Nordic Hydrology, 10, 1979, 7-24

Optimal Allocation of Regional Water Resources

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A methodology is presented for the optimal allocation of water resources in a region, considering the economic and hydrologic characteristics of the water resources system. In order to systematically represent the hydrologically interdependent regional water resources system the allocation problem is formulated and solved in an input-output framework. The optimization approach is one of iterative quadratic programming, capable of minimizing a concave objective function over a linear constraint set. For purposes of testing and illustration the methodology has been applied to the Cache la Poudre River basin in northern Colorado. Results from this application are briefly discussed.

Introduction

The purpose of this paper is to describe a methodology for the optimal planning of water resources in a region, considering the hydrologic as well as the economic characteristics of the water resources system.

The planning problem should be familiar to almost any regional water planning authority. A multitude of future demands for water in an area – for agricultural, municipal, industrial, recreational and other purposes – must be satisfied from a number of existing or potential sources: precipitation, streamflows, groundwater, imports from adjacent areas and various forms of reuse of water. Demands as well as supplies have certain spatial, temporal and quality characteristics which must be reconciled in this demand-supply "matching" process. Obviously, regardless of whether the planning area in question has water in abundance or in very short supply, an infinite number of such demand-supply combinations exists, each one with a different physical and social impact in the region. Given certain societal goals and objectives, it is the task of the planning authority to identify and select the most appropriate and desirable out of this large number of combinations.

As the number of alternative planning strategies grows very large, it is no longer possible for the planner to compare alternatives and select the best without the aid of some systematic methodology. One such methodology is presented here, which combines a comprehensive, but yet simple and understandable display of the regional water resources problem with an appropriate optimization technique.

Objectives of Study

Extensive research in the past has focused on the physical description and simulation of hydrologic processes in river basins. Similarly, considerable progress has been made in the field of socio-economic optimization of water resources systems. However, the simultaneous consideration of hydrologic and socio-economic variables in the total system's context has been largely ignored. Combined optimization-simulation studies have been reported in the literature (e.g. ReVelle, Loucks and Lynn 1968; Bishop and Grenney 1976; Maddaus and Gill 1976) but simulation in these studies have focused on particular problems, such as stream water quality and groundwater levels. Bishop and Hendricks (1971) and Bishop, Narayanan, Pratishthananda, Klemetson and Grenney (1975) in their input-output approach to comprehensive regional water resources planning account partially for the hydrologic interactions in the system. In the methodology presented here it is attempted to consider the hydrologic effects of planning decisions in an explicit and comprehensive, although very simplified manner.

A common problem in the optimization of regional water resources systems is to identify an appropriate trade-off between comprehensiveness and simplicity of description. Advanced systems analysis techniques allow an exact description of nonlinearities in the system but their application is usually restricted to fairly small size problems (Pratishthananda and Bishop 1977). Simpler techniques, such as linear programming, allow the planner to deal with large-scale problems, but only after rather drastic simplifications in their formulation. An iterative quadratic programming approach is presented here which appears to offer a compromise; the mathematics and numerics is rather simple, but nonlinearities can be considered. The problem of minimizing a concave objective function can be, if not solved then at least satisfactorily dealt with quite easily in this approach.

The basic element in the planning methodology is a single-period, single-objective allocation model in which a nonlinear (concave) total cost function is minimized subject to a set of linear system constraints. A short description of this model and its application to an area along the eastern foothills of the Rocky Mountains in Colorado follows, as well as a brief discussion of the extension of this basic model to consider multiple objectives and seasonal variation.

The Input-Output Approach to Regional Water Resources Planning

In order to ensure that the economic optimization is performed in proper hydrologic context the planning system is formulated and analyzed in an input-output framework. This approach, which is inspired by the familiar economic input-output analysis (Miernyk 1965), affords a comprehensive and convenient description of the hydrologic interdependencies that characterize water resources systems.

The recognition that the interdependence in water resources systems and economic production systems is of a similar nature, and that consequently principles of economic analysis may be relevant in the analysis of water resources systems as well, has led to a series of input-output inspired modelling studies of such systems (e.g. Bishop et.al. 1971, 1975; Hendricks and De Haan 1975; Goldbach 1977; Hendricks, Janonis, Gerlek, Goldbach and Patterson 1977).

An example of the input-output matrix display of a water resources system is presented in Fig. 1 (from the case study reported below). Origins of water in the system are displayed as rows in the matrix, while destinations for water are displayed as columns. Three types of quantitative descriptions, reflecting the three principle uses of the interaction matrix can be identified:

Water use display: Each entry represents a quantity of water transferred or used in the appropriate time period. Entries are total demands in the bottom row, and total availabilities in the right column.

Water quality display: Each entry represents the quality differential, if any, to be overcome between origin and destination. Entries are quality demands in the bottom row, and quality availabilities in the right column.

