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Topological Structure of Nonrelativistic Chern-Simons Vortices Solution*

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Abstract: By using the gauge potential decomposition, we have discussed the self-dual equation and its solution in Jackiw-Pi model. We obtained a new concrete self-dual equation and find relationship between Chern-Simons vortex solution and topological number which is determined by Hopf indices and Brouwer degrees of ϕ-mapping. To show the meaning of topological number we presented several figures with different topological numbers. In order to investigate the topological properties of many vortices, 5 parameters (two positions, one scale, one phase per vortex and one charge of each vortex) have been used to describe each vortex in many vortices solution in Jackiw-Pi model. For many vortices, three figures with different topological numbers have been drown to show the effect of the charge on the many vortices solution. We also studied the quantization of flux of those vortices related to the topological numbers in this case.

Key words: topological number; vortex; Jackiw-Pi model
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1 Introduction

Chern-Simons theories based on secondary characteristic classes discovered in Ref. [1] exhibit many interesting and important physical properties. In the early 1980s, the first physical applications of the Chern-Simons form called topologically massive gauge theory was advanced by Schonfeld[2], and many topological invariants of knots and links discovered in the 1980s could be reinterpreted as correlation functions of Wilson loop operators in Chern-Simons theory[3]. Moreover, for gauge theories and gravity in three-dimensions, they can appear as natural mass terms and will lead to a quantized coupling constant as well as a mass after quantization[4]. They have also found applications to a lot of physical problems, such as particle physics, quantum Hall effect, quantum gravity and string theory[5—10]. Chern-Simons term acquires dynamics via coupling to other fields[6,11], and gets multifarious gauge theory, non-relativistic Chern-Simons theory supports vortices solution, these static solutions can be obtained when their Hamiltonian was minimal. Vortices and their dynamics are interesting objects to be studied[11—14]. R. Jackiw and S-Y. Pi considered a gauged, nonlinear Schrödinger equation in two spatial dimensions, to describe non-relativistic matter interacting with Chern-Simons gauge fields. Then they found explicit static, self-dual solution which satisfies the Li-

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ouville equation and got an n-soliton solution depends on $4n$ parameters (two positions, one scale, one phase per solition)[6, 7]. P. A. Horváthy proved that the solution depends on $4n$ parameters without the use of an index theorem and the flux quantization[15], and indicated the regular solutions with finite degree only arises for rational functions, the topological degree of those solutions is the common number of their zeros and poles on the Riemann sphere[16].

In this paper, by using the gauge potential decomposition[17—19], we will discuss topological structure of the self-dual solution in Jackiw-Pi model. We will look for complete many vortices solution from the self-dual equation and set up the relationship between the many vortices solution and topological number which is determined by Hopf indices and Brouwer degrees. We will give several figures to show the effects of the topological number on vortices. We also study the quantization of the flux of those vortices.

2 Topological Number of Self-dual Vortex in Jackiw-Pi Model

In this section, based on the self-dual equation, we will look for complete vortex solution in Jackiw-Pi model making use of the decomposition of gauge potential. The Abelian Jackiw-Pi model in nonlinear Schrödinger systems is[6, 7]

$$L = \frac{\kappa}{4} \in^{\mu\alpha\beta} A_\mu F_{\alpha\beta} + i\hbar\psi^* \left(\partial_t + \frac{ie}{\hbar} A^0\right)\psi - \frac{\hbar^2}{2m}|\boldsymbol{D}\psi|^2 + \frac{g}{2}(\psi^*\psi)^2, \quad (1)$$

here relativistic notation with the metric diag is $(1, -1, -1)$ and $x^\mu = (ct, \boldsymbol{r})$, where $\boldsymbol{D} = \nabla - i\frac{e}{\hbar c}\boldsymbol{A}$, and ψ is "matter" field, the first term is the Chern-Simons density, which is not gauge invariant. Also m is the mass parameter, A_μ is gauge potentials, g governs the strength of nonlinearity, κ controls the Chern-Simons term and provides a cutoff at large distance greater than $1/\kappa$ for gauge-invariant electric and magnetic fields, which can be written as $E = -\nabla A^0 - (1/c) \times \partial_t A$ and $\boldsymbol{B} = \nabla\times\boldsymbol{A}$. Thus the Chern-Simons terms give rise to massive, yet gauge-invariant "electrodynamics". The last term represents a self-coupling contact term of the type commonly found in nonlinear Schrödinger systems. The magnetic fields B satisfies

