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Dynamical Masses of Quarks and Self-energy Functions*

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Abstract: Study of mass of quark is one of the most important issues in the investigation of QCD. Because masses of quarks are fundamental QCD input parameters of standard Model, and an accurate determination of these parameters is extremely important for both phenomenological and theoretical applications. Based on the parameterized fully dressed quark propagator proposed by us, the theoretical predictions of the masses of quarks are predicted in this short note. The effective quark mass is defined by the scalar self-energy function $B_f(p^2)$ and vector self-energy function $A_f(p^2)$. The results of our calculations are in agreement with the empirical values used widely in literature and also show that the parameterized form of quark propagator is an applicable and reliable approximation.

Key words: quark propagator; self-energy function; dynamical mass of quark

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1 Introduction

Studying masses of quarks and self-energy functions is a very interesting subject in the investigation of QCD. As it has been known that the values of quark masses cannot be directly measured in experiments since the quarks with three colors are confined in hadrons which are in color singlet states so that there is no free quark in nature. But, quark masses are fundamental parameters of the strong interaction theory, QCD. An accurate determination of these parameters is, in fact, an extremely important issue for both phenomenological and theoretical applications. From a theoretical point of view, an accurate determination of quark masses will give an insight on the physics of flavors, revealing relations between masses and mixed angles, or specific textures in the quark mass matrix, which may originate from still uncovered flavor symmetries.

Like all other parameters in the Standard Model Lagrangian, quark masses can be defined as effective couplings, which are both the renormalization scheme and scale dependence. The quark propagator is closely related to the determination of quark mass so that studying quark propagator is very important task in strong interaction physics.

2 Fully Dressed Quark Propagator

2.1 Quark propagator

The quark propagator $S_f(p)$ can be expressed in general by the Dyson-Schwinger equation[1,2],

$$iS_f^{-1}(p) = i[S_f^0(p)]^{-1} + C_f g_s^2 \int \frac{d^4k}{(2\pi)^4} \cdot$$

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$$\gamma^{\mu} S_f(k) \Gamma^{\nu}(k, p) G_{\mu\nu}(p-k) , \quad (1)$$

where $S_f^0(p)$ is bare propagator of quark with flavor f and momentum p, $\mathrm{i}[S_f^0(p)]^{-1} = \not{p} - m_f$. The $C_f = 4/3$ stands for color factor[3] and g_s is strongly coupling constant of QCD related the so-called running coupling constant $\alpha_s(Q^2)$ by the equation of $\alpha_s = g_s^2/4\pi$. The $G_{\mu\nu}(p-k)$ denotes an effective gluon propagator, which is known in the perturbative QCD region but it is unknown in the non-perturbative QCD region and has to be modeled.

As is seen from Eq. (1), the quark propagator, defined by DSEs Eq. (1), is determined by three ingredients: fully dressed gluon propagator $g_s^2 G_{\mu\nu}$, fully dressed quark-gluon coupling vertex $\Gamma^{\nu}(k, p)$, and the truncation of the DSEs since it is an infinite tower integration.

2.2 Rainbow approximation of DSEs

As it is impossible to solve the complete set of DESs, one has to truncate this infinite tower in a physically acceptable way to reduce them to something that is soluble. To this end, we make a further simplification by replacing the fully dressed quark-gluon vertex $\Gamma^{\nu}(k, p)$ with its bare one γ^{ν}. Under this approximation, the DSEs of Eq. (1) consequently turns out to be

$$\mathrm{i}S_f^{-1}(p) = \mathrm{i}[S_f^0(p)]^{-1} + \frac{4}{3} g_s^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \cdot \gamma^{\mu} S_f(k) \gamma^{\nu} G_{\mu\nu}(p-k) . \quad (2)$$

Eq. (2) is called "rainbow" approximation of DSEs (1)[4].

An important observation is that the general form of the inverse quark propagator $S_f^{-1}(p)$ can be also expressed in Euclidean space[5] in a new formulism as

$$S_f^{-1}(p) = \mathrm{i}\gamma p A_f(p^2) + B_f(p^2) \quad (3)$$

in a covariant gauge. The A_f and B_f are called the self-energy functions which are renormalized at space-like point μ^2 according to $A_f(\mu^2) = 1$ and $B_f(\mu^2) = m_f(\mu^2)$ with $m_f(\mu^2)$ being the current quark mass in QCD Lagrangian.

Except for the current quark mass and perturbative corrections, the functions $A_f(p^2)$ and $B_f(p^2)$ are non-perturbative quantities which are referred to the vector and scalar propagator condensates, respectively. $A_f(p^2)$ and $B_f(p^2)$ satisfy a new set of DSEs in the rainbow approximation[6] and in the Feynman gauge $\xi = 1$.

$$[A_f(p^2) - 1]p^2 = \frac{8}{3} g_s^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \cdot G(p-q) \frac{A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} p \cdot q , \quad (4)$$

$$B_f(p^2) = \frac{16}{3} g_s^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \cdot G(p-q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} . \quad (5)$$

Given a gluon propagator $G_{\mu\nu}^{ab}(q)$, we can solve the two coupled integral Eqs. (4) and (5).

