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Quantization of $SU(n)$ $N=2$ Supersymmetric Gauge Field System with Non-Abelian Chern-Simons Topological Term and Its Fractional Spin*

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Abstract: We quantize $SU(n)$ $N=2$ supersymmetric gauge field system with non-Abelian Chern-Simons topological term for constrained Hamilton system in framework of Faddeev-Senjanovic path integral quantization, deduce the total angular momentum based on the global canonical Norther theorem at quantum level, obtain the fractional spin of this supersymmetric system, and find that this anomalous fractional spin has the contribution from the group superscript components.

Key words: supersymmetry; Chern-Simons; constrained system; fractional spin; gauge field

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1 Introduction

Considerable attention has been given to a connection between extended supersymmetry and the existence of self-dual solutions[1], Ref. [2] studied supersymmetric Chern-Simons systems. A $SU(n)N=2$ supersymmetric gauge field model is constructed[3], fractional spin and statistics have important meanings in explaining the quantum Hall effects[4] and high-T_c superconductivity phenomena[5].

Gauge theories with Chern-Simons topological term may result in fractional spin[6,7]. It is interesting to study the supersymmetric anyon system, because both spinor fields and scalar fields are naturally contained in supersymmetric fields.

Refs. [6—9] investigated the angular momentum of Chern-Simons system by energy-momentum tensor and classical Noether theorem, and obtained the fractional spin character. Because the phase-space path integral method is more fundamental than the configuration-space path integral method.

In this letter we investigate quantization and fractional spin of the $SU(n)N=2$ supersymmetric gauge field system with non-Abelian Chern-Simons topological term at quantum level by using the phase-space path integral method.

2 Supersymmetric Gauge Field System with Non-Abelian Chern-Simons Term and Its Constraints

A $SU(n)N=2$ super symmetric gauge field system with non-Abelian Chern-Simons topological term in 2+1 dimensions was constructed[10]. Using Wess-Zumino gauge, the action is expressed in terms of component fields as

$$S = \int d^3x\Big[-\frac{1}{4}G^{ab,r}G^r_{ab} + \frac{1}{2}\kappa\,\varepsilon^{abc}\cdot$$

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$$(A_a^r\partial_b A_c^r+\frac{1}{3}f^{rsu}A_a^r A_b^s A_c^u)+\mathrm{i}\psi^{+\alpha}(\gamma^a)_\alpha^\rho D_a\psi_\rho+$$
$$(D^a\varphi)^+D_a\varphi+\frac{1}{2}\mathrm{i}\lambda^\alpha(\gamma^a)_\alpha^\rho D_a\lambda_\rho+$$
$$\frac{1}{2}\mathrm{i}\chi^\alpha(\gamma^a)_\alpha^\rho D_a\chi_\rho+\frac{\kappa}{2}\lambda^{\alpha,r}\lambda_\alpha^r+$$
$$\frac{\kappa}{2}\chi^{\alpha,r}\chi_\alpha^r+\frac{1}{2}(D^aN^r)D_aN_r-$$
$$U(\varphi,\varphi^+,\psi,\psi^+,\lambda,\chi,N)\Big],\tag{1}$$

where $\gamma^0=\mathrm{i}\sigma^1$, $\gamma^1=\sigma^2$, and $\gamma^2=\mathrm{i}\sigma^3$ satisfy $\gamma^a\gamma^b=g^{ab}+\mathrm{i}\varepsilon^{abc}\gamma_c$, $g_{ab}=\mathrm{diag}(+1,-1,-1)$, $D_a=\partial_a-\mathrm{i}A_a^rT^r(a=1,2,3)$, $G_{ab}^r=\partial_aA_b^r-\partial_bA_a^r+f^{rsu}A_a^sA_b^u$. T^r are generators of $SU(n)$ and satisfy $[T^r,T^s]=\mathrm{i}f^{rsu}T^u$ and $\mathrm{tr}(T^aT^b)=\delta^{ab}/2$. Fixing the potential by requiring the conservation of the fermion-number, the potential term is[10]

