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Field Investigation of the Thermal Bar in Lake Ladoga, Spring 1991

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The thermal bar in Lake Ladoga was studied during one week in May 1991. Temperature and current velocity distributions were measured. The movement of the thermal bar and the horizontal heat transport were examined.

The temperature off-shore the thermal bar was found to be constant within a vertical. The velocity distributions were complex and dependent on wind conditions. Calculations of changes in heat content showed that there was a horizontal heat transport from the near-shore warm zone towards the thermal bar. The observed off-shore progression of the thermal bar was compared with analytically computed progression rates. The progression rate is underestimated if it is assumed that surface heating and vertical mixing are the only mechanisms contributing to the thermal bar progression. Better agreement is obtained if mixing of warm and cold water near the thermal bar zone is considered.

Introduction

A thermal bar is a thermo-hydrodynamic phenomenon characteristic of temperate lakes. It is associated with freshwater being at its highest density at approximately 4°C. During early spring, just after the ice cover has disappeared, all water in a lake has s temperature below 4°C. In the spring the water in the shallow regions increases its temperature faster than the water in the deep parts of the lake. When the temperature of the water near the shore reaches and increases above 4°C, a stable temperature stratification is developed. In deep parts of the lake the water in a water column is well-mixed due to convection and its temperature is still below 4°C. Between these regions, the warm and the cold zones, or the stably stratified

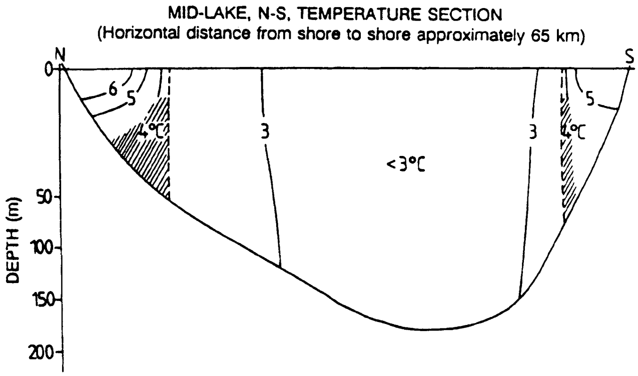


Fig. 1. Observed temperature distribution in a section of Lake Ontario (after Rodgers, 1966). "W" stands for the stably stratified near-shore region (temperature > 4°C), "B" is the thermal bar zone (temperature ≈ 4°C) and "C" is the convectively mixed deep water region (temperature < 4°C).

zone and the convectively mixed deep water region, there is a zone of sinking water with a temperature approximately equal to 4°C, corresponding to the maximum density of freshwater (see Fig. 1). This zone is called the thermal bar. When the lake water is heated further, the thermal bar is displaced towards the central deep part of the lake until all the water in the lake reaches 4°C. During the autumn cooling, a similar situation is developed as the one during the spring. A thermal bar is developed near the shore when the temperature falls down to 4°C, and vanishes when the temperature reaches this value in the deepest part of the lake.

In large lakes like the Great Lakes in North America, Lake Ladoga and Lake Onega in Russia a thermal bar can exist for a period of a couple of months. The thermal bar can inhibit horizontal exchange of water between the near-shore and the deep water mass, which affects the physical, chemical and biological environment in the lake. Pollutants may stay within the near-shore zone and cause severe pollution concentrations in the shore waters (Tikhomirov 1963; Menon *et al.* 1971; Huang 1972).

The thermal bar was first observed in Lake Geneva by Forel (1880) who described the temperature distribution pattern and explained it as a consequence of the anomalous equation of state of freshwater. The phenomenon was thereafter more or less forgotten until Tikhomirov (1959, 1963) presented measurements of the thermal structure in Lake Ladoga in a cross-section perpendicular to the thermal bar zone. The data from Lake Ladoga showed the thermal bar as a zone separating the deep vertically isothermal region from the stably stratified shallow regions. In the vicinity of the bar, sharp horizontal temperature gradients were observed. Tikhomirov described the circulation associated with the thermal bar as two circulations cells, one on each side of the bar, where the convergence zone constituted the bar zone.

In the middle of the sixties and in the seventies several field investigations of the thermal bar were performed in the Great Lakes in North America. A similar picture of the circulation patterns associated with the thermal bar as the one suggested by Tikhomirov was given by Rodgers (1965) and was partly confirmed by heat content change calculations (Rodgers 1968, 1971) which suggested heat advection from the shore areas towards the advancing thermal bar. This in turn implied a surface current from the shore towards the bar. However, Rodgers also suggested that this circulation pattern is secondary and that it should be superimposed on a geostrophic circulation along the isotherms, where the current velocities are at least one order of magnitude greater. Indications of the bar acting as a barrier between the near-shore region and deep water parts in a lake was reported by Hubbard and Spain (1973) and Spain *et al.* (1976) based on analysis of fluorescence measurements, and by Menon *et al.* (1971) based on analysis of bacterial density and biomass measurements.

Elliot and Elliot (1969, 1970) studied the thermal bar in a small laboratory tank and found good agreement between the temperature distributions in the laboratory and the ones measured in the field. They measured the density induced velocity field and found a cellular-like circulation pattern to those suggested by Tikhomirov and Rodgers. Elliott (1971) developed a mathematical model for prediction of the temperature and velocity fields. Good agreement with the laboratory experiments was obtained. However, the effect of the Earth rotation was not taken in account. Theoretical models including the Coriolis terms were proposed by Bennett (1971), Huang (1971, 1972) and Brooks and Lick (1972). The theoretical results showed that the primary density-induced circulation is cyclonic between the shore and the thermal bar zone and anticyclonic in the deep water region. The circulation cells perpendicular to the shore are secondary, with velocity magnitudes being at least an order of magnitude less.

