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One-parameter optimal systems for the nonlinearevolution equation

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Abstract: The symmetry algebras of 1+1 dimensional nonlinear evolution equation arising from the motion of plane curve in affine geometry are systematically studied. It is found that the equation admits a seven-dimensional symmetry group H_1 and there are twenty-one elements in the one-parameter optimal system of the symmetry algebras. The optimality of one-parameter optimal system θ_1 is established by finding some algebraic invariants under the ajoint actions of the group H_1.

Key words: Lie group of symmetry; optimal system; adjoint representation; nonlinear evolution equation

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One of the main applications of Lie theory of symmetry groups for differential equations is the construction of group invariant. Given any subgroup of the symmetry group, under some mild conditions one can write down the equation for the invariant solution with respect to this subgroup. This reduced equation is of fewer variables and is easier to solve generally. In fact, for many important equations arising from geometry and physics these invariant solutions are the only ones which can be studied thoroughly. Their importance lies in the fact that they usually describe the asymptotic behavior or display the structure of the singularities of a general solution.

A basic problem concerning the group invariant solution is its classification. Since a Lie group (or Lie-algebra) usually contains infinitely many subgroups (or subalgebras) of the same dimension, a classification of them up to some equivalence relation is necessary. Following Ovsiannikov[1], one calls two subalgebras θ_1 and θ_2 of a given Lie algebra equivalent if one can find some element g in the Lie group generated by θ so that $Ad_g(\theta_1)=\theta_2$, where Ad_g is the adjoint representation of g on θ. A family of r-dimensional subalgebras $\{\theta_\alpha\}\alpha\in A$ is an r-parameter optimal system if (1) any r-dimensional subalgebra is equivalent to some θ_α and (2) θ_α and θ_β are inequivalent for distinct α and β. Discussions on optimal systems can be found in [1], Olver[2] and Ibragimov. Some examples of optimal systems can also be found in Ibragimov[3].

The method in [2] is based on the observation that the Killing form of the Lie algebra is an "invariant" for the adjoint representation. We shall apply this idea to study the nonlinear equation

$$u_t=-u_{xx}^{-\frac{4}{3}}u_{xxxx}+\frac{5}{3}u_{xx}^{-\frac{7}{3}}u_{xxx}^2, \tag{1}$$

which arises from the motion of plane curves in affine geometry Sa(2)

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$$\gamma t = fn + gt, \quad (2)$$

where f and g are respectively the normal and tangent velocity, γ is the curve. Letting $f=k$, k is the affine curvature in (2), then we get equation (1).

The layout of this paper is as follows: In section 1, we calculate the lie point symmetries admitted byequation (1). In section 2, we construct the 1-para meter oprimal system θ_1 of the Lie algebra h_1 and the optimality of the optimal system is also established. Section 3 is the concluding remarks on this work.

1) Lie Point symmetries of the equation (1).

Set $$\Delta = u_t + u_{xx}^{-\frac{4}{3}} u_{xxxx} - \frac{5}{3} u_{xx}^{-\frac{7}{3}} u_{xxx}^2, \quad (3)$$

we determine the infinitesimal transformations of the form

$$X = x + \varepsilon\xi(x,t,u) + 0(\varepsilon^2),$$
$$T = t + \varepsilon\tau(x,t,u) + 0(\varepsilon^2),$$
$$U = u + \varepsilon\Phi(x,t,u) + 0(\varepsilon^2). \quad (4)$$

Equation(1)admit Lie point transformations of the form (4) if and only if

$$V^{(4)}(\Delta) = 0, \quad \text{whenever} \quad \Delta = 0。 \quad (5)$$

where $V^{(4)}$is the fourth extended generator of

$$V = \xi(x,t,u)\frac{\partial}{\partial x} + \tau(x,t,u)\frac{\partial}{\partial t} + \Phi(x,t,u)\frac{\partial}{\partial u},$$

which is given by the following relation

$$V^{(4)} = V + \frac{\Phi^x\partial}{\partial u_x} + \frac{\Phi^t\partial}{\partial u_t} + \frac{\Phi^{xx}\partial}{\partial u_{xx}} + \frac{\Phi^{xxx}\partial}{\partial u_{xxx}} + \frac{\Phi^{xxxx}\partial}{\partial u_{xxxx}}.$$

Here $\Phi^x, \Phi^t, \Phi^{xx}, \Phi^{xxx}$and $\Phi^{xxxx}$are given explicitly in terms of ξ, τ, Φ and their derivatives[4].