$$U(\varphi,\varphi^+,\psi,\psi^+,\lambda,\chi,N)=$$
$$f^{rsu}\chi^{\alpha,r}\lambda_\alpha^sN^u-\mathrm{i}(\psi^{+\alpha}\lambda_\alpha^rT^r\varphi-\varphi^+T^r\lambda^{\alpha,r}\psi_\alpha)-$$
$$\psi^{+\alpha}\chi_\alpha^rT^r\varphi-\varphi^+T^r\chi^{\alpha,r}\psi_\alpha-\frac{n-1}{2kn}\nu^2\psi^{+\alpha}\psi_\alpha-$$
$$\psi^{+\alpha}N^rT^r\psi_\alpha+\varphi^+(N^rT^r+\frac{n-1}{2kn}\nu^2)\cdot$$
$$(N^sT^s+\frac{n-1}{2kn}\nu^2)\varphi+\frac{1}{2}(\varphi^+T^r\varphi+\kappa N^r)\cdot$$
$$(\varphi^+T^r\varphi+\kappa N^r),\tag{2}$$

in which ν is the expected value of vacuum state.

One can see that Eq. (1) is singular in Dirac method. We first analyze the constraints of this system in phase space. The canonical momenta are defined as

$$\pi_\alpha=\frac{\partial_R\mathscr{L}}{\partial\dot\phi^\alpha},\tag{3}$$

where ϕ^α stands for the component fields, the subscript "R" denotes the right derivative for ϕ^α. It is easy to get the relative momenta as

$$\pi^{a,r}=\frac{\partial_R\mathscr{L}}{\partial\dot A_a^r}=-G^{0a,r}+\frac{1}{2}\kappa\varepsilon^{0ab}A_b^r\quad,$$
$$\pi_{\Psi_\rho}=\frac{\partial_R\mathscr{L}}{\partial\dot\Psi_\rho}=\mathrm{i}\Psi^{+\alpha}(\gamma^0)_\alpha^\rho\quad,$$
$$\pi_{\Psi^{+\alpha}}=\frac{\partial_R\mathscr{L}}{\partial\dot\Psi^{+\alpha}}=0,$$
$$\pi_\varphi=\frac{\partial_R\mathscr{L}}{\partial\dot\varphi}=D_0\varphi^+,$$
$$\pi_{\varphi^+}=\frac{\partial_R\mathscr{L}}{\partial\dot\varphi^+}=D_0\varphi,$$
$$\pi_{\lambda_\rho}=\frac{\partial_R\mathscr{L}}{\partial\dot\lambda_\rho}=\frac{1}{2}\mathrm{i}\lambda^\alpha(\gamma^0)_\alpha^\rho\quad,$$
$$\pi_{\chi_\rho}=\frac{\partial_R\mathscr{L}}{\partial\dot\chi_\rho}=\frac{1}{2}\mathrm{i}\chi^\alpha(\gamma^0)_\alpha^\rho\quad,$$
$$\pi_{N^r}=\frac{\partial_R\mathscr{L}}{\partial\dot N^r}=D_0N^r\quad.\tag{4}$$

According to Dirac-Bergmann procedure[11], the primary constraints of the system should includes π_0^r, π_{Ψ_ρ}, $\pi_{\Psi^{+\alpha}}$, π_{λ_ρ}, π_{χ_ρ}. The constraints referring to fermion fields have novel feature, and can be handled in a different procedure[12]. According to Dirac-Bergmann procedure, the primary constraints of the system are given by

$$\Gamma_1^r=\pi^{0,r}\approx0,$$
$$\Gamma_2^\rho=\pi_{\psi_\rho}-\mathrm{i}\psi^{+\alpha}(\gamma^0)_\alpha^\rho\approx0,$$
$$\Gamma_3^\alpha=\pi_{\psi^{+\alpha}}\approx0,$$
$$\Gamma_4^\rho=\pi_{\lambda_\rho}-\frac{1}{2}\mathrm{i}\lambda^\alpha(\gamma^0)_\alpha^\rho\approx0,$$
$$\Gamma_5^\rho=\pi_{\chi_\rho}-\frac{1}{2}\mathrm{i}\chi^\alpha(\gamma^0)_\alpha^\rho\approx0,\tag{5}$$