Three different analytical models have been developed for describing the progression of the thermal bar. In the first one, by Elliott and Elliott (1970) it is assumed that horizontal advection and diffusion is of minor importance, and that the heat content change in a convectively-mixed water column is determined by surface heating only. The weakness of this linear approach is that the mixing of warm and cold water in the bar zone, *i.e.* heat transport from the warm zone towards the bar, is not accounted for. Analytical models incorporating this mixing mechanism were presented by Zilitinkevich *et al.* (1991) and Zilitinkevich and Malm (1993). The first of these two models was developed for an idealized infinite wedge-shaped basin, while in the latter the effect of shore line curvature is accounted for by using a circular lake as a model. In all the models the effect of wind mixing is neglected. It is further assumed that the heat flux through the water surface is constant in time.

Although some investigations have been devoted to the thermal bar, lack of data is preventing a more clear understanding of the phenomenon. Therefore, a field investigation was carried out in Lake Ladoga in the spring of 1991. The objectives of the measurement program were:

- to obtain a spatial and temporal field, especially in the thermal bar zone;
- to study the circulation pattern associated with the thermal bar, with special attention to the importance of the density induced circulation;
- to determine the horizontal heat transport perpendicular to the thermal bar;
- to study the progression of the thermal bar movement. Observed progression rates will be compared with simple analytical model predictions in order to distinguish the most important factors that determine the thermal bar movement.

Description of Measurements

Measurements of temperature and current velocity and direction were performed between May 24-30, 1991, at two sections (see Fig. 2) in the southern part of Lake Ladoga, both chosen because of their comparatively simple bottom topography with a slight and relatively constant bottom slope. Three surveys were made at the west-east (W-E) section and two surveys at the south-north (S-N) section. In Table 1 the dates and durations are given for each survey. The time needed to complete the measurements was approximately 10.5 hours for the W-E section and 12 hours for the S-N section. The numbering of the surveys introduced in the table will be used in the following to distinguish between them.

Temperature and current velocity data were measured from a research vessel, Talan. There were no fixed stations along the sections. The position of each measurement point was determined by a satellite navigation system, Raystar 590 GPS, with a spatial accuracy of approximately 100 metres. The distance between mea-

Table 1 – Date and time (local time) from beginning to end of each survey during the field investigation of the thermal bar 1991.

Survey	Section	Period of field measurements*	Number of stations along section
1	W-E	24-25/5, 23^{00}-09^{00}	12
2	S-N	25-26/5, 23^{30}-11^{30}	19
3	W-E	28-29/5, 16^{30}-03^{00}	14
4	S-N	29/5, 10^{30}-23^{00}	19
5	W-E	30/5, 11^{00}-21^{30}	15

*In survey number 1, 4 and 5 the measurements started close to the shore and continued towards the outer edge of the section. In survey number 2 and 3 the measurements were made in the reverse direction.

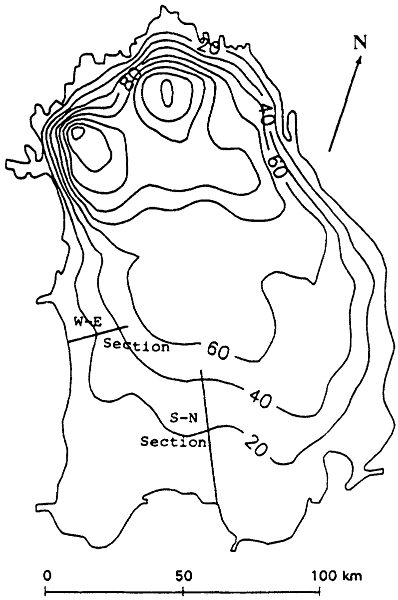


Fig. 2. Schematic map over Lake Ladoga showing the bottom topography and locations of the measurement sections.

surement points was about 5 km in the part of the sections with water temperatures below 4°C as the temperature did not change much there. In the near-shore part of the sections with water temperatures above 4°C, the distance between stations was approximately 2 km at the W-E section and 2.5-3 km at the S-N section. The shorter distance used at the W-E section was mainly due to the larger bottom slope which was expected to cause a faster change in temperature. A more narrow spacing (approximately 0.5 km) between measurements was used near the bar to get a good resolution of the temperature distribution in this zone.

The temperature at each station was recorded with a thermistor probe on a 50 metre cable. The accuracy of the probe is 0.1°C. Horizontal current velocities were measured with pendulum current meters. The pendulum current meters allow measurements of current velocity and direction at a desired number of depths at one location without anchoring the vessel. The current meter consists of a specially designed fin that is directed and inclined by the current. Current speed and direction is registered using a compass needle in a small plexiglass box mounted in the fin. The plexiglass box contains a gelatine solution which is in liquid phase when the meter is lowered into the water, but solidifies after a short time in the water so that the compass needle is fixed in a position giving an instantaneous registration of direction and magnitude of the current. The accuracy is about 2 cm/s.

Measurements were also made of meteorological parameters for calculation of the heat flux through the water surface. The net radiation (*i.e.* the algebraic sum of the incoming short-wave radiation from the sun and sky including reflection, long-wave terrestrial radiation and back radiation from the water surface) was measured with a Siemen Ersking radiometer. The average error of the net radiation measured with the Siemen Ersking radiometer lies within 10 % of the measured value (Lindroth 1978). Wind speed was measured as an average over 100 s at 5 m height above the water surface using a hand anemometer with the accuracy of 0.3 m/s. The wind direction was evaluated using compass and is correct within 10°. The air temperature and that of a wet bulb thermometer were measured with an aspiration psychrometer at the height 2 m above the surface, and the atmospheric pressure with a barometer-aneroid. The accuracies of these devices are 0.2°C and 1.7 mb, respectively. Sensible and latent heat fluxes were determined from wind speed, air temperature, water vapor pressure (determined using wet bulb temperature), atmospheric pressure and water surface temperature using the gradient method (*e.g.* Mironov 1991). All data from the measurements are presented in a report by Malm *et al.* (1991).