$$B = -\varepsilon^{ij}\partial_i A_j = \varepsilon^{ij}\partial_i A^j = -\frac{e}{k}\rho, \quad (2)$$

where

$$\rho = \psi^*\psi, \quad (3)$$

with $g = \mp e^2\hbar/2mck$, and sufficiently well-behaved fields so that the integral over all space of $\nabla\times\boldsymbol{J}$ vanishes, the energy is

$$H = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,|(D_1 \pm iD_2)\psi|^2, \quad (4)$$

this is non-negative and vanishes. So it is obvious that ψ satisfies a self-dual equation

$$D_1\psi = \mp iD_2\psi. \quad (5)$$

To solve Eq. (5), we note that when ψ is decomposed into two scalar fields

$$\psi = \psi^1 + i\psi^2. \quad (6)$$

We can define a unit vector field $\boldsymbol{n}$ as follows

$$n^a = \frac{\psi^a}{|\psi|^2}, \quad a = 1, 2. \quad (7)$$

It is easy to prove that $\boldsymbol{n}$ satisfies the constraint conditions

$$n^a n^a = 1. \quad (8)$$

From Eq. (5), and making use of the decomposition of $U(1)$ gauge potential in terms of the two-dimensional unit vector field[19], we can obtain

$$A^i = \frac{\hbar c}{e}\left(\varepsilon^{ab} n^a \partial_i n^b \pm \frac{1}{2}\varepsilon^{ij}\partial_j \ln\rho\right). \quad (9)$$

From Eq. (2) and Eq. (9) we get

$$B = \frac{\hbar c}{e}\varepsilon^{ij}\varepsilon^{ab}\partial_i n^a \partial_j n^b \pm \frac{\hbar c}{2e}\nabla^2 \ln\rho, \quad (10)$$

i. e.

$$-\frac{e}{k}\rho = \frac{\hbar c}{e}\varepsilon^{ij}\varepsilon^{ab}\partial_i n^a \partial_j n^b \pm \frac{\hbar c}{2e}\nabla^2 \ln\rho. \quad (11)$$

This equation can be rewritten as

$$\nabla^2 \ln\rho = \pm \frac{2e^2}{\hbar ck}\rho \pm 2\varepsilon^{ij}\varepsilon^{ab}\partial_i n^a \partial_j n^b \ , \qquad (12)$$

with the help of the ϕ-mapping method[19], Eq. (12) can be written as

$$\nabla^2 \ln\rho = \pm \frac{2e^2}{\hbar c\kappa}\rho \pm 4\pi\delta^2(\boldsymbol{\psi}) J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right) \ , \qquad (13)$$

in which $\boldsymbol{J}(\boldsymbol{\phi}/\boldsymbol{x})$ is Jacobian

$$J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right) = \frac{1}{2}\varepsilon^{ab}\varepsilon^{ij}\frac{\partial\psi_a}{\partial x^i}\frac{\partial\psi_b}{\partial x^j} \ , \quad i, j = 1, 2 \ . \qquad (14)$$

When $\rho \neq 0$, Eq. (13) will be the Liouville equation,

$$\nabla^2 \ln\rho = \pm \frac{2e^2}{\hbar c\kappa}\rho \ , \qquad (15)$$

as we all know, Eq. (15) has the general real solution as follows

$$\rho(r) = \frac{4\hbar c\kappa}{e^2}\frac{|f'(z)|^2}{[1 + |f(z)|^2]^2} \ , \qquad (16)$$

in which

$$\boldsymbol{r} = (r\cos\theta, \ r\sin\theta) = \boldsymbol{x} \ , \qquad (17)$$

where $z = r\exp(\mathrm{i}\theta)$ and $f(z)$ is an arbitrary function[6].