Cost display: Each entry represents appropriate cost information associated with the water treatment or transfer.

The matrix representation of water use in the region provides an easy tool for assuring mass balance in the system. The sum of total demands represented in the bottom row must equal the sum of total availabilities represented in the right column. The breakdown of demands and availabilities also makes the water use matrix a convenient tool for formulating constraints for optimization purposes.

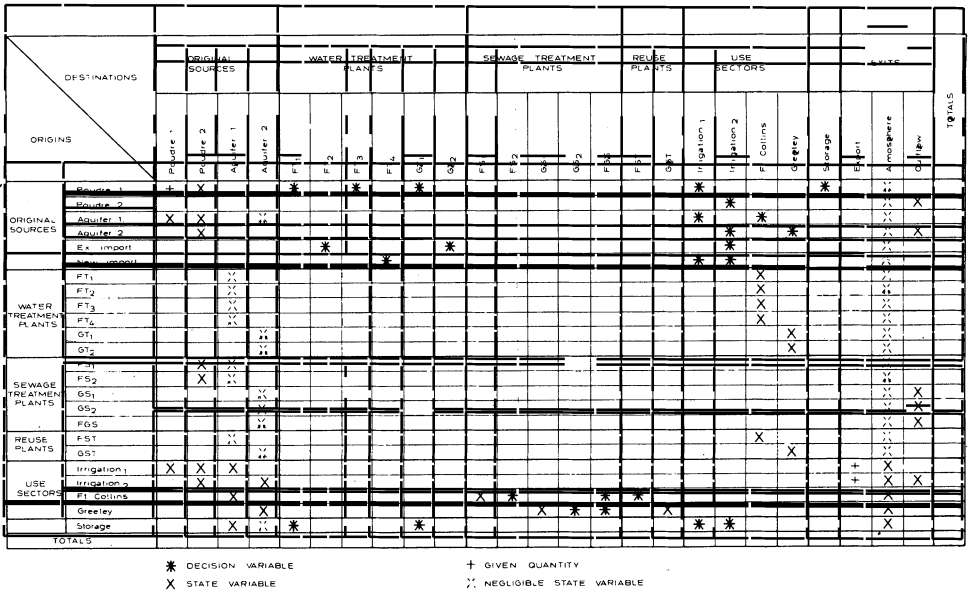

Fig. 1. Input-output matrix display of the Cache la Poudre system.

In situations where the water quality information as such is of no significant interest, but where the water quality differentials to be overcome between origins and destinations affect the costs associated with these transfers, the water quality aspects can be considered indirectly through the appropriate entries in the cost information matrix. This is the approach to water quality taken here.

Description of the Basic Model

A generalized flowchart of the basic allocation model is shown in Fig. 2. The left half of this chart describes the steps to be taken by the user of the planning model, while the right half outlines the model operations.

Input-Output Formulation of the Planning System

An input-output matrix depicting all feasible transfers of water in the planning system is established in a process of conceptualization and aggregation into water producing and water consuming sectors. Each of the transfers in this matrix is a planning variable whose value depends on the system configuration and operation. Thus, alternative planning solutions are characterized by a set of variables $\{t_{mn}\}$, where t_{mn} represents the transfer from row m to column n.