Results from the Measurements

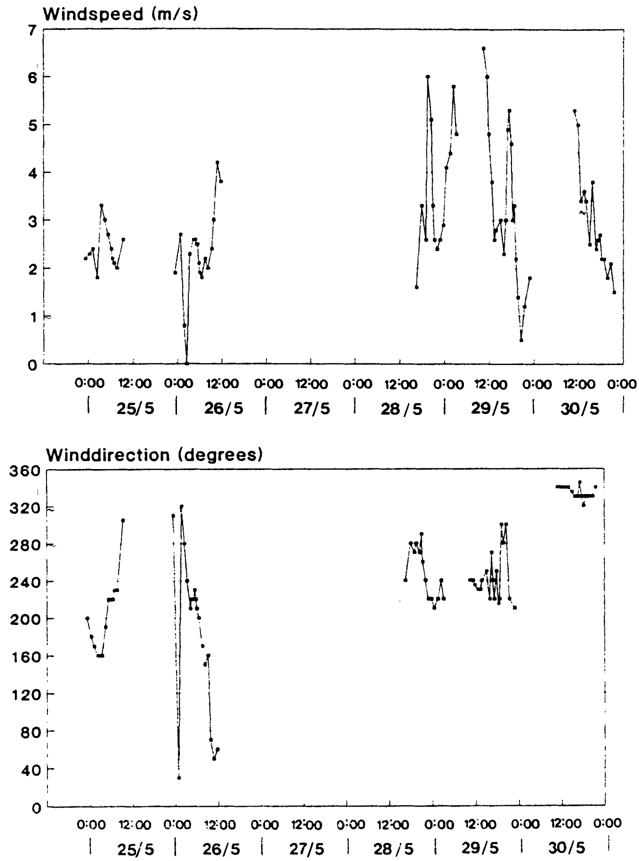
Meteorological Observations

The weather was quite calm during May 24 – Maj 27 with a wind speed of a few metres per second. During the morning and noon May 28 the wind increased to about 7 m/s coming from north-west. Later, the wind decayed and varied between 0 to 7 m/s during the rest of the field campaign. The wind speed and wind direction registered during the surveys are shown in Figs. 3a,b. The local time is given in the figures (1 p.m. corresponds to actual noon). The net heat flux through the water surface (positive downwards) is shown in Fig. 4. It is determined as the sum of the net radiation and the latent and sensible heat fluxes. The average net heat flux for the period was estimated to 205 W/m^2. A representative value for the end of May over Lake Ladoga is 209 W/m^2 (Tikhomirov 1982).

The net radiation dominated the net heat flux during daytime being approximately 100 times larger than the sensible and latent heat fluxes. Such a situation is typical for spring conditions over Lake Ladoga, since calm weather and stable stratification of the near-surface air layer do not favor intensive turbulent heat and mass exchange between air and water.

Temperature and Current Velocity Distributions

The temperature distributions for the W-E section were measured on three occasions, May 24-25, May 28-29 and May 30, and for the S-N section on two occasions, May 26 and May 29 (locations of the cross-sections are given in Fig. 2). As the duration of one survey could be about 10-12 hours, it would be interesting to get some idea of the temporal changes of the temperature within the section. The sections were quite well mixed during the five surveys, with no or very weak stratification in the near-shore zone. The temperature change with time can there-



a) Wind speed and

b) wind direction.

Fig. 3. Data from the five surveys during the period 24/5-30/5, 1991.

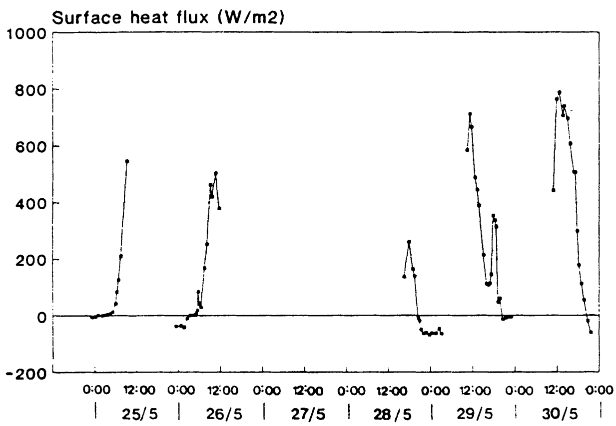


Fig. 4. Heat flux through the water surface for the five surveys during the period 24/5-30/5, 1991.