By solving the determining equations dereved by equation (5) we obtain

$$\xi = c_1 u + c_2 x + c_3,$$
$$\Phi = c_4 u + c_5 x + c_6,$$
$$\tau = \frac{4c_4 t}{3} + \frac{4c_2 t}{3} + c_7.$$

So the Lie point symmetries of equation (1) form a seven-dimensional Lie algebra which is generated by the following vector fields

$$v_1 = \partial x, v_2 = \partial u, v_3 = \partial t, v_4 = x\partial u - u\partial x, v_5 = u\partial u + x\partial x + \frac{8}{3}t\partial t, v_6 = x\partial x - u\partial u, v_7 = x\partial u + u\partial x.$$

We denote the Lie algebra spanned by $\{v_1, v_2, \cdots v_7\}$by h_1.

2) The 1-parameter optimal system for equation(1).

In this section we give an optimal system for the Lie algebra h_1. To obtain the optimal system of h_1, we calculate the commutation realtions of the Lie algebra h_1 and the adjoint of the Lie group H_1 on the Lie algebra h_1, The results are listed in table 1and table 2 respectively.

Theorem 1 An one-dimemsional optimal system θ_1 of h_1 is given by

$$w_1 = v_5, w_2 = v_4, w_3 = v_3 + v_4,$$
$$w_4 = v_4 - v_3, w_5 = v_5 + av_4 (a \neq 0)$$
$$w_6 = v_3, w_7 = v_1, w_8 = v_2 + v_3,$$
$$w_9 = v_4 + v_7, v_{10} = v_4 + v_7 + v_3,$$
$$w_{11} = v_4 + v_7 - v_3, w_{12} = v_4 + v_7 + v_1,$$
$$w_{14} = v_4 + v_7 + v_1 + v_3,$$
$$w_{15} = v_4 + v_7 + v_1 - v_3, v_{18a} = v_4 + v_7 + v_5,$$
$$v_{18b} = v_4 + v_7 - v_5, w_{19} = v_7, w_{20} = v_3 + v_7,$$
$$w_{22} = v_5 + av_7 (a > 0, a \neq 0),$$
$$w_{23} = v_5 + v_7, w_{24} = v_5 + v_7 + v_2.$$

Tab. 1 The commutation relations for h_1(the$(i,j)^{th}$-entry is$[v_i, v_j]$)

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	0	0	v_2	v_1	v_1	v_2
v_2	0	0	0	$-v_1$	v_2	$-v_2$	v_1
v_3	0	0	0	0	$8v_3/3$	0	0
v_4	$-v_2$	v_1	0	0	0	$-2v_7$	$2v_6$
v_5	$-v_1$	$-v_2$	$-8v_3/3$	0	0	0	0
v_6	$-v_1$	v_2	0	$2v_7$	0	0	$2v_4$
v_7	$-v_2$	$-v_1$	0	$-2v_6$	0	$-2v_4$	0

Proof Let $w = \sum_1^7 a_i v_i$ is a general vector in h_1 and $A = a_4^2 + a_5^2 - a_6^2 - a_7^2$. Later we shall see that $A(A d(g)v) = A(v)$for all $g \in H_1$ and $v \in h_1$. That's, A is an invariant of h_1. To simplify$w$, we consider two cases separately.