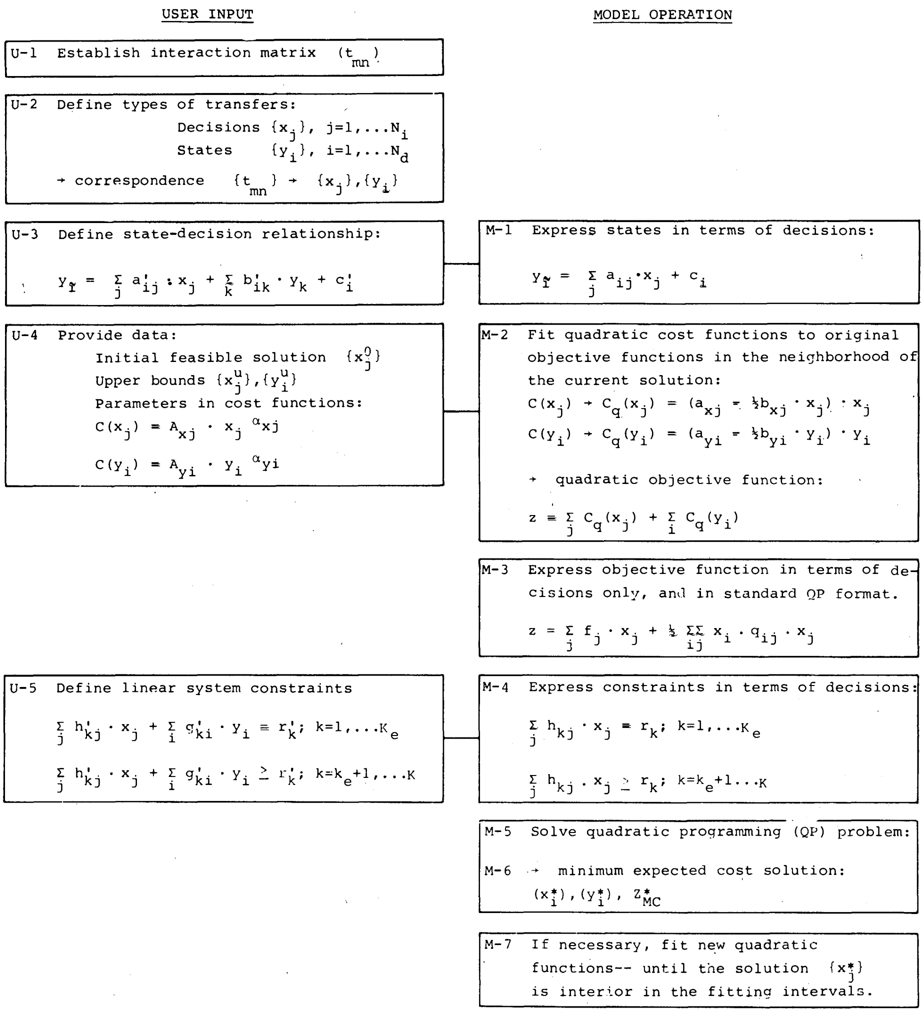

Fig. 2. Generalized flowchart of the basic allocation model.

Definition of Variables

Two basically different kinds of water transfers characterize water resources systems. Certain transfers, such as the amount of water diverted for surface irrigation, are strictly controlled by man, whereas other transfers, such as the evapotranspiration from cropped surfaces, take place naturally, controlled only indirectly by man. Thus the total array of water transfers $\{t_{mn}\}$ is in fact composed of a set of independent (decision) variables $\{x_j\}$ and a set of dependent (state) variables $\{y_i\}$, in the sense that once a set of primary planning decisions

$\{x_j\}$ has been defined as a possible regional plan a set of states $\{y_i\}$ can be deduced as a result.

The mass balance of the system, expressed in a series of linear continuity equations and hydrologic relations (irrigation efficiencies, etc.), provide a simple definition of the state-decision relationship:

$$y_i = \sum_j a'_{ij} x_j + \sum_k b'_{ik} y_k + c'_i \qquad (1)$$

which by means of a Gaussian elimination procedure can be expressed as:

$$y_i = \sum_j a_{ij} x_j + c_i \qquad (2)$$

a', b' and c' in Eq. (1) are constants to be determined in a hydrologic study.

Thus the planning problem is reduced considerably in size by transforming the formulation in the original transfer variables $\{t_{mn}\}$ to a formulation in the decisions $\{x_j\}$ only, eliminating the states $\{y_i\}$ whenever they appear by means of the state-decision relation Eq. (2). Not only is the number of variables in the problem minimized this way, the number of constraints is drastically reduced as well. In the traditional transportation problem formulation (e.g. Bishop et.al. 1971) all the mass balance equations (1) appear as constraints in the optimization problem, but these constraints are eliminated a priori in this approach. Hence, in the illustrative case study reported below (the Cache la Poudre River basin in Colorado) an original problem with 60 variables and 38 constraints is reduced to a problem with 23 variables and 13 constraints.

Definition of Objective Function

The objective function in the basic single-objective model is an expression of total annual costs (capital, operation and maintenance costs). Costs associated with transport and treatment of water typically exhibit economies of scale, marginal costs decreasing with the total quantity of water transported or treated. This fact is of crucial importance in economic water resources planning. It may be economically more advantageous to build and operate a few large facilities than many smaller ones (e.g. regional rather than local treatment plants), and foresight in capacity expansion by building large for the future may prove to be much cheaper than several minor expansions.

Cost functions most frequently encountered in the water resources literature (e.g. Klemetson and Grenney 1975; Bishop et.al. 1975) are of the type:

$$C(x_j) = A_j x_j^{\alpha_j}, \quad A_j > 0, \quad 0 < \alpha_j < 1 \qquad (3)$$

where $C(x_j)$ is the cost associated with treatment (at a prescribed level) or transport of the water quantity x_j, given as a flow rate, and A_j and α_j are constants. However, in a planning situation reliable information on the actual cost structure is often very difficult to obtain, and constant unit costs, possibly revised in successive optimizations, are often used in planning models (Bishop et.al. 1971; Maddaus and Gill 1976). A reasonable compromise is to work with quadratic cost functions:

$$C(x_j) = (a_j - \frac{1}{2} b_j x_j)\, x_j \qquad (4)$$

assuming that a simple linear decrease in marginal costs with quantity can be defined. The three types of cost functions, in order of increasing sophistication and realism: linear – quadratic – power are illustrated in Fig. 3.