3 Dynamical Mass of Quarks

In order to study masses of quarks, we turn to a form of the quark propagator which define the quark mass and can be written as

$$S_f^{-1}(p) = \not{p} - m_f - \Sigma_f(p) \equiv Z^{-1}(p^2) \cdot [\not{p} - M(p^2)] , \quad (6)$$

where $Z^{-1}(p^2) \equiv A_f(p^2)$ is the momentum dependence of the quark wave function renormalization. The quark effective mass is defined by the fully dressed quark propagator and is then given explicitly in terms of DSEs solutions $A_f(p^2)$ and $B_f(p^2)$ via

$$M(p^2) \equiv \frac{B_f(p^2)}{A_f(p^2)} . \quad (7)$$

4 Parameterized Form of Quark Propagator

It should be pointed out that the price to pay for using the rainbow approximation, $\Gamma^{\nu} = \gamma^{\nu}$, is that the gauge invariance of DSEs has been lost. Therefore, we should use another form of the confined quark propagator. Numerical solution of the quark propagator given by solving DSEs is quite complicated. An extensive study shows that the quark propagator could be approached by a parameterized form in an acceptable way. An important observation is that the general form of the inverse quark propagator can be well represented by the following algebraic parameterization:

$$S_f(p) = -i\gamma p\, \sigma_v^f(p^2) + \sigma_s^f(p^2), \tag{8}$$

where the σ_v^f and σ_s^f are given by

$$\sigma_s^f = \frac{\bar{\sigma}_s^f}{\Lambda}, \quad \sigma_v^f = \frac{\bar{\sigma}_v^f}{\Lambda^2} \tag{9}$$

with

$$\bar{\sigma}_s^f(x_1) = \frac{[1-\exp(-b_1^f x_1)]}{b_1^f}\frac{[1-\exp(-b_3^f x_1)]}{b_3^f x_1}\cdot \left[b_0^f + b_2^f\frac{1-\exp(-\Lambda')}{\Lambda' x_1}\right] + \bar{m}_f\frac{1-\exp[-2(x_1+\bar{m}_f)]}{x_1+\bar{m}_f^2}, \tag{10}$$

$$\bar{\sigma}_v^f(x_1) = \frac{2(x_1+\bar{m}_f^2) - 1 + \exp[-2(x_1+\bar{m}_f^2)]}{2(x_1+\bar{m}_f^2)^2}, \tag{11}$$

with $\bar{m}_f = m_f/\Lambda$, $x_1 = p^2/\Lambda^2$, $\Lambda' = 10^{-4}$ and $\Lambda = 0.566$ GeV. The parameters b_i^f ($i = 0, 1, 2, 3$; and m_f ($f =$ u, d, s, $\cdots$) are listed in Table 1[2, 7].

Table 1 The parameters of the confined quark propagators and the current mass m_f with flavor f in QCD Lagrangian

Flavor (f)	b_0^f	b_1^f	b_2^f	b_3^f	m_f/MeV
u	0.131	2.90	0.603	0.185	5.1
d	0.131	2.90	0.603	0.185	5.1
s	0.105	2.90	0.740	0.185	127.5

Comparing Eq. (8) with Eq. (3) we can set up a relation between A_f, B_f and σ_v^f, σ_s^f the parameterized quark propagator. After some algebraic calculations, we then arrive at

$$A_f(p^2) = \frac{\sigma_v^f}{(\sigma_s^f)^2[p^2(\sigma_v^f/\sigma_s^f)^2+1]}, \tag{12}$$

$$B_f(p^2) = \frac{1}{\sigma_s^f[p^2(\sigma_v^f/\sigma_s^f)^2+1]}. \tag{13}$$

Therefore, $A_f(p^2)$ and $B_f(p^2)$ can be evaluated from σ_v^f and σ_s^f.

5 Predictions of Quark Self-energy Functions and Dynamical Quark Masses by Our Parameterized Quark Propagator of Eqs. (12) and (13)

Based on the parameterized fully dressed quark propagator, the theoretical predictions for the vector self energy function $A_f(p^2)-1$ and scalar self-energy function $B_f(p^2)$ are shown in Fig. 1 and Fig. 2, respectively.

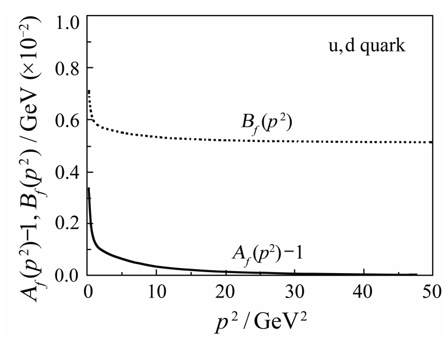

Fig. 1 p^2-dependence of the self-energy functions, $A_f(p^2)-1$ and $B_f(p^2)$ for u, d quark.