Because ρ is the charge density of the vortex, it must be positive, the Liouville equation is

$$\nabla^2 \ln\rho = -\frac{2e^2}{\hbar c|\kappa|}\rho \ , \qquad (18)$$

so Eq. (16) should be

$$\rho(r) = \frac{4\hbar c|\kappa|}{e^2}\frac{|f'(z)|^2}{[1 + |f(z)|^2]^2} \ , \qquad (19)$$

and Eq. (13) can rewritten as

$$\nabla^2 \ln\rho = -\frac{2e^2}{\hbar c|\kappa|}\rho - (\mathrm{sgn}\kappa)4\pi\delta^2(\boldsymbol{\psi}) J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right) \ . \qquad (20)$$

It is a Liouville equation with a δ function on its right side. For $\boldsymbol{r} = 0$, this equation is also right. To show the meaning of the δ function of this equation, we will integrate Eq. (20) in section 3, and discuss its singular point. For one vortex

$$f(z) = \left(\frac{c}{z - z_0}\right)^N, \qquad (21)$$

in which $c = r_c\exp(\mathrm{i}\theta_c)$ and $z_0 = r_0\exp(\mathrm{i}\theta_o)$ is a complex constant, so there are 5 real parameters involved in this solution: 2 real parameters z_0 describing the locations of the vortices, 2 real parameters c corresponding to the scale and phase of each vortex, 1 real parameter N describing the charge of the vortex, and it is easy to obtain the radially symmetric solutions[20]

$$\rho = \frac{4\hbar c|\kappa| N^2}{e^2 r_c^2}\frac{(r - r_0/r_c)^{2(N-1)}}{\{1 + [(r - r_0)/r_c]^{2N}\}^2} \ , \qquad (22)$$

where

$$N - 1 = -\frac{\kappa}{|\kappa|}Q \ , \qquad (23)$$

$$Q = \beta\eta \ , \qquad (24)$$

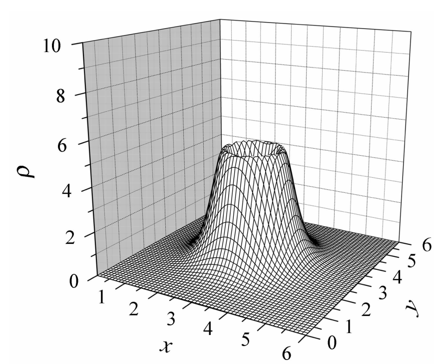

Fig. 1 Density ρ for solution (22) representing one vortex with $Q = 1$.

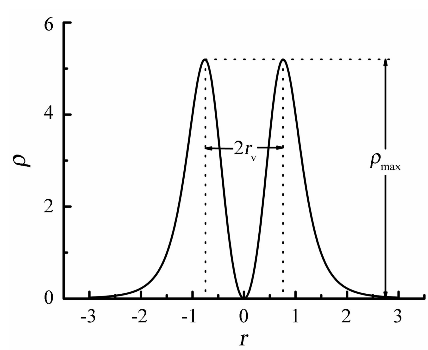

Fig. 2 The section plane of Fig. 1, this section plan includes the center of this vortex, as is shown in this figure, we can define the hight ρ_{max} and radius r_v of the vortex.

where Q is the topological number of the vortex, those numbers are determined by Hopf indices and Brower degrees of ψ. Particularly, ρ is invariant when change N to $-N$, see Fig. 1 for a plot of the one vortex case, in this case, the center of the vortex is $z_0 = 3 + 3\mathrm{i}$. In order to study the shape of the vortex, we slice off Fig. 1 through it's center, and define the height and radius of the vortex, as is shown in Fig. 2, we can obtain

$$r_{\mathrm{v}} = r_{\mathrm{c}}\left(1 - \frac{2}{-\operatorname{sgn}\kappa Q + 1}\right)^{\frac{1}{-2\operatorname{sgn}\kappa Q + 2}} . \quad (25)$$

The height of the vortex

$$\rho_{\max} = \frac{\hbar c\,|\kappa|}{r_{\mathrm{c}}^2 e^2}\left(\frac{\operatorname{sgn}\kappa Q}{\operatorname{sgn}\kappa Q - 2}\right)^{\frac{1}{\operatorname{sgn}\kappa Q - 1}} \times [(\operatorname{sgn}\kappa Q - 1)^2 - 1] . \quad (26)$$

From Eqs. (25) and (26) we can see that the height and radius of the vortex are depend on topological number and r_{c}[21].