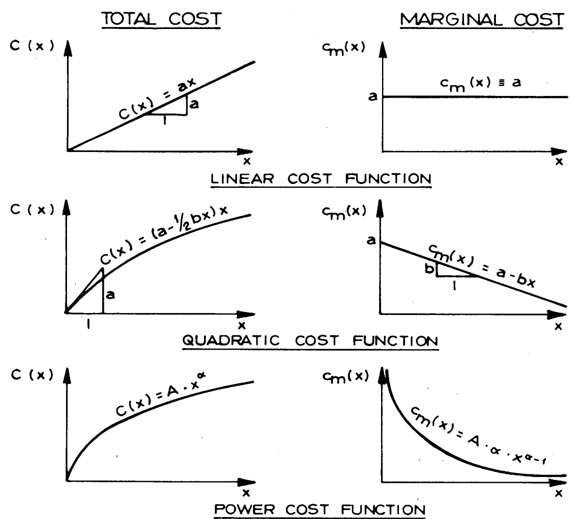

Fig. 3. Types of cost functions.

In the methodology presented here any of these types of cost functions may be used, depending on the type of cost information available to the planner. The optimization procedure is based upon the minimization of a quadratic objective function, defined as the sum of quadratic cost functions in the decision variables. (Cost functions in terms of state variables are automatically transformed to functions of decision variables by means of the state-decision relation Eq. (2)). If power cost functions are provided as input to the model, these will be approximated by quadratic functions of the type given by Eq. (4) in prescribed intervals around current values of the variables. This fitting procedure is performed by linear regression on the cost function derivatives, yielding the parametres a_j and b_j in Eq. (4) as simple analytical functions of the original parameters A_j and α_j in Eq. (3). The entire procedure (approximation, optimization) is repeated until the optimal solution is interior in the prescribed fitting intervals, usually in a few iterations.

Definition of Constraints and Variable Bounds

In order to ensure that future demands for water in all sectors are satisfied within the limitations of available water resources a set of linear constraints and variable bounds are defined. Constraints may be equalities (e.g. demands) or inequalities (e.g. availabilities), and the formulation may be in terms of states and/or decisions (see Fig. 2). Physical, environmental and social limitations to be observed by the planning authorities must be part of this formulation.

An important aspect of the constraint formulation is the uncertainty inherent in water resources planning. It is almost a historical fact that demand projections for municipal water supply are bound to fail, and the stochastic nature of natural hydrologic processes necessitates the definition of water availability in probalilistic terms. Numerous approaches to these problems are suggested in the literature, including chance-constraints (Askew 1974; Bishop et. al. 1975; Knudsen and Rosbjerg 1977), penalty functions (Knudsen and Rosbjerg 1977), and simple exploration of a wide range of demand and supply scenarios. Any one of these approaches may be used in connection with the methodology presented here, depending on the planners philosophy on risk and uncertainty.

The Optimization Approach

As indicated in the previous discussion and the generalized flow chart in Fig. 2 the optimization approach is one of repeated applications of quadratic programming.

The computer routine for solving the quadratic programming problem has been developed using an algorithm known as the General Differential Algorithm. The theory behind this algorithm is presented in Wilde and Beightler (1967) and in more computational details by Morel-Seytoux (1972).

Quadratic programming is a convex programming procedure, which means that only minimization of a convex objective function over the linear (convex) constraint set will guarantee a global minimum. The cost functions considered here, as well as their quadratic approximations, are concave, not convex, and consequently the criterion of optimality (Kuhn-Tucker conditions) may be satisfied at a local, non-global minimum in the cost minimization. In order to solve this problem, and try to obtain global solutions when minimizing concave objective functions, a special procedure has been added to the standard quadratic programming algorithm which in almost all cases ensures a global solution, or at least a good and very consistent local one. The principle in this procedure is to search in all directions from the local minimum for a better solution. If indeed in this process better solutions are identified the best of these is taken as the starting point for a new quadratic programming procedure, the process being repeated until no improvement results in the search for better than local solutions. The search procedure is discussed in detail in Jønch-Clausen (1978).

