axz + u \\ \dot{z} = -bz + xy \end{cases} \quad (6)$$

is asymptotically stable with the controller$u = -cx - y$.

Theorem 2.1 The origin of the controlled T-system (6) with parameters $a = 2.1, b = 0.6, c = 30$ is asymptotically stable.

Proof Choose the following Lyapunov candidate:

$$V = x^2 + y^2 + az^2.$$

The differential of the Lyapunov function is

$$\begin{aligned}\frac{1}{2}\dot{V} &= x\dot{x} + y\dot{y} + az\dot{z} = -ax^2 - abz^2 + y(u + cx) \\ &= -ax^2 - abz^2 + y(-y - cx + cx) = -ax^2 - y^2 - abz^2\end{aligned}$$

Since V is positive definite and $\dot{V}(x, y, z)$ is negative definite with controller$u$, according to Lyapunov stability theory, the equilibrium $(0, 0, 0)$ of the system (6) is asymptotically stable, namely, the controlled system (6) can asymptotically converge to the equilibrium $(0, 0, 0)$. The T-system is stabilized to its unstable equilibrium by use only one linear controller, comparison with other control methods, linear feedback controller more simple in practice, numerical simulations demonstrate the effectiveness and feasibility.

3 Numerical simulations

In simulation, we select the parameters of T-system as $a = 2.1, b = 0.6, c = 30$, the initial value of the controlled T-system is $(x_0, y_0, z_0) = (0.1, -0.3, 0.2)$. The horseshoe chaos is shown in Fig. 1, the controlled T-system is stabilized to equilibrium $(0, 0, 0)$ in Fig. 2.

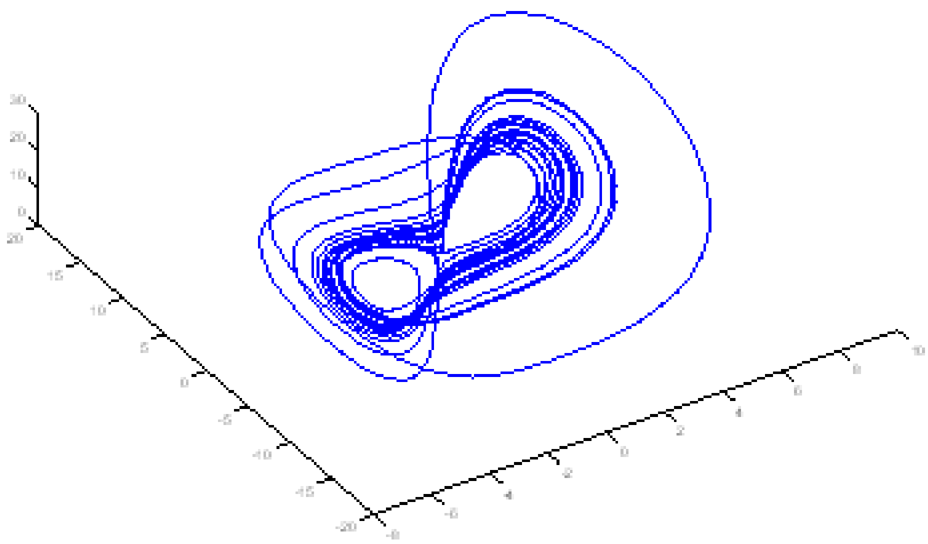

Fig. 1 Horseshoe chaos

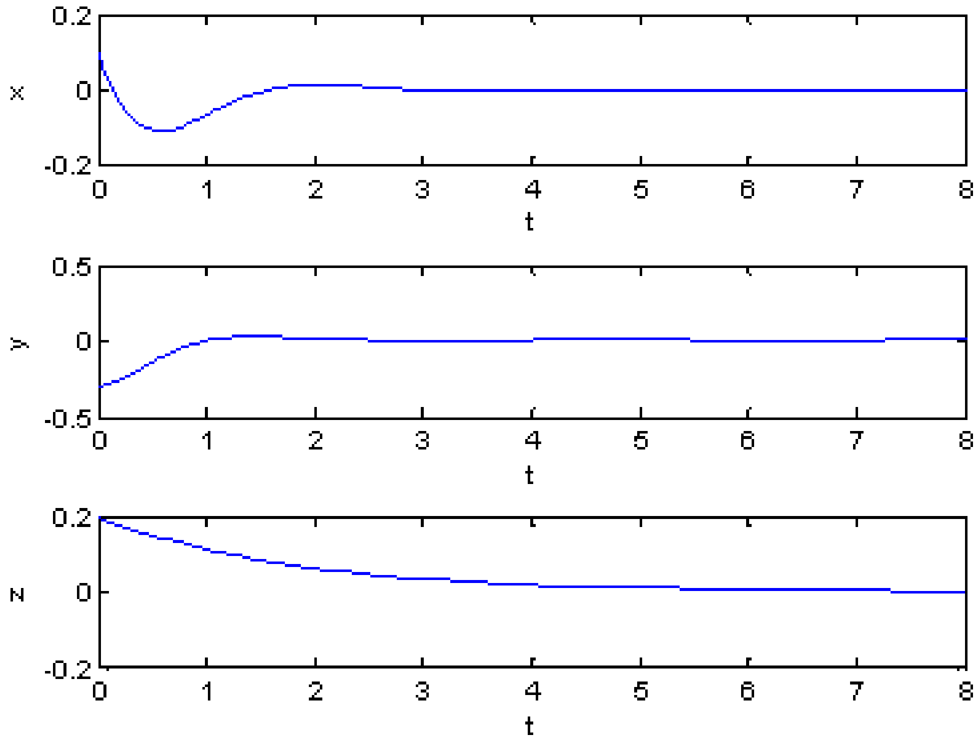

Fig. 2　The controlled T-system is stabilized equilibrium $(0, 0, 0)$

4　Conclusions

In this paper, by constructing a new Lyapunov function, the solutions of the T-system are globally bounded was proved, which include the horseshoe chaos. Linear feedback controller is designed to control the T-system to the equilibrium$(0, 0, 0)$.

[References]

[1] LORENZ E N. Deterministic non-periods flows [J]. Journal of the Atmospheric Science, 1963, 20: 130-141.

[2] CHRN G R, UETA T. Yet another chaotic attractor [J]. International Journal of Bifurcation and Chaos, 1999, 9: 1465–1466.

[3] LU J, CHEN G. A new chaotic attractor coined [J]. International Journal of Bifurcation and chaos, 2002 , 12: 659-661.

[4] TIGAN G H. Analysis of a dynamical system derived from the Lorenz system[J]. Sci Bull Politehnica university of Timisoara, 2005, 50(64); 61-72.

[5] TIGAN G H, DUMITRU O. Analysis of a 3D chaotic system[J]. Chaos Solitons Fractals 2008(36): 1315-1319.

[6] TIGAN G H. Bifurcation and the stability in a system derived from the Lorenz system[C]// Proceedings of the 3rd international colloquium: mathematics in engineering and numerically physics. Bucharest: [s.n], 2004: 265-72.

[7] POGROMSKY A, SANTOBONI G, NIJMEIJER H. An ultimate bound on the trajectories of the Lorenz system and its applications [J]. Nonlinearity, 2003, 16: 1597-1605.

[8] QIN W X, CHEN G R. On the boundedness of solutions of the Chen system[J]. J Mathematical Analysis and Applications, 2007, 329(1): 445-451.