where symbol "$\approx$" means weak equality in Dirac sense[11]. The canonical Hamilton density corresponding to action (1) is given by

$$\mathscr{H}_c=\pi_\varphi\dot\varphi+\pi_{\varphi^+}\dot\varphi^++\pi^{a,r}\dot A_a^r+\pi_{\psi_\rho}\dot\psi_\rho+\dot\psi^{+\alpha}\pi_{\psi^{+\alpha}}+$$
$$\pi_{\chi_\rho}\dot\chi_\rho+\pi_{\lambda_\alpha}\dot\lambda_\alpha+\pi_{N^r}\dot N^r-\mathscr{L}$$
$$=\frac{1}{4}G^{ij,r}G_{ij}^r-\frac{1}{2}\kappa\varepsilon^{0ij}A_0^r\partial_iA_j^r-\frac{1}{2}\kappa\varepsilon^{0ij}A_i^r\pi_j^r-$$
$$A_0^r\partial_i\pi^{i,r}-\frac{1}{2}\pi^{i,r}\pi_i^r-f^{rsu}\pi^{i,r}A_0^sA_i^u-$$
$$\frac{1}{8}\kappa^2A_i^uA^{i,u}+\mathrm{i}A_0^r\pi_{\psi_\rho}T^r\psi_\rho+\pi_{\varphi^+}\pi_\varphi+$$
$$\mathrm{i}A_0^r(\pi_\varphi T^r\varphi-\pi_{\varphi^+}T^r\varphi^+)+\mathrm{i}A_0^r\pi_{\lambda_\rho}T^r\lambda_\rho+$$
$$\mathrm{i}A_0^r\pi_{\chi_\rho}T^r\chi_\rho+\frac{1}{2}\pi_{N^r}\pi_{N^r}+\mathrm{i}A_0^r\pi_{N^s}T^rN^s-$$
$$\mathrm{i}\psi^{+\alpha}(\gamma^j)_\alpha^\rho D_j\psi_\rho-(D^j\varphi)^+D_j\varphi-$$
$$\frac{1}{2}\mathrm{i}\lambda^\alpha(\gamma^j)_\alpha^\rho D_j\lambda_\rho-\frac{1}{2}\mathrm{i}\chi^\alpha(\gamma^j)_\alpha^\rho D_j\chi_\rho-$$
$$\frac{\kappa}{2}\lambda^{\alpha,r}\lambda_{\alpha,r}-\frac{\kappa}{2}\chi^{\alpha,r}\chi_{\alpha,r}-\frac{\kappa}{2}(D^jN^r)D_jN^r+$$

$$U(\varphi,\ \varphi^+,\ \psi,\ \psi^+,\ \lambda,\chi,\ N)\ . \tag{6}$$

Then, the total Hamiltonian is

$$H_{\mathrm{T}} = \int_V \mathrm{d}^2x(\mathscr{H}_{\mathrm{c}} + \eta_1^r\Gamma_1^r + \eta_2^\rho\Gamma_2^\rho + \eta_3^\alpha\Gamma_3^\alpha + \eta_4^\alpha\Gamma_4^\alpha + \eta_5^\rho\Gamma_5^\rho)\ , \tag{7}$$

where η_1^r, η_2^ρ, η_3^α, η_4^α, and η_4^ρ are the conrrespondence multipliers. Using Possion bracket[13] and the consistency conditions $\dot{\Gamma}_1^r = \{\Gamma_1^r, H_{\mathrm{T}}\}_{PB} \approx 0$, we get the secondary constraints

$$\Gamma_6^r = \partial_i\pi^{i,r} - f^{rsu}\pi^{i,s}A_i^u + \frac{1}{2}\kappa\varepsilon^{0lm}\partial_l A_m^r - \mathrm{i}(\pi_\varphi T^r\varphi + \pi_{\varphi^+}T^r\varphi^+ + \pi_{\psi_\rho}T^r\psi_\rho + \pi_{\lambda_\rho}T^r\lambda_\rho + \pi_\chi T^r\chi + \pi_{N^s}T^r N^s) \approx 0\ . \tag{8}$$