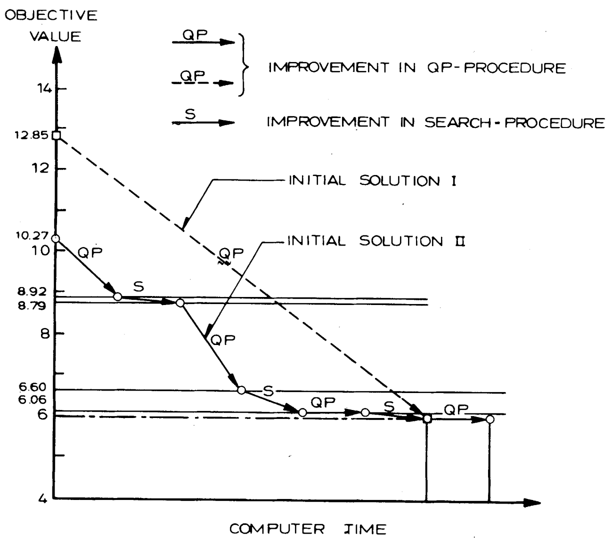

Fig. 4. Example of optimization procedure.

An illustration of results from this optimization procedure is provided in Fig. 4, which is taken from the case study reported below. In one case an initial planning solution with an annual cost of 12.85 mill. $ is taken as the starting point for the optimization procedure, while a "better" initial solution costing "only" 10.27 mill. $ is used in the second case. The same optimal solution with an annual cost of 6 mill. $ results in both cases, but whereas in the first case the final solution is obtained directly in the quadratic programming procedure, several local minima are encountered in the process in the second case. The example further illustrates that the "best" initial solution may not in fact be the most effective one from a programming point of view.

Other Approaches to the Concave Minimization Problem

The concave minimization problem has been addressed by a number of water resources planners, and practically all contributions to the solution of the problem represent different approaches.

Deininger (1966) initially uses iterative linear programming with different initial starting points. Recognizing that this procedure might not yield a global solution, Deininger and Su (1973) applies Murty's extreme point ranking approach (Murty 1968). Although this approach ensures a global minimum, it is very time consuming: as much as 40 percent of the extreme points may have to be enumerated in order to identify the global minimum (Deininger and Su 1973).

Convex programming procedures are used in some cases with no apparent attempt to identify the global minimum, but the majority of studies reported in the literature presents attempts to identify at least a good local minimum. The most common approach towards this objective is to generate local minima from a number of different initial feasible solutions. Thus, Maddaus and Gill (1976) use an out-of-kilter algorithm combined with a recosting procedure and several starting points; Pratishthananda and Bishop (1977) apply nonlinear programming techniques in a multilevel optimization scheme, again from a number of different initial solutions; and Chiang and Lauria (1977) obtain minimum solutions within $\pm 0.25\%$ of each other by starting from 20 different initial solutions in a heuristic optimization algorithm. Their minimum is identical to the one obtained by Deininger and Su (1973), while in other cases their solutions agree closely with those obtained by using mixed integer programming techniques. Mulvihill and Dracup (1974) obtain an improved local solution in a successive linear programming procedure.

Whereas approaches such as extreme point ranking and mixed integer programming from a theoretical point of view are known to yield a global solution to the concave minimization problem (although possibly at a high computer cost), the repeated application of convex programming and heuristic procedures can only guarantee good local solutions. However, computational experience may be so encouraging that the planner tends to regard a good and consistent local minimum as global – and with a fairly high probability he may be right. This is also the case here: starting the quadratic programming routine from a number of different initial feasible solutions generally results in a number of different minima. However, adding the search procedure to the algorithm changes this, good and very consistent local minima resulting in every instance.

Cost Variance Minimization

The basic planning model described above can be extended to address the important question of cost uncertainty. Minimum expected cost is the objective