3 Topological Structure of Many Vortices Solution and Its Magnetic Flux

In this section, making use of Eq. (20), we will discuss the topological structure of the many vortices solution, then we will study the magnetic flux of the vortices. The meromorphic function $f(z)$ yields a regular many vortices solution with finite magnetic flux if and only if $f(z)$ is a rational function,

$$f(z) = \frac{P(z)}{T(z)} , \quad (27)$$

subject to

$$\deg P < \deg T . \quad (28)$$

In particular, when all roots of $T(z)$ are simple, $f(z)$ can be developed in to partial fractions[15],

$$f(z) = \sum_{a=1}^{M} \frac{c_a}{z - z_a} , \quad (29)$$

in which

$$z_a = r_a \exp(\mathrm{i}\theta_a) , \; c_a = r_{0a}\exp(\mathrm{i}\theta_{0a}) , \quad a = 1, 2, \cdots M . \quad (30)$$

So there are $4M$ real parameters involved in this solution: $2M$ real parameters r_a and θ_a describing the locations of the vortices, $2M$ real parameters r_{0a} and θ_{0a} corresponding to the scale and phase of each vortex. However, there is non evidence to believe the charge of each vortex equals to 1, in order to study the charge and topological structure of each vortex in this solution, we suppose the charge of vortex z_a is N_a (vortex z_a is the vortex whose center is z_a), then we can rewrite $f(z)$ as

$$f(z) = \sum_{a=1}^{M}\left(\frac{c_a}{z - z_a}\right)^{N_a} , \quad (31)$$

whick describes M separated charge vortices, and $N_a > 0$ because of Eq. (28), then we add M real parameters N_a in our solution for describing the charge of each vortex and in the following we will see that N_a is related to the topological number of each vortex. Under the radially symmetric, $\nabla^2 \ln\rho$ can be expressed as

$$\nabla^2 \ln\rho = \frac{\partial^2}{\partial^2 r}\ln\rho + \frac{1}{r}\partial_r \ln\rho . \quad (32)$$

Integrating Eq. (20)

$$\int \nabla^2 \ln\rho \mathrm{d}\boldsymbol{r} = \int\left[-\frac{2e^2}{\hbar c\,|\kappa|}\rho + \operatorname{sgn}\kappa 4\pi\delta^2(\boldsymbol{\psi})J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right)\right]\mathrm{d}\boldsymbol{r} . \quad (33)$$

The Eq. (33) can be rewritten as

$$\int_{\boldsymbol{r}_a}^{\boldsymbol{r}_a+\boldsymbol{r}_\varepsilon} \nabla^2 \ln\rho \mathrm{d}\boldsymbol{r} = \int_{\boldsymbol{r}_a}^{\boldsymbol{r}_a+\boldsymbol{r}_\varepsilon}\left[-\frac{2e^2}{\hbar c\,|\kappa|}\rho + \operatorname{sgn}\kappa 4\pi\delta^2(\boldsymbol{\psi})J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right)\right]\mathrm{d}\boldsymbol{r} , \quad (34)$$

where $\boldsymbol{r}_\varepsilon = (\boldsymbol{r}_\varepsilon\cos\theta, r_\varepsilon\sin\theta)$, $0 \leqslant \theta \leqslant 2\pi$, and $\boldsymbol{r}_\varepsilon$ is a infinitesimal scalar, so $\int_{\boldsymbol{r}_a}^{\boldsymbol{r}_a+\boldsymbol{r}_\varepsilon}$ is an integral in the infinitesimal region neighbouring the $\boldsymbol{r}_a$ point. The left side of this equation is

$$\int_{\boldsymbol{r}_a}^{\boldsymbol{r}_a+\boldsymbol{r}_\varepsilon} \nabla^2 \ln\rho \mathrm{d}\boldsymbol{r} = 4\pi(N_a - 1) . \quad (35)$$