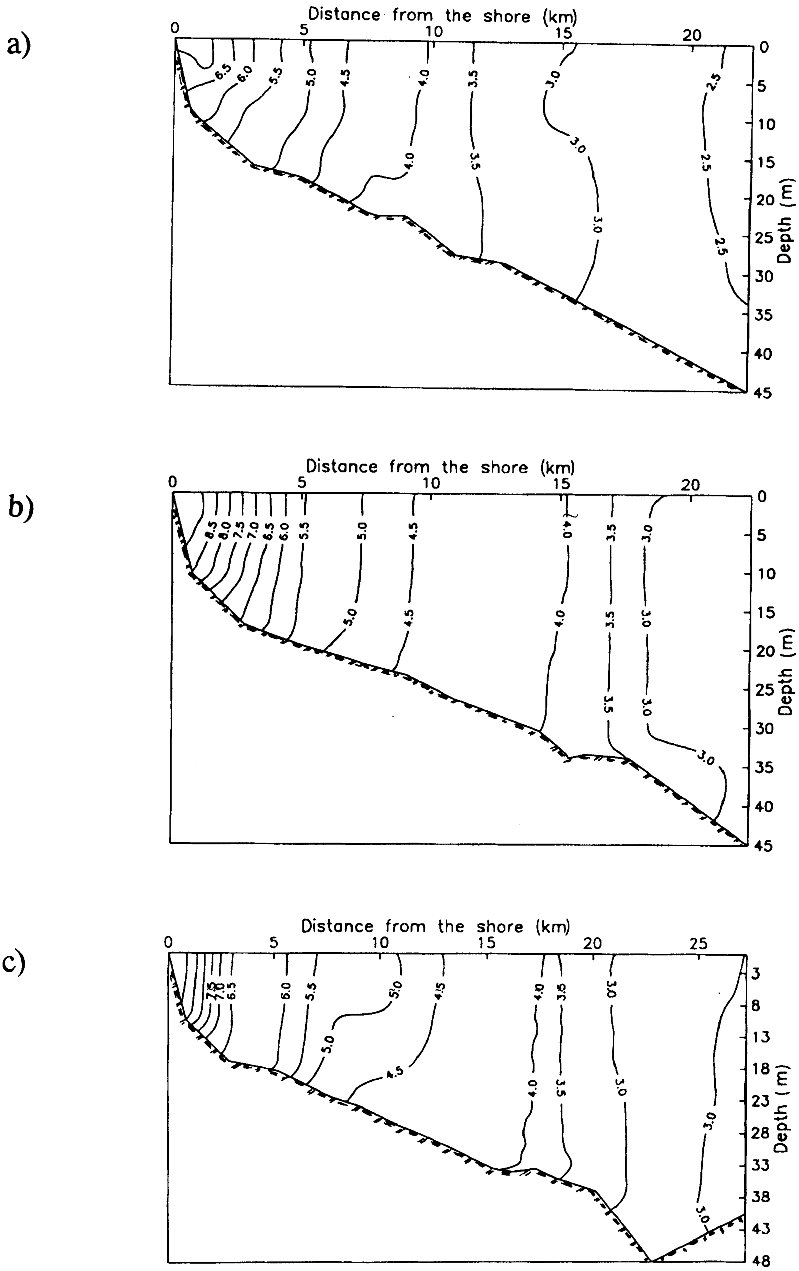


Fig. 5. Temperature distribution in the W-E section during a) the first survey, May 24-25, b) third survey, May 28-29 and c) fifth survey May 30. Temperature distribution in the S-N section during d) the second survey, May 26 and e) fourth survey, May 29.

d)

e)

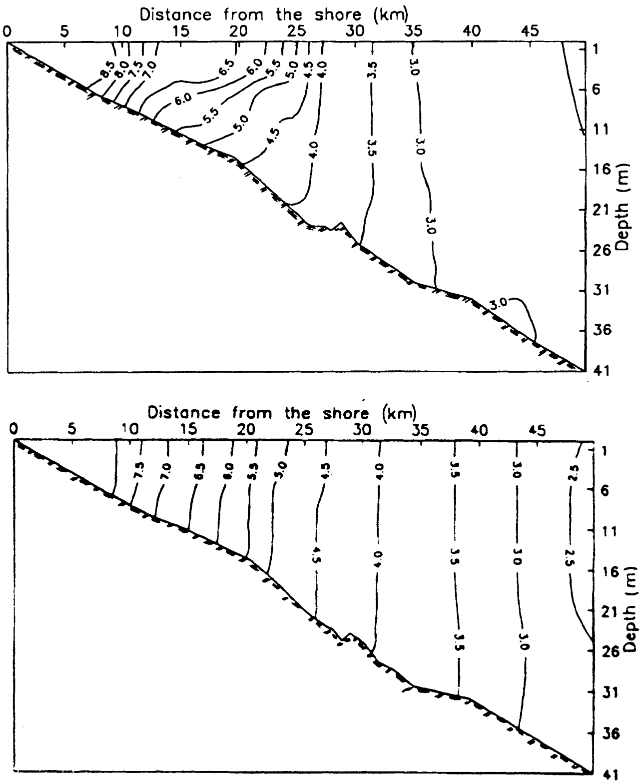


Fig. 5. cont.

fore roughly be estimated as the average temperature change in a vertical between two surveys. The maximum average temperature change in a vertical per day between survey number 1 and 5 (first and last survey at the W-E section) was about 0.4°C. This means that the temperature changes with time are relatively slow, why the temperature distributions obtained during the five surveys should be rather representative for a moment in time view. The results from the measurements are shown in Figs. 5a,b,c,d,e.

The horizontal temperature gradients near the 4°C isotherm are small. Rodgers (1966) and Tikhomirov (1968) found large horizontal temperature gradients in the shore region, especially close to the thermal bar, and only small gradients off-shore the bar. However, in the present study there are no big differences between horizontal temperature gradients in different parts of the sections.