1) $a_6^2+a_7^2=0$. A direct calculation shows that

$$Ad\exp(\varepsilon v_1)Ad\exp(\varepsilon_2 v_2)(w)=\sum_1^7\tilde{a}_i v_i,\text{ where}$$

$$\tilde{a}_1=a_1-\varepsilon_1 a_5+\varepsilon_2 a_4,\ \tilde{a}_2=a_2-\varepsilon_1 a_4-\varepsilon_2 a_5.$$

1.1) $a_4^2+a_5^2\neq 0$. We can eliminate $\tilde{a}_1$ and $\tilde{a}_2$ by choosing

$$\varepsilon_1=(a_1a_5+a_2a_4)/(a_4^2+a_5^2),$$

$$\varepsilon_2=(a_1a_4-a_2a_5)/(a_4^2+a_5^2).$$

Tab. 2　Actions of the adjoint representation of H_1 on (the $(i,j)^{th}$ − entry is $Ad(\exp\varepsilon v_i)v_j$)

Ad	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	v_1	v_2	v_3	$v_4-\varepsilon v_2$	$v_5-\varepsilon v_1$	$v_6-\varepsilon v_1$	$v_7-\varepsilon v_2$
v_2	v_1	v_2	v_3	$v_4+\varepsilon v_1$	$v_5-\varepsilon v_2$	$v_6+\varepsilon v_2$	$v_7-\varepsilon v_1$
v_3	v_1	v_2	v_3	v_4	$v_5-8\varepsilon v_3/3$	v_6	v_7
v_4	$v_1\cos\varepsilon+v_2\sin\varepsilon$	$v_2\cos\varepsilon-v_1\sin\varepsilon$	v_3	v_4	v_5	$v_6\cos 2\varepsilon+v_7\sin 2\varepsilon$	$v_7\cos 2\varepsilon-v_6\sin 2\varepsilon$
v_5	$e^{\varepsilon}v_1$	$e^{\varepsilon}v_2$	$e^{8\varepsilon/3}v_3$	v_4	v_5	v_6	v_7
v_6	$e^{\varepsilon}v_1$	$e^{-\varepsilon}v_2$	v_3	$v_4\cosh 2\varepsilon-v_7\sinh 2\varepsilon$	v_5	v_6	$v_7\cosh 2\varepsilon-v_4\sinh 2\varepsilon$
v_7	$v_1\cosh\varepsilon+v_2\sinh\varepsilon$	$v_2\cosh\varepsilon+v_1\sinh\varepsilon$	v_3	$v_4\cosh 2\varepsilon+v_6\sinh 2\varepsilon$	v_5	$v_6\cosh 2\varepsilon+v_4\sinh 2\varepsilon$	v_7

Then w is reduced to: $w=a_3v_3+a_4v_4+a_5v_5$.

Now we consider three subcases:

i) If $a_4=0$, $a_5\neq 0$, we take $a_5=1$ and use $Ad\exp(\varepsilon v_3)$ to kill v_3 term by choosing $\varepsilon=\frac{3a_3}{8a_5}$.

Then we get an equivalent form for w

$$w_1=w_5.$$

ii) If $a_4\neq 0, a_5=0$ applying $Ad\exp(\varepsilon v_5)$ on w, we have the following inequivalent form

$$w_2=v_4,\ (a_3=0),$$

$$w_3=v_4+v_3,\ (a_3a_4>0),$$

$$w_4=v_4-v_3.\ (a_3a_4<0).$$

iii) If $a_4\neq 0, a_5\neq 0$. using $Ad\exp(\varepsilon v_3)$ acting on w by choosing $\varepsilon=\frac{3a_3}{8a_5}$, w is equivalentto: $w_5=v_5+av_4\ (a\neq 0)$.

1.2) $a_6^2+a_7^2=0, a_4^2+a_5^2=0$. In this case is reduced to

$$w=a_1v_1+a_2v_2+a_3v_3.$$

If $a_1^2+a_2^2=0$, we obtain $w_6=w_3$. $w_6=v_3$.