While the consistencies $\dot{\Gamma}_{12}^r$, $\dot{\Gamma}_3^\alpha$, $\dot{\Gamma}_4^\alpha$ and $\dot{\Gamma}_5^\rho$ of the primary constraints lead to the equations for determining the Lagrange multipliers, then no further constraint occurs. We further deduce that constraint Γ_1^r are the first class constraint, constraints Γ_2^r, Γ_3^α, Γ_4^ρ, Γ_5^ρ and Γ_6^r are the second class. Therefore, we can find a set of the first class constraints

$$\begin{aligned}\Lambda_1^r &= \Gamma_1^r = \pi^{0,r} \approx 0\ ,\\ \Lambda_2^r &= -\pi_{\psi_\rho}T^r(\gamma^0)_\rho^\alpha\pi\varphi^{+\alpha} + \partial_i\pi^{i,r} - f^{rsu}\pi^{i,s}A_i^u + \frac{1}{2}\kappa\varepsilon^{0lm}\partial_l A_m^r -\\ &\quad \mathrm{i}(\pi_\varphi T^r\varphi + \pi_{\varphi^+}T^r\varphi^+ + \pi_{\psi_\rho}T^r\psi_\rho + \pi_{\lambda_\rho}T^r\lambda_\rho + \pi_{\chi_\rho}T^r_{\chi_\rho} + \pi_{N^s}T^r N^s) \approx 0\ ,\end{aligned} \tag{9}$$

here Λ_1^r and Λ_2^r are also the gauge transformation generators, the second-class constraints are

$$\begin{aligned}\theta_1^\rho &= \Gamma_2^\rho = \pi_{\psi_\rho} - \mathrm{i}\psi^{+\alpha}(\gamma^0)_\alpha^\rho \approx 0\ ,\\ \theta_2^\alpha &= \Gamma_3^\alpha = \pi_{\psi^{+\alpha}} \approx 0\ ,\\ \theta_3^\rho &= \Gamma_4^\rho = \pi_{\lambda\rho} - \frac{1}{2}\mathrm{i}\lambda^\alpha(\gamma^0)_\alpha^\rho \approx 0\ ,\\ \theta_4^\rho &= \Gamma_5^\rho = \pi_{\chi_\rho} - \frac{1}{2}\mathrm{i}\chi^\alpha(\gamma^0)_\alpha^\rho \approx 0\ .\end{aligned} \tag{10}$$

We thus complete the classification of the constraints.

3 Quantization of the System in Framework of Path Integral

We consider the Coulomb gauge

$$\Omega_1^r = \partial_i A_i^r \approx 0\ . \tag{11}$$

Because of the existence of two first-class constraints Λ_1^r and Λ_2^r, another gauge-fixing condition should be chosen as the consistent condition

$$\Omega_1' = \dot{\Omega}_1 = \{\Omega_1, H_{\mathrm{T}}\}_{\mathrm{PB}} = \nabla^2 A_0^r - \partial_i\pi^{i,r} - f^{rsu}(\partial_l A_0^s)A_i^u = 0\ . \tag{12}$$

On the other hand, because general physical processes should satisfy quantitative causal relation[14], some changes (cause) of some quantities in condition (12) must lead to some relative changes (result) of the other quantities in condition (12), so that the right side of condition (12) keeps zero, namely, condition (12) also satisfies the quantitative causal relation, which just makes the different quantities form a useful expression.