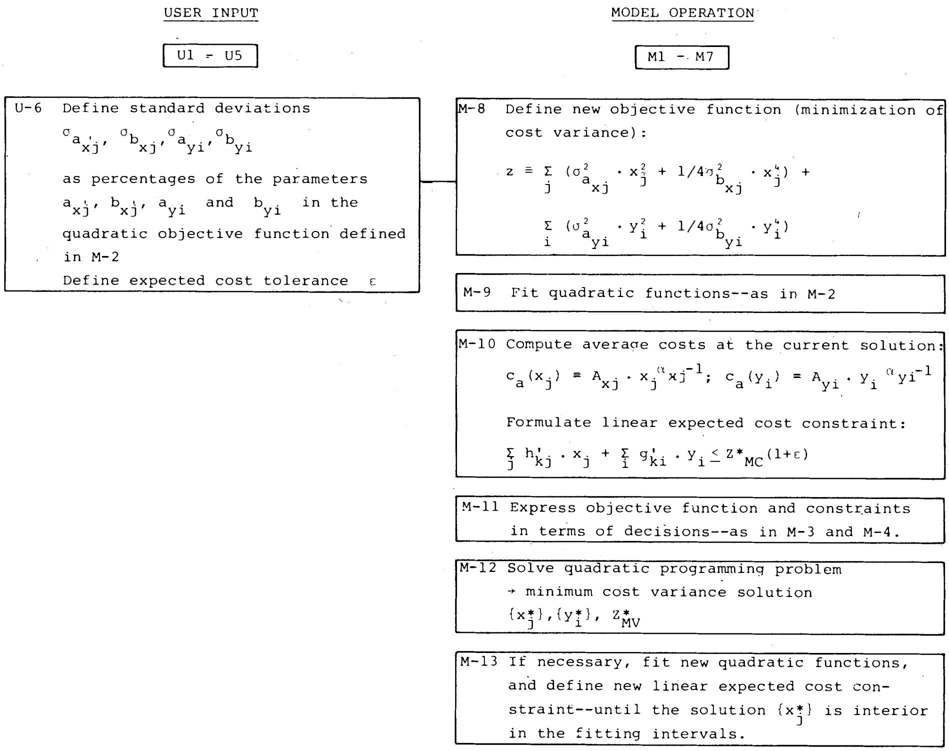

Fig. 5. Generalized flowchart of the extended allocation model (continuation of Fig. 2).

most frequently pursued in water resources planning, but as implied in the term "expected", costs are in fact random variables, and the strategy which results in minimum expected cost may eventually turn out to be very expensive because of heavy reliance on very uncertain cost estimates. On the other hand, plans with higher expected cost may depend little on uncertain cost estimates and consequently be much more reliable and attractive to decision makers, who would rather settle for higher costs and little risk than vice versa.

The methodology has been developed with the capability of accomodating several ranked objectives. Having minimized total system cost as the primary objective, the minimization of total cost variance, subject to a prescribed maximum increase in expected cost, may be taken as a secondary objective. Other objectives could be substituted for or added to the two just mentioned, and consequently the methodology has a multiple objective capability.

Fig. 5 (extension of the generalized flowchart in Fig. 2) illustrates the cost variance minimization procedure, which proceeds from estimates of coefficients of variation associated with parameters in the quadratic cost function approximations.

Seasonal Model

Few regional water resources optimization studies consider the seasonal variation of water demand and supply in a dynamic framework. Bishop et. al. (1975) consider two seasons in their approach, summer and winter, as two separate time periods with no mutual interaction. The single-period model described above can be applied in a multi-season framevork, in which inter-seasonal interactions are considered. The reason for this is that spring and early summer cannot be optimized independently from late summer and early fall in the semi-arid western United States where this methodology has been developed. Water is diverted into storage during the high spring runoff season for subsequent use in irrigation later on, when runoff is low but irrigation requirements are at their maximum, and this carry-over storage must be considered in the optimization procedure.

In the application of the methodology to the Cache la Poudre river basin in Colorado four seasons are considered, two of which are interdependent. With storage providing the linkage between seasons a simple dual-period allocation model based on decomposition and multilevel optimization as illustrated in Fig. 6 has been developed and applied.

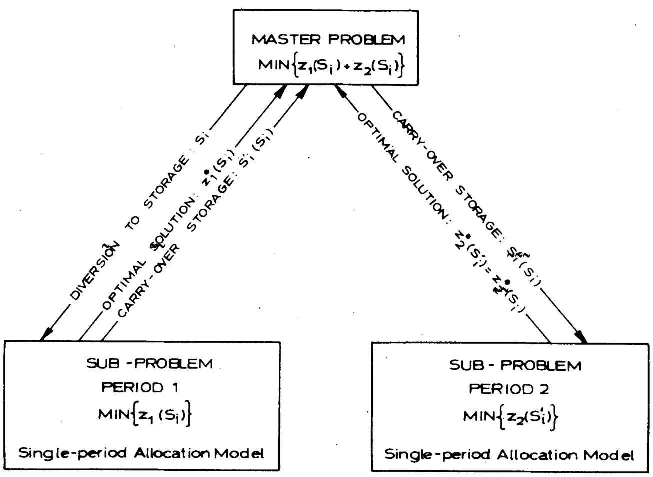

Fig. 6. Principle in multi-season allocation model.

In addition to better represent the water supply and demand variation over the year, the seasonal approach provides some insight into the optimal management and operation of the system. Thus, in the case study presented below the optimal operation of inter-seasonal storage facilities is a direct result from the seasonally based approach, whereas storage is not even considered in an annual model.

Case Study

Description of the Problem

The methodology has been applied to the Cache la Poudre River basin in Colorado, U.S.A., on the eastern slope of the Rocky Mountains (Fig. 7). Draining an area of 4900 km², the river basin ranges in elevation between some 1400 m on the plains to 3700 m at the Continental Divide. The mountainous part of the basin, covering little more than half the area, support few people and activities other than recreation, whereas a sizeable and fast growing population (125,000 in 1975) inhabits the plains in the lower part of the basin. Economically the area is primarily agricultural with only little industry. With an average annual rainfall of only 280-350 mm, irrigation has been practiced in the plains area since 1860, and today some 90,000 ha of irrigated land is served by a highly developed canal and reservoir system, supplemented by pumping from groundwater sources (Evans 1971; Janonis and Gerlek 1977).