If $a_1^2+a_2^2\neq 0$, after using $Ad\exp(\varepsilon v_4)$ for suitable ε, w is equivalent to

$$w_7=v_1,\ (a_3\neq 0),\ w_8=v_2\pm v_3(a_3\neq 0).$$

It is also pointed out that v_3-v_2 is mapped into v_3+v_2 by using $Ad\exp(\pi v_4)$.

2) $a_6^2+a_7^2\neq 0$, $w:\sum_1^7 a_iv_i$. Firstly, using $Ad\exp(\varepsilon v_4)$ to kill v_6 term, the coefficient of new w is

$$\tilde{a}_1=a_1\cos\varepsilon-a_2\sin\varepsilon,$$

$$\tilde{a}_2=a_2\cos\varepsilon-a_1\sin\varepsilon,$$

$$\tilde{a}_6=a_6\cos 2\varepsilon-a_7\sin 2\varepsilon,$$

$$\tilde{a}_7=a_7\cos 2\varepsilon+a_6\sin 2\varepsilon.$$

if $a_6\neq 0$, we choose $\varepsilon=\frac{1}{2}\arctan\frac{a_7}{a_6}$; if $a_6=0$, we choose $\varepsilon=\frac{\pi}{4}$. then $\tilde{a}_6=0$. Moreover, using $Ad\exp(\varepsilon v_6)$, we can arrange the coefficients of v_4 and v_7 to be equal, so w is equivalent to the following three cases

$$w=a_1v_1+a_2v_2+a_3v_3+a_4v_4+a_5v_5,\ (a_4^2>a_7^2),$$

$$w=a_1v_1+a_2v_2+a_3v_3+a_4(v_4+v_7)+a_5v_5,\ (a_4^2=a_7^2),$$

$$w=a_1v_1+a_2v_2+a_3v_3+a_5v_5+a_7v_7.\ (a_4^2<a_7^2).$$

For the first case, we use $Ad\exp(\varepsilon_1v_1)$ and $Ad\exp(\varepsilon_2v_2)$ to eliminate a_1 and a_2. w is reduced to

$$w=a_3v_3+a_4v_4+a_5v_5.$$

This case has been classified before.

For the second case

$$w=a_1v_1+a_2v_2+a_3v_3+a_4(v_4+v_7)+a_5v_5.$$

If $a_5=0$, we choose $\varepsilon=\frac{a_2}{2a_4}$ using $Ad\exp(\varepsilon v_1)$ to kill v_2 term, w becomes

$$a_1v_1+a_3v_3+a_4(v_4+v_7).$$

Now we consider the following several different cases:

i) $a_1=a_3=0$, $w_9=v_4+v_7$;

ii) $a_1=0, a_3\neq 0$. $w_{10}=v_4+v_7+v_3$. $(a_3a_3>0)$

$w_{11}=v_4+v_7-v_3$. $(a_3a_4<0)$;

iii) $a_3=0, a_1\neq 0$. $w_{12}=v_4+v_7+v_1$, $(a_1a_4>0)$

$w_{13}=v_4+v_7-v_1$. $(a_1a_4<0)$;

Since $Ad(\exp\pi v_4)(v_4+v_7+v_1)=v_4+v_7-v_1$, w_{12} is equivalent to w_{13}.

iv) $a_1\neq 0, a_3\neq 0$. Acting on w by $Ad\ \exp(\varepsilon_1 v_5)$ and $Ad\ \exp(\varepsilon_2 v_6)$ for the suitable $\varepsilon_1,\varepsilon_2$, we get

$w_{14}=v_4+v_7+v_1+v_3$, (a_1 and a_3 have the same sign),

$w_{15}=v_4+v_7+v_1-v_3$, (otherwise).