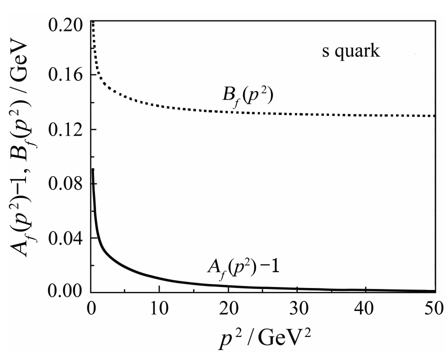

Fig. 2 p^2-dependence of the self-energy functions $A_f(p^2)-1$ and $B_f(p^2)$ for s quark.

The theoretical predictions for the dynamical masses of quarks $M(p^2)$ by the parameterized quark propagator for u, d quark and s quark are shown in Fig. 3 and Fig. 4, respectively.

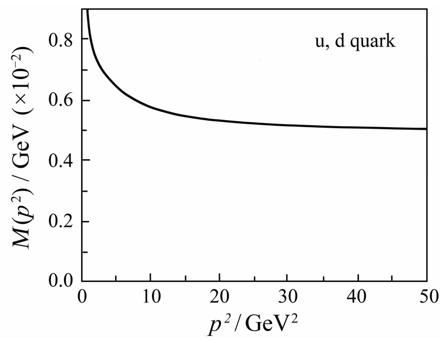

Fig. 3 p^2-dependence of the effective masses of quarks, $M(p^2)$, predicted by the parameterized quark propagator for u, d quarks.

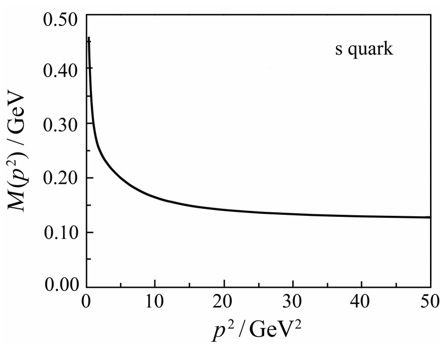

Fig. 4 p^2-dependence of the effective mass of s quark, $M(p^2)$, predicted by the parameterized quark propagator.

6 Discussions

From these figures, we can see the resulting A_f and B_f defined by the parameterized quark propagator have the following features:

(1) In the limit of that the quark momentum p^2 becomes large and space-like, the functions A_f and B_f approach an asymptotic limit

$$\lim_{p^2\to\infty} A_f(p^2) = 1.0 \ , \qquad (14)$$

$$\lim_{p^2\to\infty} B_f(p^2) = m_f \ . \qquad (15)$$

Hence, the confined quark propagator, $S_f(p)$, reduces to a free quark propagator (of course, free quark never comes up in our vision),

$$\lim_{p^2} S^{-1}(p) = i\not{p} - m_f \ . \qquad (16)$$

For the quark self-energy, $\Sigma_f(p)$, we can see that when $p^2\to\infty$, $\Sigma_f(p)\to 0$ due to the renormalization requirements of $A_f(\infty) = 1.0$ and $B_f(\infty) = m_f$. This means that the self-energy vanishes, and the free quark propagator appears. We have an asymptotically free quark. Therefore, the quark propagator $S_f(p)$ has a correct behavior. On the other hand, when $p^2\to 0$, by definition, $S_f^{-1}(0) = -B_f(0)$, $\Sigma_f(0) = B_f(0) - m_f$, and $M_f(0) = B_f(0)/A_f(0)$ reflect the value of constituent quark mass. In fact, when p^2 runs from 0 to ∞, $M_f(p^2)$, changes from constituent quark mass to current quark mass, the dynamical transition of the effective quark mass is naturally obtained.

(2) For small quark momentum, the quark propagator becomes quite different compared to the free quark propagator $S_f^0(p^2)$. In this momentum region, there is a strong non-perturbative enhancement of the mass function $B_f(p^2)$. This enhancement is a manifestation of dynamical chiral symmetry breaking and quark confinement. We can see the change from the transition of constituent quark mass to current quark mass, which defined by $M_f = B_f(p^2)/A_f(p^2)$.

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自能函数和夸克的动力学质量*

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摘 要：研究夸克的质量是 QCD 研究中的一个非常重要问题。因为，夸克质量是标准模型的基本输入参数，准确地确定这些参数无论对于唯象的应用还是对于理论的应用都是极其重要的。基于参数化的完全穿衣服的夸克传播子，研究了自能函数和夸克的动力学质量。理论预言了夸克质量和自能函数，其结果与文献中的经验值相符合，也与 Dyson-Schwinger 方程解一致。反过来这也说明了参数化的夸克传播子是成功和可靠的。

关键词：夸克传播子；自能函数；动力学的夸克质量

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