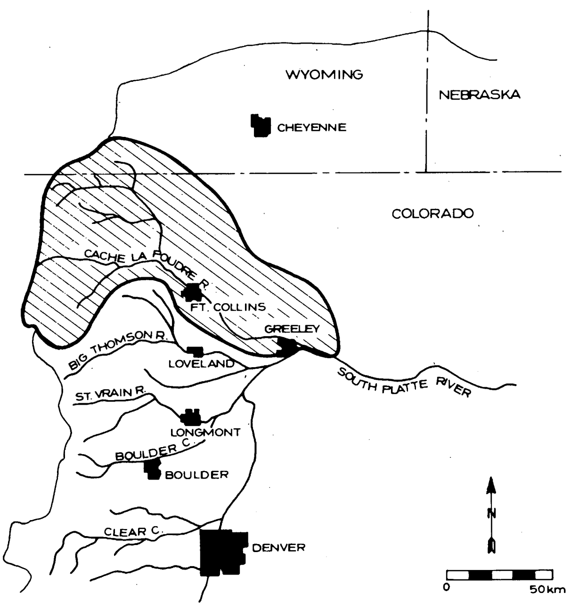

Fig. 7. The Chache la Poudre River basin.

Already, water resources in the area are scarce, and although intensive reuse of return flows from agriculture and cities effectively increase the amount of water available for use in the area, 23% of the total water supply in 1970 was derived from interbasin water transfer schemes (Evans 1971), primarily from the western slope of the Rocky Mountains. The growing environmental and political controversies surrounding these water transfer schemes, in combination with the rapidly increasing demands for urban water supply in the area, creates the kind of water resources planning problem for which this methodology has been designed.

Application of the Allocation Model

The Cache la Poudre system interaction matrix is shown in Fig. 1. The planning model covers the lower portion of the river basin which is further subdivided into two areas, with the city of Ft. Collins located in the upstream, and the city of Greeley in the downstream one. Irrigation and urban water supply demands may be satisfied from surface – or groundwater sources, or from existing or potential imports from adjacent areas. A number of existing and potential water and sewage treatment plants are considered, including a regional sewage treatment facility (secondary treatment) and local municipal reuse facilities (tertiary treatment).

State and decision variable designations are indicated in Fig. 1. Fig. 8 summarizes the hydrologic information requirements for the formulation of the state-decision relationship Eq. (1). This information has been obtained from existing literature on the hydrology and water resources of the Cache la Poudre River basin. (Hershey and Schneider 1964; Evans 1971; Thaemert 1976; Hendricks et. al. 1977) Cost functions have been established on the basis of local information (Wicke 1976), supplemented with general cost information in the literature (e.g. Klemetson et.al. 1975; Bishop et.al. 1975). In addition to the obvious demand, availability and capacity constraints, restrictions have been imposed to ensure certain minimum river flows, as well as positive net recharge to the aquifer (safe yield groundwater policy).

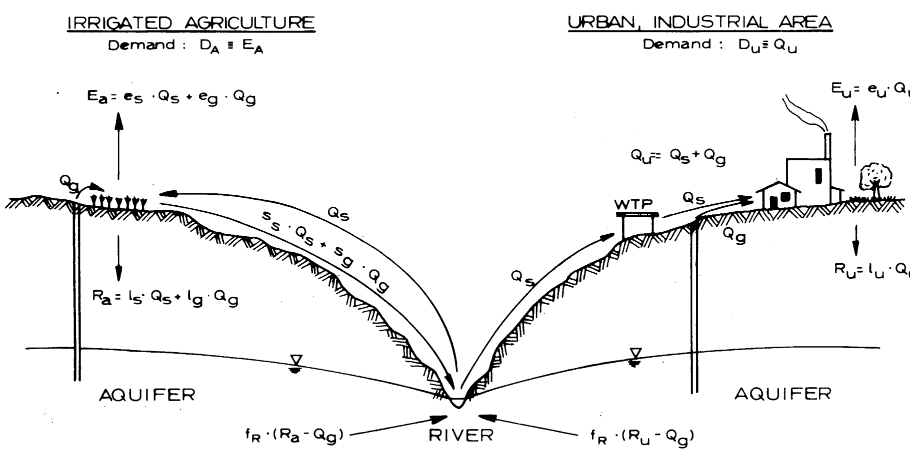

Fig. 8. Summary of hydrologic information requirements.

Results and Discussion

The study of the Cache la Poudre River basin should be considered illustrative only. However, the results are indicative of the kinds of conclusions to be generated from a detailed model based on a more specific and comprehensive data base.