Suppose that the vector field $\boldsymbol{\psi}$ possesses M isolated zero which is in $\boldsymbol{r} = \boldsymbol{r}_a$, according to the δ-function theory, we can obtain

$$\delta^2(\boldsymbol{\psi}) = \sum_{a=1}^{M}\frac{\beta_a}{|J(\boldsymbol{\psi}/\boldsymbol{x})|_{\boldsymbol{r}=\boldsymbol{r}_a}}\delta^2(\boldsymbol{r} = \boldsymbol{r}_a) , \quad (36)$$

and then we can obtain

$$\int_{\boldsymbol{r}_a}^{\boldsymbol{r}_a+\boldsymbol{r}_\varepsilon}\left[4\pi\delta^2(\boldsymbol{\psi})J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right)\right]\mathrm{d}\boldsymbol{r} = 4\pi\int_{\boldsymbol{r}_a}^{\boldsymbol{r}_a+\boldsymbol{r}_\varepsilon}\beta_a\eta_a\delta^2(\boldsymbol{r} - \boldsymbol{r}_a)\mathrm{d}\boldsymbol{r} , \quad (37)$$

where β_a is the positive integer (the Hopf index of the zero point) and η_a, the Brouwer degree of the vector field $\boldsymbol{\psi}$,

$$\eta_a = \mathrm{sgn} J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right)\Big|_{r=r_a} = \pm 1 . \qquad (38)$$

The meaning of the Hopf index β_a is that while $\boldsymbol{r}$ covers the region neighbouring the z_a point once, the vector field $\boldsymbol{\psi}$ covers the corresponding region β_a times. Hence, β_a and η_a are the topological numbers which show the topological properties of the vortex solution. We have

$$\delta^2(\boldsymbol{\psi}) J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right) = \beta_a \eta_a \delta^2(\boldsymbol{r} - \boldsymbol{r}_a) . \qquad (39)$$

If we define the topological number Q_a of the vortex whose center is z_a as

$$Q_a = \int \delta^2(\boldsymbol{\psi}) J\left(\frac{\boldsymbol{\psi}}{\boldsymbol{x}}\right) \mathrm{d}\boldsymbol{x} = \beta_a \eta_a , \qquad (40)$$

from Eq. (34) we can get

$$N_a - 1 = (-\mathrm{sgn}\boldsymbol{\kappa}) Q_a , \qquad (41)$$

in which $N_a - 1 > 0$, so $\boldsymbol{\kappa}$ must satisfy $\mathrm{sgn}\boldsymbol{\kappa} = -\mathrm{sgn}J$ and we can set $N_a - 1 = |Q_a|$, then the total charge of those vortices

$$N = \sum_{a=1}^{M} N_a = (-\mathrm{sgn}\boldsymbol{\kappa}) Q + M , \qquad (42)$$

where Q is the total topological number and can be defined as

$$Q = \sum_{a=1}^{M} Q_a . \qquad (43)$$

Substituting Eq. (41) into Eq. (31), we can obtain

$$f(z) = \sum_{a=1}^{M} \left(\frac{c_a}{z - z_a}\right)^{|Q_a|+1} , \qquad (44)$$

it is obviously that Eq. (44) and Eq. (19) are the solutions of Eq. (20). On the other hand, this means vortices density ρ relates to its topological number Q_a. We now see that N_a must be an integer. If we note the unit magnetic flux $\Phi_0 = 2\pi\hbar c/e$, we can get

$$\Phi = \int_0^{\infty} B\mathrm{d}\boldsymbol{r} = -2(\mathrm{sgn}\boldsymbol{\kappa})\Phi_0 |-\mathrm{sgn}\boldsymbol{\kappa} Q + M| , \qquad (45)$$

from this equation we know the magnetic flux is quantized. When the total topological number equal to zero, the magnetic flux of this vortex is

$$\Phi = \int_0^{\infty} B\mathrm{d}\boldsymbol{r} = -2(\mathrm{sgn}\boldsymbol{\kappa}) M\Phi_0 . \qquad (46)$$

For example, when set $M = 1$ in Eq. (31), we can get the one vortex solution Eq. (22). When set $M = 2$ in Eq. (31), we can get the two vortices solution with $z_1 = -3$, $z_2 = 3$, $c_1 = c_2 = 1$, i. e.

$$f(z) = \left(\frac{1}{z+3}\right)^{|Q_1|+1} + \left(\frac{1}{z-3}\right)^{|Q_2|+1} , \qquad (47)$$

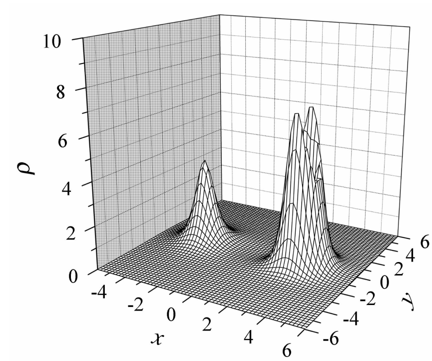

Fig. 3 Density ρ for solution (47) representing two separated vortices with $Q_1 = 0$, $Q_2 = 1$.