From the five figures showing the temperature distributions, it is seen that the temperature in the offshore region is almost depth constant, which indicates strong convective mixing. This is in accordance with the results from other studies (see, *e.g.* Tikhomirov 1959; Rodgers 1966; Hubbard and Spain 1973). In the first two surveys there was a tendency towards a stable temperature stratification in the near-shore zone, as has been found in other studies. During surveys No. three and

No. four the water in the near-shore zone was found to be almost vertically isothermal, similar to the temperature pattern in the deep parts. The wind, which had been very calm at the time of the first surveys, increased to about 7 m/s, blowing from the north-west about half-a-day before the second surveys were started. A coast-parallel current with a maximum speed of 10-15 cm/s developed, see Figs. 6a,b. This current should have generated turbulent mixing and diffusive heat flux to large depths. The order of magnitude of the vertical heat flux can be estimated using the following parameterization of the turbulent heat conductivity, K_T (see *e.g.* Rodi 1980)

$$K_T \sim 10^{-2}\, U\, D \tag{1}$$

where U is a representative velocity difference and D is the depth. The downward turbulent heat flux is estimated as

$$c_w \rho_w K_T \frac{\partial T}{\partial z} \sim 10^{-2} c_w \rho_w\, U\, D \frac{\delta T}{D} \equiv 10^{-2} c_w \rho_w\, U\, \delta T \tag{2}$$

where δT is a representative vertical temperature difference; ϱ_w *and* c_w are density and specific heat of water at constant pressure respectively. From the observed data in Figs. 5a and 6a the following estimates are made: $U = 10$ cm/s and $\delta T = 1$°C, which gives the heat flux 4,190 W/m^2, *i.e.* well above the daily average surface heat flux, 205 W/m^2. It is reasonable to assume that the temperature homogeneity in the vertical is due to the large turbulent vertical heat flux.

The temperature field from survey No. 5 once again showed a weak tendency of stable temperature stratification in the near-shore zone.

The measured current velocities and directions during all surveys indicate a complex circulation system. The currents seems to be strongly affected by present and previous wind conditions. It also seems like wind-induced currents dominate over the density induced currents, even during quite calm conditions as experienced in survey No. 1 with wind velocities between two and three m/s. An example of the wind-induced coast-parallel circulation mentioned above, obtained during survey No. 3, is shown in Figs. 6a,b. The time difference between the first and last current measurement was about 5 hours, why the presented current system should at least roughly be representative as a moment in time view.

Heat Content Change Calculations

During spring the temperature distribution in large dimictic lakes is strongly related to depth, with the isotherms following the depth contours rather well (see *e.g.* Rodgers 1966 and Csanady 1974). This seems also to be the case for Lake Ladoga as is indicated by the surface temperature distributions obtained from satellite images (Malm and Jönsson 1993). Therefore, the heat transport along the depth contours should be of secondary importance. As both sections investigated in the field campaign during 1991 were perpendicular to the isobaths and located in an

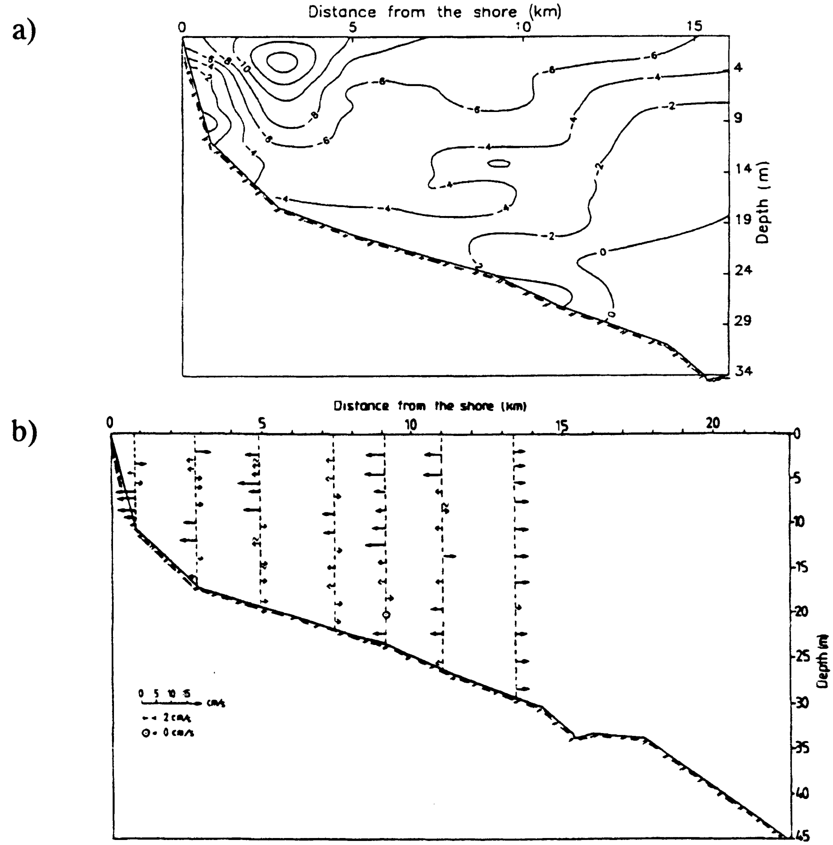


Fig. 6. a) – constant current speed contours (in cm/s) for the velocity component perpendicular to the cross-section in the W-E section during the third survey (May 28-29); positive values mean that the velocity component is directed into the paper and negative values that it is directed out from the paper. b) – measured velocity components parallel to the section in the W-E section during the third survey (May 28-29); the direction is shown with arrows, where the magnitude of speed (in cm/s) is proportional to the arrow length.

area where the depth contours were relatively straight, it seems reasonable to assume that the main transport of heat occurs along the sections.