According to Faddeev-Senjanovic quantization formulation, the phase space generating functional of Green function for the supersymmetric system is given by[15]

$$Z[0] = \int\mathscr{D}\phi^\alpha\mathscr{D}\pi_\alpha\prod_{i=1}^2\delta(\Lambda_i)\sum_{j=1}^2\delta(\theta_j)\cdot\sum_{k=1}^2\delta(\Omega_k)\det|\{\Lambda_i,\Omega_k\}|(\det|\{\theta_j,\theta_{j'}\}|)^{1/2}\cdot\exp\left\{i\int\mathrm{d}^3x(\pi_\alpha\dot{\phi}^\alpha - \mathscr{H}_c)\right\}\ , \tag{13}$$

where $\phi^\alpha = (A_a^r,\ \varphi,\ \varphi^+,\ \psi,\ \psi^+,\ \lambda,\chi,\ N)$, $\pi_\alpha = (\pi^{a,r},\ \pi_\psi,\ \pi_\varphi,\ \pi_{\varphi^+},\ \pi_\lambda,\ \pi_\chi,\ \pi_N)$. Therefore, we can obtain

$$|\{\Lambda_i,\ \Omega_k\}| = [\nabla^2\delta^{rs}\delta^{(2)}(x-y)]^2, \tag{14}$$

$$|\{\theta_j,\ \theta_{j'}\}| = [\delta^{\alpha\beta}\delta^{(2)}(x-y)]^4. \tag{15}$$

Thus, we find that both $|\{\Lambda_i,\ \Omega_k\}|$ and $|\{\theta_j,\ \theta_{j'}\}|$ are independent of field variables and they can be ignored in the generating functional. The condition (12) coming from the consistent condition very naturally eliminates the gauge arbitrariness. Using the properties of δ-function, we finally write out the phase space generating functional of Green function

$$Z[0] = \int\mathscr{D}\phi^\alpha\mathscr{D}\pi_\alpha\mathscr{D}\lambda_i\mathscr{D}\mu_j\mathscr{D}\omega_k\exp\{i\int\mathrm{d}^3x(\mathscr{L}_{\mathrm{eff}}^p)\}\ , \tag{16}$$

where

$$\mathscr{L}^{\mathrm{p}}_{\mathrm{eff}} = \mathscr{L}^{\mathrm{p}} + \lambda^r_i \Lambda^r_i + \omega^s_j \Omega^s_j + \mu^u_k \theta^u_k \quad ,$$

$$\mathscr{L}^{\mathrm{p}} = \pi_\varphi \dot{\varphi} + \pi_{\varphi^+}\dot{\varphi}^+ + \pi^{a,r} A^r_a + \pi_\psi \dot{\psi} + \dot{\psi}^+ \pi_{\psi^+} + \pi_\chi \dot{\chi} + \pi_\lambda \dot{\lambda} + \pi_N \dot{N} - \mathscr{H}_{\mathrm{c}} \quad ,$$

in which λ^r_i, ω^s_j and μ^u_k are multipliers of the first class constraints Λ^r_i, the gauge fixing conditions Ω^s_j and the second class constraints θ^u_k, respectively.

4 Quantal Angular Momentum and Fractional Spin

We first formulate the quantal canonical Noether theorem[13]: If the effective action $I^P_{\mathrm{eff}} = \int \mathrm{d}^2 x \mathscr{L}^P_{\mathrm{eff}}$ is invariant in extended phase space under the following global transformation

$$x^{\mu'} = x^\mu + \Delta x^\mu = x^\mu + \varepsilon_\sigma \tau^{\mu\sigma}(x, \phi, \pi) \,, \tag{17}$$

$$\phi^{\alpha'}(x') = \phi^\alpha(x) + \Delta\phi^\alpha(x) = \phi^\alpha(x) + \varepsilon_\sigma \xi^{\alpha\sigma}(x, \phi, \pi) \,, \tag{18}$$

$$\pi'_\alpha(x') = \pi_\alpha(x) + \Delta\pi_\alpha(x) = \pi_\alpha(x) + \varepsilon_\sigma \eta^\sigma_\alpha(x, \phi, \pi) \,, \tag{19}$$

where ε_σ are the global infinitesimal arbitrary parameters ($\sigma = 1, 2, \cdots, r$), $\tau^{\mu\sigma}$, $\xi^{\alpha\sigma}$ and η^σ are functions of canonical variables and space time, and if the Jacobian of the transformations (17—19) is unity, then at the quantum level there are conserved laws

$$Q^\sigma = \int_V \mathrm{d}^2 x [\pi(\xi^\sigma - \phi_k \tau^{k\sigma}) - \mathscr{L}_{\mathrm{eff}} \tau^{0\sigma}] = \mathrm{const}, \quad \sigma = 1, 2, \cdots, r \,. \tag{20}$$