If $a\neq 0$, $w=a_1v_1+a_2v_2+a_3v_3+a_4(v_4+v_7)+a_5v_5$, Choosing $\varepsilon_3=3a_3/8a_5$, $\varepsilon_2=a_2/a_5$ and $\varepsilon_1=a_1/a_5$, we use $Ad\ \exp(\varepsilon_1v_1)$, $Ad\ \exp(\varepsilon_2v_2)$ and $Ad\ \exp(\varepsilon_3v_3)$ to kill v_1, v_2 and v_3, then apply $Ad\ \exp\ (\varepsilon v_6)$ on w, so w is equivalent to the following form

$w_{18a}=v_4+v_7+v_5$, $(a_5a_4>0)$,

$w_{18b}=v_4+v_7-v_5$, $(a_5a_4<0)$.

For the third case

$w=a_1v_1+a_2v_2+a_3v_3+a_5v_5+a_7v_7$.

We will consider three subcases:

i) $a_5=0, a_3=0$. After acting on w by $Ad\ \exp(\varepsilon_1v_1)$ and $Ad\ \exp(\varepsilon_2v_2)$ for suitable ε_1 and ε_2, we see that w is equivalent to: $w_{19}=v_7$.

ii) $a_5=0, a_3\neq 0$. We get the following equivalent form of w by using the action of $Ad\ \exp(\varepsilon_1v_1)$ $Ad\ \exp(\varepsilon_2v_2)$ on w

$w_{20}=v_7+v_3$, $(a_7a_3>0)$, $w_{21}=v_7-v_3$, $(a_7a_3<0)$.

Since $Ad\ \exp(\frac{\pi}{2}v_4)(w_{20})=-w_{21}$, w_{21} is equivalent to w_{20}.

iii) $a_5\neq 0$. We use $Ad\ \exp(\varepsilon v_3)$ to kill term v_3, then w is reduced to

$w=a_2v_2+a_5v_5+a_7v_7$.

If $a_2=0$, we have

$w_{22}=v_5+av_7$, $(a>0, a\neq 1)$, $w_{23}=v_5+v_7$.

It is also pointed out $v_5+av_7 (a>0)$ is equivalent to a_5-av_7, since $Ad\ \exp(\frac{\pi}{2}v_4)(v_5+av_7)=v_5-av_7$.

If $a_2\neq 0$, we get

$w_{24}=v_5+v_7+v_2$,

$w_{25}=v_5+v_7-v_2$,

$w_{26}=v_5-v_7+v_2$,

$w_{27}=v_5-v_7-v_2$.

It is easy to find that

$Ad\ \exp(\pi v_4)(v_5+v_7+v_2)=v_5+v_7-v_2$,

$Ad\ \exp(-\frac{1}{2}v_1)Ad\ \exp(\frac{1}{2}v_2)(v_5+v_7+v_2)=v_5+v_7-v_1$,

$Ad\ \exp(-\frac{\pi}{2}v_4)(v_5+v_7-v_1)=v_5-v_7+v_2$,

$Ad\ \exp(\pi v_4)(v_5-v_7+v_2)=v_5-v_7-v_2$.

From these expressions, we conclude that w_{24}, w_{25}, w_{26} *and* w_{27} are equivalent to each other.

Thus, we have shown that every one-dimensional subalgebra of h_1 is equivalent to one member in θ_1. It remains to prove θ_1 is optimal, we shall accomplish this by introducing some adjoint invariant in addition to A.

Lemma 1 $A=a_{24}+a_{25}-a_{26}-a_{27}$ is an invariant.

Proof A straightforward calculation shows

$$K(V,W)=10(a_4\tilde{a}_4+a_5\tilde{a}_5-a_6\tilde{a}_6-a_7\tilde{a}_7)-\frac{172}{9}a_5\tilde{a}_5$$

is the Killing form of Lie algebra h_1, where

$$V=\sum_{i=1}^{7}a_iv_i,\quad W=\sum_{i=1}^{7}\tilde{a}_iv_i.$$

A well-known fact is that the Killing form is invariant under the adjoint action. Hence $K(V,V)$ is invariant under the adjoint action. From Lemma 2, we see A is an invariant.