The horizontal heat transport along a section can be estimated by calculating the change of heat content in water columns between two surveys. The change of heat content in a water column is caused by surface heating and horizontal heat transport (see Fig. 7)

$$\frac{\Delta \underset{\sim}{H}_i}{\Delta t} \equiv Q_s - \frac{1}{\Delta x}\left(Q_{i,i+1} - Q_{i-1,i}\right) \qquad (3)$$

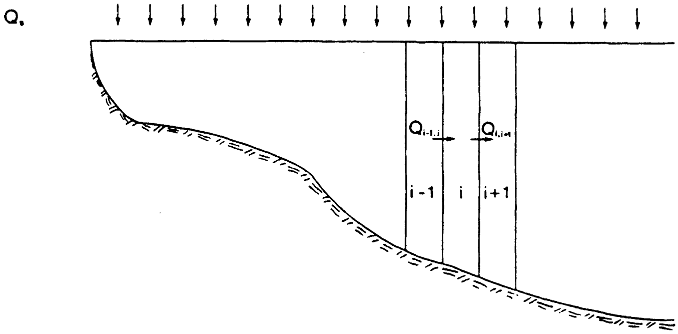


Fig. 7. Schematic pattern of heat content change in a water column. The heat content change in the i-th column is due to surface heating (Q_s) and horizontal transport from (to) neighboring columns ($Q_{i\text{-}1,i}$ and $Q_{i,i+1}$). Heat flux through the basin bottom is neglected.

Here

$$H_i = \frac{1}{\Delta x} \int_{x_i - \frac{\Delta x}{2}}^{x_i + \frac{\Delta x}{2}} \int_0^{D(x)} \rho_w c_w T(x,z,t)\, dz\, dx \tag{4}$$

is the heat content of the i-th water column whose width is equal to Δx; Δt is the time interval between surveys; Q_s is the heat flux through the water surface that is considered to be horizontally homogeneous; $Q_{i\text{-}1,i}$ and $Q_{i,i+1}$ are the horizontal heat fluxes (Q_h) between the neighboring columns integrated over the depth

$$Q_{i-1,i} = \int_0^{D(x_i - \frac{\Delta x}{2})} Q_h\left(x_i - \frac{\Delta x}{2}, z\right) dz\ , \quad Q_{i,i+1} = \int_0^{D(x_i + \frac{\Delta x}{2})} Q_h\left(x_i + \frac{\Delta x}{2}, z\right) dz \tag{5}$$

The heat flux through the basin bottom is neglected.

The change of heat content is expressed in terms of a heat flux (in W/m^2). If there is no horizontal heat transport, then the change of heat content in a water column should be equal to the rate of surface heating Q_s. By comparing the heat content change computed from the observed temperature distributions with the heat flux through the water surface calculated from the meteorological measurements, the horizontal heat transport within a section can be determined.

As the temperature profiles were not measured at fixed stations, interpolated temperatues at a number of fixed verticals at each section have been used. The changes of heat content between surveys expressed as a surface heat flux *versus* distance from the shore are shown in Figs. 8, 9. The position of the thermal bar (*i.e.* the location of the 4°C isotherm) at each survey is also indicated.

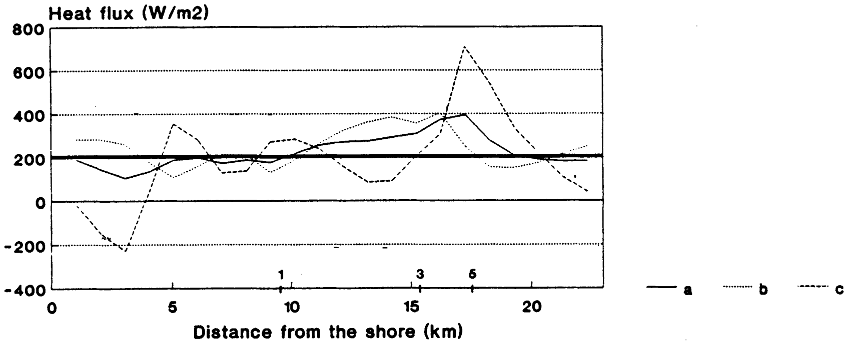


Fig. 8. Calculated heat content changes for complete water columns *versus* distance from the shore at the W-E cross-section: a – between survey number one and five, b – between survey number one and three, c – between survey number three and five. The heat content changes are expressed as a heat flux through a unit area of the lake surface. Numbers above the x-axis indicate the positions of the thermal bar. The horizontal line indicates the heat flux through the water surface.

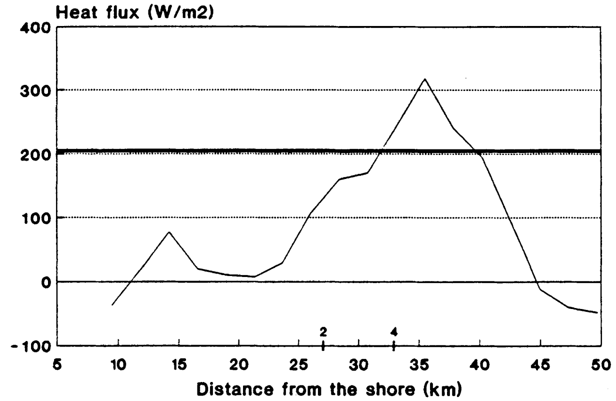


Fig. 9. The same as in Fig. 8, for the period between survey number two and four at the S-N cross-section.

The heat content near the thermal bar at the W-E section increased much more than what can be explained by the incoming surface heat flux 205 W/m^2. In the warm near-shore zone the heat content change was found to be less than the heat flux through the water surface as determined from meteorological measurements. Thus, heat is transported to the bar from the warm near-shore zone. The heat content of the water in the S-N section was found to increase most, well above the surface heat flux, in a region just off-shore the thermal bar. The heat content in the warm zone at the S-N section increased at a rate lower than the rate of surface heating, indicating heat transport towards the bar zone as at the W-E section.

The average increase of heat content in the cold off-shore zone in the W-E section (see Fig. 8) is almost the same as the heat flux through the water surface, while there is a heat loss in the S-N section. However, since the temperature was measured in a limited part of the cold off-shore zone, no firm conclusions about the horizontal heat transport off-shore the thermal bar can be made.