We now deduce the angular momentum by using the conserved quantities in 2 + 1 dimensions. Consider the Lorentz transformation

$$\Delta x^\mu = \delta\, \omega^{\mu\nu} x_\nu \quad ,$$

$$\Delta\phi^\alpha = \frac{1}{2}\delta\omega^{\mu\nu}(\Sigma^{\mu\nu})^\alpha_\beta \phi^\beta \quad ,$$

$$\Delta\pi_\beta = \frac{1}{2}\delta\omega^{\mu\nu}(\Sigma^{\mu\nu})^\alpha_\beta \pi_\alpha \quad . \tag{21}$$

Under the spatial rotation, the effective canonical action $I_{\mathrm{eff}} = \int \mathrm{d}^3 x \mathscr{L}^P_{\mathrm{eff}}$ is invariant, and the Jacobian of the spatial rotation transformation is unity. Using Eq. (20) we can obtain the conserved angular momentum

$$J = \int \mathrm{d}^2 x\, \varepsilon^{0ij} [x_i \pi^{a,r} \partial_j A^r_a + \pi^r_i A^r_j + x_i \pi_{\psi_\rho} \partial_j \psi_\rho + x_i \pi_\varphi \partial_j \varphi + x_i \pi_{\varphi^+} \partial_j \varphi^+ + x_i \pi_{\lambda_\alpha} \partial_j \lambda_\alpha + x_i \pi_{\chi_\rho} \partial_j \chi_\rho + x_i \pi_{N^s} \partial_j N^s + \frac{1}{2\mathrm{i}}(\pi_\psi \gamma_i \gamma_j \psi + \pi_{\lambda_\alpha} \gamma_i \gamma_j \lambda_\alpha + \pi_{\chi_\rho} \gamma_i \gamma_j \chi_\rho)] \,. \tag{22}$$

The last term is related to spinor fields and is coincide with the result[6, 7] obtained by classical Noether theorem. One can observe that the partial angular momentum given by non-Abelian Chern-Simons topological term is

$$J^N_{\mathrm{CS}} = \int \mathrm{d}^2 x\, \varepsilon^{0ij} [x_i \pi^{0,r} \partial_j A^r_0 + x_i \pi^{k,r} \partial_j A^r_k + \pi^r_i A^r_j] = \int \mathrm{d}^2 x [-\varepsilon^{ij} x_i G^{0k,r} \partial_j A^r_k - \varepsilon^{ij} G^r_{o\,i} A^r_j + \frac{\kappa}{2}\varepsilon^{ij}\varepsilon^{kl} x_i A^r_l \partial_j A^r_k + \frac{\kappa}{2}\varepsilon^{ij}\varepsilon_{il} A^{l,r} A^r_j] \,, \tag{23}$$

where Eqs. (4) and (5) have been used, and the total angular momentum can be rewritten as

$$J = \int \mathrm{d}^2 x\, \varepsilon^{0ij} x_i [\pi_{\psi_\rho} \partial_j \psi_\rho + \pi_\varphi \partial_j \varphi + \pi_{\varphi^+} \partial_j \varphi^+ + \pi_{\lambda_\alpha} \partial_j \lambda_\alpha + \pi_{\chi_\rho} \partial_j \chi_\rho + \pi_{N^s} \partial_j N^s + G^{k0,r} \partial_j A^r_k + \int \mathrm{d}^2 x\, \varepsilon^{0ij} \Big[\frac{1}{2\mathrm{i}}(\pi_{\psi_\rho} \gamma_i \gamma_j \psi_\rho + \pi_{\lambda_\alpha} \gamma_i \gamma_j \lambda_\alpha + \pi_{\chi_\rho} \gamma_i \gamma_j \chi_\rho) + G^r_{i0} A^r_j \Big] + \frac{\kappa}{2} \int \mathrm{d}^2 x (\varepsilon^{ij}\varepsilon^{kl} x_i A^r_l \partial_j A^r_k + \varepsilon^{ij}\varepsilon_{il} A^{l,r} A^r_j) = J_0 + J_{\mathrm{S}} + J_{\mathrm{F}} \quad . \tag{24}$$