Lemma 2 a_5 is an invariant.

Proof: This can be easily seen from Tab. 2.

Lemma 3 Define

$$x=\begin{bmatrix}a_1 & \cdots\cdots & -a_5 & -a_6\\ a_2 & \cdots\cdots & -a_4 & -a_7\end{bmatrix},$$

$$y=\begin{bmatrix}a_1 & \cdots\cdots & a_4 & -a_7\\ a_2 & \cdots\cdots & a_6 & -a_5\end{bmatrix}.$$

$$B=\begin{cases}1 & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases},$$

Then B is an invariant provided $A=0$.

Proof The new (x,y) under the adjoint actions are listed in Tab. 3, it is clear from this table that B is invariant if $A=0$.

Tab. 3 New (x,y) under adjoint actions

$Ad(\varepsilon)$	New(x,y)
v_1	$(x,y+\varepsilon A)$
v_2	$(x-\varepsilon A,y)$
v_3	(x,y)
v_4	$(x\cos\varepsilon-y\sin\varepsilon, x\sin\varepsilon+y\cos\varepsilon)$
v_5	$e^{\varepsilon}(x,y)$
v_6	$(e^{-\varepsilon}x,e^{\varepsilon}y)$
v_7	$(x\cosh\varepsilon-y\sin\varepsilon, y\cosh\varepsilon-x\sinh\varepsilon)$

Lemma 4 Let $C=\begin{cases}a_3, & (a_5=0)\\ 0, & (a_5\neq 0)\end{cases}$,

Then C is an invariant.

Proof From Tab. 1, it is sufficient to check the invariance of C under $Ad\exp(\varepsilon v_i)$ for $i=1,2,4,5,6,7$. In fact under $Ad\exp(\varepsilon v_3)$, the newcoefficient: $\tilde{a}_3=a_3-\frac{8}{3}\varepsilon a_5$. Hence $\tilde{a}_3=a_3$ if and only if $a_5=0$. The lemma has been proved.

Lemma 5 Let $D=\begin{cases}\mathrm{sign}a_4, & A-a_5^2\geqslant 0,\\ 0, & \text{otherwise},\end{cases}$

then D is an invariant.

Proof Since a_5, A are invariants, we only need to prove the lemma in the case $a_5=1, a_4\neq 0, a_4^2-a_6^2-a_7^2\geqslant 0$. When we use $Ad\exp(\varepsilon v_i)(i=1,2,3,4,5)$, the value of a_4 is not changed. So $\mathrm{sign}a_4$ is preserved. For $Ad\exp(\varepsilon v_6)$, we find

$$\tilde{a}_4=a_4\cosh 2\varepsilon(1-\frac{a_7}{a_4}\tanh 2\varepsilon).$$

Since $a_4^2\geqslant a_6^2+a_7^2\geqslant a_7^2$, so $|\frac{a_7}{a_4}|\leqslant 1$ and $|\tanh 2\varepsilon|<1$, so we know $1-\frac{a_7}{a_4}\tanh 2\varepsilon\geqslant 0, \cos 2\varepsilon>0$. So we can get $\mathrm{sign}\tilde{a}_4=\mathrm{sign}a_4$.

For $Ad\exp(\varepsilon v_7)$, we have $\tilde{a}_4=a_4\cosh 2\varepsilon(1+\frac{a_6}{a_5}\tanh 2\varepsilon)$. Equivalently we can prove $\mathrm{sign}\tilde{a}_4=\mathrm{sign}a_4$. Then the lemma has been proved.

We find that $\{A,B,C,D,a_5,F\}$ is enough to distinguish all the vectors and yields $\{w_1,w_2,w_3\cdots w_{24}\}$. Now we evaluate them at $w_i(i=1,2,3\cdots,24)$, the results are put in Tab. 4. From this table one can see that all w_i are mutually inequivalent. Hence the theorem has been proved.