Progression of the Thermal Bar

In the course of the spring the thermal bar progresses off-shore from the shallow shore region to the deeper part of the lake. In this section the progression rate of the thermal bar is evaluated from observed temperatures. The position of the thermal bar is defined as the distance from the shore to the position of the 4°C isotherm. The movement of the thermal bar between successive surveys is then computed as the ratio of the bar displacement to the time interval between surveys. The observation results are given in Table 2.

Table 2 – Observed positions of the thermal bar, l, and time of measurements, t, taken from the beginning of survey No. 1.

Cross section	W-E			S-N	
Survey	1	3	5	2	4
l, km	9.5	15.3	17.7	27.0	33.0
t, hours	6.8	91.5	139.5	30.0	116.0

The observed progression rates of the thermal bar, $\Delta l/\Delta t$, are within a range of 1.3-2.0 cm/s (see Table 3), which is not far from the speed 0.6 cm/s determined by Tikhomirov (1982) in Lake Ladoga during one month in spring 1959. Two main factors that determine the movement of the thermal bar are surface heat flux and bottom topography. These are accounted for in an analytical model suggested by Elliott and Elliott (1970), where the heat flux is considered to be constant in time and space. The model assumes that horizontal heat fluxes are of secondary importance, that complete vertical mixing results from convection in the unstably stratified region (including the thermal bar zone) and that the bottom is thermally insulated. The temperature distribution in the region with water temperatures below or equal to 4°C is then described by

$$T(x,t) = T_0(x) + \frac{Q_s(t-t_0)}{\rho_w c_w D(x)} \tag{6}$$

where t is time; T is the temperature; $T_0(x)$ is the temperature distribution at the initial moment of time, t_0; Q_s is the heat flux through the water surface; and $D(x)$ is the depth at a distance x from the shore.

The position of the thermal bar $x=l$ can be obtained by inserting $T(l)=T_m \approx 4$°C into Eq. (6). The initial temperature distribution and the bottom slope can be approximated with simple linear functions

$$T_0(x) = T_m - \left(\frac{\partial T}{\partial x}\right)_0 (x - l_0) \; ; \quad D(x) = D_0 + \mu (x - l_0) \tag{7}$$

where D_0 is the depth at the initial position of the thermal bar $l = l_0$, and $(\partial T/\partial x)_0$ is the initial horizontal temperature gradient in the cold zone. The bottom slope μ is taken to be constant in the horizontal direction. This leads to the expression

$$l(t) \equiv l_0 - \frac{D_0}{2\mu} + \sqrt{\left(\frac{D_0}{2\mu}\right)^2 + \frac{Q_s (t - t_0)}{\rho_w c_w \mu (\partial T/\partial x)_0}} \tag{8}$$

In another analytical approach to describe the thermal bar movement by Zilitinkevich *et al.* (1991) the mixing of warm and cold water in the bar zone is accounted for. The importance of including this mechanism is also indicated in the above analysis of the horizontal heat fluxes along the section, where a heat transport was observed from the near-shore region towards the thermal bar zone. In this model the governing equation for the thermal bar movement is derived on basis of heat budget equations for the following three zones in an idealized wedge-shaped basin: 1) warm stably stratified near-shore zone; 2) thermal bar zone; 3) convectively mixed deep-water zone. It is here assumed that the horizontal heat transport directed perpendicular to the section is of secondary importance as discussed above in the section concerning heat content change calculations. For the linear initial temperature distribution given above the governing equation becomes (detailed description of derivation is given in Zilitinkevich *et al.* 1991)

$$\left[\left(\frac{\partial T}{\partial x}\right)_0 (l - l_0) - \frac{Q_s (t - t_0)}{\rho_w c_w (D_0 + \mu (l - l_0))} \right] \frac{dl}{dt} = F_l \tag{9}$$

where F_l represents the vertically averaged horizontal dynamic heat flux through the vertical $x = l$. It is noted that for $F_1 = 0$ Eqs. (8) and (9) coincide since the expression in brackets in the left-hand side of Eq. (9) should be equal to 0 in this case. Thus, the Zilitinkevich *et al.* approach reduces to the Elliott and Elliott heat balance model if the horizontal dynamical heat flux from the warm zone to the bar zone is neglected. Zilitinkevich *et al.* used dimensional analysis to describe the descending movement in the bar zone and to determine the horizontal dynamical heat flux due to entrainment of warm water from the near-shore zone to the thermal bar zone. The following expression was derived

$$F_l = C_t (g a \mu l)^{\frac{1}{2}} (T_w - T_m)^2 \tag{10}$$

where C_t is an empirical dimensionless constant; T_w is the temperature averaged over the warm zone; g is the acceleration due to gravity; $a = 1.6509 \times 10^{-5}$ (°C)$^{-2}$ is the coefficient in the square form of the freshwater equation of state

$$\rho_w \equiv \rho_m \left(1 - \frac{1}{2} a (T - T_m)^2\right) \tag{11}$$

where ϱ_m is the maximum freshwater density. The equation for the mean temperature of the warm near-shore zone, T_w, is derived from the two-dimensional heat transfer equation, averaged over this zone, which gives

$$\frac{\frac{1}{2}D_0 l_0 + D_0(l-l_0) + \frac{1}{2}\mu(l-l_0)^2}{D_0+\mu(l-l_0)}\,\frac{dT_w}{dt}$$

$$= \frac{Q_s}{\rho_w c_w}\,\frac{l}{D_0+\mu(l-l_0)} - C_t(ga\mu l)^{\frac{1}{2}}(T_w-T_m)^2 - (T_w-T_m)\frac{dl}{dt} \qquad (12)$$

The initial conditions for Eqs. (9) and (12) are

$$l = l_0\,, \quad T_w = T_{w0} \quad \text{at} \quad t = t_0 \qquad (13)$$

where T_{w0} is the initial value of the temperature averaged over the warm zone.