A range of future scenarios for the development in the area has been investigated in combination with a range of hydrologic conditions. The extremes considered are intermediate demand projections for the year 2000 under average hydrologic conditions, and high demand projections for the year 2020 combined with drought conditions.

Based on these scenarios the presently available water supplies in the Cache la Poudre River basin (native plus current water importations) are adequate in the year 2000. The minimum expected cost solution for that year indicates maximum use of the relatively cheap supplies, Cache la Poudre River water and present imports, whereas the more expensive groundwater resource accounts for only 16% of the total supply. It is important to note, however, that groundwater and surface water are inseparable resources, in the sense that every unit of water pumped from the alluvial aquifer ultimately is drawn from the river, and vice versa. Thus groundwater is not a supply source per se, rather the aquifer is a vehicle for maximizing the beneficial use of available system water. In fact, from a system mass balance point of view water "lost" through canal seepage and deep percolation is actually conserved in groundwater storage; and only through evapotranspiration and system outflow is water really lost to the system.

The year 2020 scenario represents a more serious situation from a water supply point of view. Utilizing river water and present imports fully (still meeting low flow requirements), and pumping from the ground the maximum amount possible within a safe yield groundwater policy (47% of the total supply), additional water from new transbasin water transfer projects is still required if system demands are to be satisfied. In this situation the efficiency in water use is the maximum possible; system water is being used and reused more than twice.

Whereas municipal water reuse is not required in the year 2000 scenario, the water reuse alternative may represent a crucial water conservation measure in year 2020. As long as municipal water reuse contributes to a reduction in system outflow, and the average cost associated with this alternative is lower that the average cost of water from new transbasin water projects, the municipal water reuse alternative is feasible and economically attractive. However, if outflow requirements dictated by agreements with downstream water users exceed the sum of the minimum required streamflow plus the maximum amount of urban effluent that can leave the system, no water savings can be realized by reclaiming municipal effluents. Reductions in the amount of such effluents only result in corresponding increases in the amount of outflow to be produced from other sectors. In this situation municipal reuse is economically infeasible.

Sensitivity analysis on the scale parameter associated with a regional sewage treatment facility illustrates the effect of economies of scale in treatment and transportation costs. Under an assumption of relatively modest economies of scale characteristics the minimum expected cost solution favors expansion of local sewage treatment plants, but with increased scale effects a shift in strategy towards a regional facility occurs.

The most significant feature of the minimum cost variance solutions is their tendency to distribute activities on many transfer and treatment processes, rather than trying to rely on few activities, as dictated by the economies of scale in the minimum expected cost solutions. Although the results obtained in the case study tend to favor the minimum expected cost solutions, it is concluded that repeated generation of minimum cost variance solutions under different cost uncertainty and expected cost exceedance criteria should provide the basis for possible adjustments of the minimum expected cost solution towards increased reliability.

In addition to the already mentioned representation of storage, comparisons of the annual and seasonal solutions point to some important shortcomings of the annual approach as compared to the more realistic seasonal one. Thus, in the annual allocation model water supplies are considered useful regardless of their time of occurrence, while in fact, in the lower Cache la Poudre River basin, winter return flows from the aquifer to the river can neither be beneficially used, nor diverted to storage. Furthermore, the annual solutions yield all-or-nothing policies with respect to the utilization of water and sewage treatment facilities, while the seasonal solutions – in accordance with the actual operation of the present system – indicates a variation over the year of the most economic use of these facilities. Other aspects of the planning problem, such as the representation of municipal peak demand factors and low flow criteria for river segments, are considered more realistically in the seasonal approach as well.

It is evident that valuable insight into the weaknesses and pitfalls in annually based modelling can be gained from the seasonal approach, and lead to improvements in the much cheaper and easier annual approach. Thus, it is concluded that the two approaches should not be considered mutually exclusive, but rather highly complementary. A host of different assumptions and conditions may be explored through the annually based approach with the purpose of identifying an appropriate formulation of the more accurate, but also more complex and time consuming seasonal approach.

Conclusion

Efficient and economic use of water resources in a region requires the simultaneous consideration of hydrologic and economic factors. The allocation models described here meet this requirement by maintaining hydrologic mass balance in the economic optimization of the regional water resources system.

In the application of the models to a river basin in the semi-arid western United States optimal plans have been generated for a number of future scenarios with respect to water demand and availability. While such plans in the short term tend to agree with the present operation of the system, in the urban as well as in the agricultural sector, long term optimal plans indicate alternative ways of allocating scarce water supplies under different assumptions about the hydrologic and economic characteristics of the regional water resources system. Considering the uncertainties inherent in projections of future water demands and availabilities this is the perspective in which results from planning methodologies such as the one presented here should be viewed.

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Received: 16 January, 1979

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