Finally it is found some algebraic invariants of the symmetry algebra of the 1+1-dimensional nonlinear evolution equation arising from the motion of plane curve in affine geometry under the inner autorphism of the Lie group H_1. These invariants are used to establish the optimality of the one-parameter optimal systems of the symmetry algebras of the 1+1-dimensional nonlinear evolution equation. It is of great interest to classify all subalgebras of h_1 and to extend the results to the higher-dimensional differential equations.

Tab. 4 Evaluation of the invariants

	A	B	C	D	a_5	F
w_1	1	0	0	0	1	0
w_2	1	0	0	1	0	0
w_3	1	0	1	1	0	0
w_4	1	0	−1	1	0	0
w_5	$1+a^2$	0	0	a'	1	$0(a\neq 0)$
w_6	0	0	1	0	0	0
w_7	0	0	0	0	0	1
w_8	0	0	1	0	0	1
w_9	0	0	0	1	0	0
w_{10}	0	0	1	1	0	0
w_{11}	0	0	−1	1	0	0
w_{12}	0	1	0	1	0	0
w_{14}	0	1	1	1	0	0
w_{15}	0	1	−1	1	0	0
w_{18a}	1	0	0	1	1	0
w_{18b}	1	0	0	−1	1	0
w_{19}	−1	0	0	0	0	0
w_{20}	−1	0	1	0	0	0
w_{22}	$1-a^2$	0	0	0	0	$a(a\neq 1)$
w_{23}	0	0	0	0	1	0
w_{24}	0	1	0	0	1	0

References:

[1] OVSIANNIKOV L V. Group Analysis of Differential Equations[M]. New York:Academic Press,1982.

[2] OLVER P J. Applications of Lie Groups to Differential Equation[M]. New York:Springer Verlag,1986.

[3] IBRAGIMOV N H. Transformation Groups Applied to Mathematical Physics [M]. Reider Dordrechet, 1985.

[4] IBRAGIMOV N H. Lie Group Analysis of Differential Equations[M]. Boca Raton:CRC Press,1994.

[5] BLUMAN G W,Cole J D. Similarity Methods for Differential Equations[M]. Berlin:Springer,1974.

(编　辑　曹大刚)

非线性发展方程的一维最优系统

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摘要:系统地研究了来自于射影几何中平面曲线运动的 1+1 维非线性方程的对称代数。发现此方程有一个七维对称群并且其对称代数的一维最优子代数有 21 个元素,通过寻找在群的伴随作用下的代数不变量,证明了该最优系统的最优性。

关　键　词:对称李群;最优系统;伴随表示;非线性发展方程

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$$\frac{\Phi^4(q-1)}{q-1}\prod_{p|q-1}\left(1+\frac{1}{(p-1)^3}\right),$$

及

$$\sum_{c\in F_q^*}V^2(c,q)=$$

$$\sum_{c\in F_q^*}\left(\frac{1}{q-1}\sum_{\chi}\bar{\chi}(c)\left(\sum_{a\in A}\chi(a)\right)^3\right)^2=$$

$$\frac{1}{q-1}\sum_{\chi}\left|\sum_{a\in A}\chi(a)\right|^6=$$

$$\frac{\Phi^6(q-1)}{q-1}\sum_{k|q-1}\sum_{h=1}^{k}{}'\frac{\mu^6(k)}{\Phi^6(k)}=$$

$$\frac{\Phi^6(q-1)}{q-1}\prod_{p|q-1}\left(1+\frac{1}{(p-1)^5}\right)。$$

于是完成定理 2 的证明。

参考文献:

[1] WIADYSLAW N. Classical Problems in Number Theory [M]. Warszawa: PWN-Polish Scientific Publishers,1987.

[2] APOSTOL T M. Introduction to Analytic Number Theory[M]. New York:Springer-Verlag,1976.

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On the properties of the generators in a finte field

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Abstract: The properties of the generators in a finite field are studied and something interesting are obtained.

Key words: finite field; generators; mean value theorem