The constant C_t was evaluated in Zilitinkevich *et al.* (1991) to 8×10^{-3} by comparing theoretical predictions with empirical data from Lake Ladoga. The initial temperature distribution in the lake was not known. The value T_0=0°C was taken in *op.cit.*, which must have resulted in an underestimation of the constant C_t. Using data from Lake Ontario (three surveys in 1965 and four surveys in 1970 by Rogers 1966; 1971), Zilitinkevich and Malm (1993) found $C_t=3\times10^{-2}$ on the average. The separate estimates varied within one order of magnitude. Since the initial temperature distributions were known in Lake Ontario, this seems to be more appropriate than the one for Lake Ladoga and is used here.

The physical processes determining the thermal bar movement can be analyzed by comparing model predictions with observed progression rates. If there is a good agreement between predictions and observations it indicates that surface heat flux and bottom topography are the two major factors that influence the thermal bar movement. If there is also a comparatively better agreement for the Zilitinkevich *et al.* model this means that it is necessary to include mixing at the bar zone for a more correct description. Both models are, however, very simple and do not account for the action of wind, which probably will cause a more or less clearly marked deviation between observations and predictions, at least for short-time perspectives. The magnitude of this deviation can be considered to be a measure of the influence of wind on the thermal bar progression rate.

The observed progression rates of the thermal bar, the introduced parameters (*i.e.* the observed D_0, l_0, T_{w0}, μ and $(\partial T/\partial x)_0$ values) and the theoretically computed progression rates (using the average value of heat flux given above) are presented in Table 3. The initial average temperatures of the near-shore warm zone are calculated from the observed temperature distributions.

The Elliott and Elliott model underestimates the rate at which the thermal bar progresses with approximately 30 % at the W-E section and 17 % at the S-N section. The predictions made by the model suggested by Zilitinkevich *et al.* are a bit closer to observations. This suggests that the water near the thermal bar gains heat faster than only due to heat flux through the water surface, which was indicated in the heat content change calculations above. The deviations between the computed thermal bar movement by the Zilitinkevich *et al.* model and the observed movement are not systematic as for the Elliot and Elliot model. The progression rate is underestimated in the W-E section with approximately 10% and overestimated with 30% in the S-N section. The discrepancies between analytical computations and the observations are as mentioned above probably due to wind-driven currents. As the wind was quite weak to moderate during the field campaign it seems reasonable to assume that these discrepancies will increase if short time perspectives and high wind velocities are considered.

Another factor that can influence the thermal bar progression is river inflow and outflow. As the total inflow to Lake Ladoga during a year is only about 8% of the total lake volume it is most likely that the effect on the thermal bar movement is mainly during the earliest stages of the thermal bar existence and then quite locally. As the distance from the sections to the closest rivers was quite large (shortest distance being 25 km between the W-E section and Vuoksa River) and the thermal bar had progressed relatively far from the shore at the first survey, the influence of river inflow and outflow on the progression rates must be considered to be minimal. The local influence on the thermal bar dynamics by river inflow has been studied by Noble and Anderson (1968) in Lake Michigan at the mouth of Grand River.

Table 3 – Governing parameters and observed and computed (models by Elliott and Elliott – E&E, Zilitinkevich *et al.* – ZK&T) thermal bar progression rates.

Cross-section	W-E			S-N
Surveys	1-3	3-5	1-5	2-4
D_0, m	24.5	33.5	24.5	23.0
l_0, km	9.5	15.3	9.5	27.0
T_{w0}, °C	4.79	4.96	4.79	5.46
$(\partial T/\partial x)_0$, 10^4°C/m	1.2	1.57	1.2	1.1
$\mu \cdot 10^3$	1.88	0.21	1.16	1.0
$\Delta l/\Delta t$, cm/sec (E&E)	1.29	0.90	1.30	1.61
$\Delta l/\Delta t$, cm/sec (ZK&T)	1.64	1.27	1.56	2.51
$\Delta l/\Delta t$, cm/sec (Observed)	1.93	1.33	1.72	1.94

Although the field data from Lake Ladoga are limited, they show that the movement of the thermal bar over short periods only in a rough way can be described by the two simple theoretical models discussed above. For a more correct description, the effect of wind-driven currents has to be accounted for. However, for longer periods both the suggested theoretical approaches have given results which are in rather good agreement with observations (see *e.g.* Zilitinkevich *et al.* 1991; Tikhomirov 1982). This may be due to the fact that wind-induced currents towards and off the bar zone counteract each other.

Conclusions

i) The temperature distribution offshore the 4°C isotherm in Lake Ladoga showed depth-constant temperatures, which indicates intensive convective mixing. Near the shore there was a weak stable stratification, which, however, temporarily could break down due to wind mixing. No large horizontal temperature gradients were found on the on-shore side of the thermal bar, as have been the case in several other investigations.

ii) The measured current velocity distributions were complex, strongly dependent on wind conditions. The density induced currents seemed to be of secondary importance in comparison with wind-driven currents, even during conditions with weak wind.

iii) Calculations of changes in heat content showed that there was a horizontal heat transport from the near-shore warm zone towards the thermal bar.

iv) When comparing predictions from analytical theoretical models describing the movement of the thermal bar with observations, it was found that the progression of the thermal bar was underestimated when only surface heating and vertical mixing was considered. The agreement was better when mixing of cold and warm water in the bar zone was accounted for. There were non-systematic discrepancies between computations and observations, which are thought to be due to the influence of wind-induced currents.

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