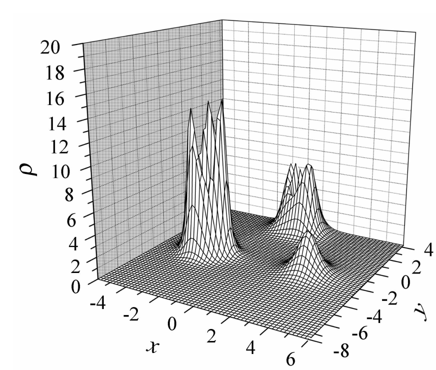

Fig. 4 Density ρ representing three separated vortices with $Q_1 = 0$, $Q_2 = 1$, $Q_3 = 2$.

with Eq. (19), we can give the solution of two vortices. See Fig. 3 for a plot of the two vortices case, see Fig. 4 for a plot of the three vortices case and Fig. 5 for the four vortices case. From the figures we can see the shape of those vortices is different when their topological numbers are different. So it is not enough to use $4M$ parameters to describe M vortices, we have to introduce a charge parameter N, from Eqs. (25) and (26) we can see, the height and radius of the vortex depend on Q.

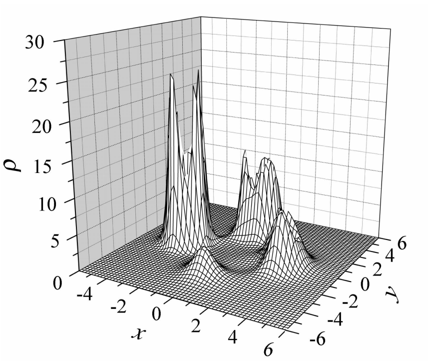

Fig. 5　Density ρ representing four separated vortices with $Q_1 = 0$, $Q_2 = 1$, $Q_3 = 2$, $Q_4 = 3$.

4　Conclusions

In this paper, We discussed the self-dual equation and its solution in Jackiw-Pi model by using the gauge potential decomposition and ϕ-mapping method, we got a Liouville equation with a δ function, then we also obtained the solution of this equation, and the δ function will not change the character of the solution when $\rho \neq 0$. We added M parameters to the M-vortices solution, those parameters describing the charge of each vortex, i. e., use $5M$ parameters (two positions, one scale, one phase per vortex and one charge per vortex) to describe M-vortices solutions in Jackiw-Pi model. We studied the topological structure of Chern-Simons vortices in Jakiw-Pi model by calculating the integral of the Liouville equation, and found the charges of those vortices were determined by the topological numbers of those vortices, those topological numbers were determined by Hopf indices and Brouwer degrees of ϕ-mapping. We also gave some figures with different topological numbers to show the relationship between the shape of those vortices and topological numbers. In many vortices solution, in order to show the shape of those vortices is different when only their topological numbers are different, we showed some vortices only with different positions and different topological numbers in many vortices figures (see Figs. 3—5). We also found the relationship between the quantization of the flux and the topological numbers from the integral value of the solution in the whole space. However, the flux is non-vanishing when the topological number equals to zero. So does the angular momentum.

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非相对论 Chern-Simons 多涡旋解的拓扑结构*

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摘　要：运用规范势分解理论研究了 Jackiw-Pi 模型中的自对偶方程，得到一个新的自对偶方程，发现了 Chern-Simons 多涡旋解与拓扑荷之间的联系。为了研究 Jackiw-Pi 模型多涡旋解的拓扑性质，构造了一个新的静态自对偶 Chern-Simons 多涡旋解，每个涡旋由5个实参数描述。2个实参量用来描述涡旋的位置，2个实参量用来描述涡旋的尺度和相位，还有一个实参量描述涡旋的荷。为了研究拓扑数对涡旋形状的影响，给出了具有不同拓扑数的多涡旋解。另外还研究了该涡旋解的磁通量的拓扑量子化。

关 键 词：拓扑数；涡旋；Jackiw-Pi 模型

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