The first part J_0, the second part J_{S} and the third part J_{F} stand for the orbital angular momentum, the spin angular momentum, and the fractional spin angular momentum, respectively. And we can have

$$J_{\mathrm{F}} = \frac{\kappa}{2} \int \mathrm{d}^2 x (\varepsilon^{ij}\varepsilon^{kl} x_i A^r_l \partial_j A^r_k + \varepsilon^{ij}\varepsilon_{il} A^{l,r} A^r_j) = -\kappa \int \mathrm{d}^2 x (\varepsilon^{ij} x_i A^s_j \varepsilon^{lm} \partial_l A^s_m) \,. \tag{25}$$

Using the following asymptotic form[16] of the non-abelian vortex configuration

$$A_i^r = \frac{2}{\kappa}\varepsilon_{0ij}\partial_x^j\int d^2y\frac{1}{2\pi}\ln\frac{1}{|x-y|}\mathscr{J}^{0,r}(y), \qquad (26)$$

and substituting Eq. (26) into Eq. (25), we first find

$$J_F = \frac{Q^S Q^S}{\pi\kappa}. \qquad (27)$$

This term is the "anomalous one" which is interpreted as fractional spin. For consistent quantum mechanics, the coefficient κ should be quantized so that $\kappa = m/4\pi$ with nonzero integer m[17]. Contrary to the abelian case, the result has the contribution of group component values. When Q^S is replaced by the abelian charge Q, this result is reduced to the common result[6—9]. If there is no gauge field strength term in the Lagrangian density (1), we also obtain the anomalous J_F, but the orbital angular momentum and spin angular momentum of the field A_μ^r will disappear, which can be seen from Eq. (24).

5 Summary and Conclusion

In terms of path integral quantization for the canonical constrained system in Faddeev-Senjanovic scheme, this paper quantizes the $SU(n)$ $N=2$ supersymmetric non-Abelian system with Chern-Simons topological term. We analyze the constraints of the system in phase space, take the Coulomb gauge and use its consistency of the gauge to deduce another gauge condition. We further obtain the phase space generating functional of Green function.

Using the global canonical Noether theorem, we deduce the angular momentum of this system, concretely get the partial angular momentum given by non-Abelian Chern-Simons topological term, and find it to possess the "anomalous spin" part. We also find that the total angular momentum in this paper is different from that of the system without gauge field strength term, the results deduced from the system without gauge field strength term are missing the orbital angular momentum and spin angular momentum of the field A_μ^r. Different with the abelian case, we discover that this anomalous spin term has the contribution from the group superscript components.

In our method, the total angular momentum plays the role of the canonical angular momentum as in Ref. [6]. Furthermore, we systemically deduce the total angular momentum and find that the orbital angular momentum, spin angular momentum, and fractional spin angular momentum all appear in the total angular momentum.

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具有非阿贝尔 Chern-Simons 拓扑项的 $SU(n)N=2$ 超对称规范场系统的量子化和分数自旋*

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摘　要：基于路径积分的 Faddeev-Senjanovic 量子化，先将具有非阿贝尔 Chern-Simons 拓扑项的 $SU(n)N=2$超对称规范场系统量子化，然后利用整体正则 Noether 定理得到了系统的总角动量，在量子水平下导出了该系统的分数自旋性质，并发现其分数自旋有来自非阿贝尔规范群分量的贡献。

关 键 词：超对称；Chern-Simons；约束系统；分数自旋；规范场

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