

## BLOCK 型代数的量子化 \*

<sup>1,2</sup> 李军波 <sup>2</sup> 苏育才

(<sup>1</sup> 常熟理工学院数学系 常熟 215500; <sup>2</sup> 中国科技大学数学系 合肥 230026)

**摘要:** 文献 [1] 研究了一类 Block 型代数的李双代数结构, 本文对此代数进行了量子化.

**关键词:** 量子化; Block 型代数; 量子群; 李双代数.

**MR(2000) 主题分类:** 17B05; 17B56; 17B65 **中图分类号:** O152.5 **文献标识码:** A

**文章编号:** 1003-3998(2009)06-

### 1 引言和结果

对不同类型的代数进行量子化可以构造出新的量子群. Ng 和 Taft<sup>[5]</sup> 给出并分类了 Witt 和 Virasoro 型李双代数的结构, Grunspan<sup>[2]</sup> 对此类代数进行了量子化. Hu 和 Wang<sup>[3]</sup> 对广义 Witt 型代数进行了量子化, 后来 Song 等<sup>[4]</sup> 对广义 Virasoro-like 型代数进行了量子化. 作者<sup>[1]</sup> 证明了 Block 型代数的李双代数结构都是上三角余边缘的. 本文将对 Block 型代数进行量子化. 记  $\mathbb{F}$  为特征 0 的域,  $\mathcal{G}$  为  $\mathbb{F}$  上含  $\mathbb{Z}$  的非零加法子群,  $\mathbb{Z}_+$  为非负整数集.

本文考虑的 Block 型代数  $\mathcal{B} = \mathcal{B}(\mathcal{G})$ , 具有基  $\{\hbar, x^{a,i} | a \in \mathcal{G}, i \in \mathbb{Z}_+\}$  满足如下 Lie 运算

$$[\hbar, x^{b,j}] = bx^{b,j}, [x^{a,i}, x^{b,j}] = ((a-1)j - (b-1)i)x^{a+b,i+j-1}.$$

代数  $\mathcal{B}$  跟 Virasoro 代数、Virasoro-like 代数、Cartan  $S$  型和  $H$  型代数都密切相关.

设  $\mathcal{A}$  是环  $\mathcal{R}$  上带单位元的代数. 对  $\forall y \in \mathcal{A}, \alpha \in \mathcal{R}, n \in \mathbb{Z}$ , 记

$$y_\alpha^{\langle n \rangle} = (y + \alpha)(y + \alpha + 1) \cdots (y + \alpha + n - 1), y_\alpha^{\langle n \rangle} = (y + \alpha)(y + \alpha - 1) \cdots (y + \alpha - n + 1).$$

**引理 1.1** (Grunspan<sup>[2]</sup>) 设  $\mathbb{F}$  代数  $\mathcal{A}$  带单位元, 对  $\forall y \in \mathcal{A}, \alpha, \beta \in \mathbb{F}, m, n, k \in \mathbb{Z}$ ,

$$y_\alpha^{\langle m+n \rangle} = y_\alpha^{\langle m \rangle} y_{\alpha+m}^{\langle n \rangle}, y_\alpha^{\langle m+n \rangle} = y_\alpha^{\langle m \rangle} y_{\alpha-m}^{\langle n \rangle}, y_\alpha^{\langle m \rangle} = y_{\alpha-m+1}^{\langle m \rangle},$$

$$\sum_{m+n=k} \frac{(-1)^n}{m!n!} y_\alpha^{\langle m \rangle} y_\beta^{\langle n \rangle} = \binom{\alpha - \beta}{k}, \sum_{m+n=k} \frac{(-1)^n}{m!n!} y_\alpha^{\langle m \rangle} y_{\beta-m}^{\langle n \rangle} = \binom{\alpha - \beta + k - 1}{k}.$$

**定义 1.2** (Drinfel'd<sup>[8]</sup>) 令  $(H, \mu, \tau, \Delta', \epsilon', S')$  是一量子包络代数, 且  $H/tH \cong \mathcal{U}(L)$ ,  $L$  为李代数,  $t$  为形变参数. 称  $\mathcal{D} \in H \otimes H$  为 Drinfeld 扭, 如果

$$(\mathcal{D} \otimes 1)(\Delta' \otimes Id)(\mathcal{D}) = (1 \otimes \mathcal{D})(Id \otimes \Delta')(\mathcal{D}), (\epsilon' \otimes Id)(\mathcal{D}) = 1 \otimes 1 = (Id \otimes \epsilon')(\mathcal{D}).$$

收稿日期: 2007-12-08; 修订日期: 2008-08-06

E-mail: sd.junbo@163.com; ycsu@ustc.edu.cn

\* 基金项目: 国家自然科学基金 (10671027, 10825101) 和中国科学技术大学“百人计划”资助

**定理 1.3** (Drinfel'd<sup>[8]</sup>) 设  $(H, \mu, \tau, \Delta', \epsilon', S')$  是一交换环上 Hopf 代数, 则

(1)  $\mathcal{U} = \mu \cdot (S' \otimes Id)(\mathcal{D})$  在  $H$  中的逆元为  $\mu(Id \otimes S')(D^{-1})$  (其中  $\mathcal{D}$  为 Drinfeld 扭).

(2) 代数  $(H, \mu, \tau, \Delta, \epsilon, S)$  是一个新的 Hopf 代数, 其中  $\epsilon = \epsilon'$ , 定义  $\Delta : H \rightarrow H \otimes H, \Delta(h) = \mathcal{D}\Delta'(h)\mathcal{D}^{-1}; S : H \rightarrow H, S(h) = \mathcal{U}^{-1}S'(h)\mathcal{U}$ .

记  $\mathcal{B}$  的普遍包罗代数为  $U(\mathcal{B})$ . 设  $(U(\mathcal{B}), \mu, \tau, \Delta', \epsilon', S')$  是一 Hopf 代数, 即

$$\Delta'(x) = x \otimes 1 + 1 \otimes x, S'(x) = -x, \epsilon'(x) = 0, \quad \forall x \in \mathcal{B}.$$

对  $\forall a_0 \neq 0, i_0 \geq 1$ , 记  $X = x^{a_0, i_0} \in \mathcal{B}$ , 取  $\partial = \frac{\hbar}{a_0} \in \mathbb{C}\hbar$ , 使得  $[\partial, X] = X$ . 对  $\forall \alpha \in \mathbb{F}$ , 记  $\mathcal{U}_\alpha = \mu(S' \otimes Id)\mathcal{D}_\alpha, \mathcal{V}_\alpha = \mu(Id \otimes S')\mathcal{D}_\alpha, \mathcal{D}_\alpha = \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\alpha^{(k)} \otimes X^k t^k, \mathcal{D}'_\alpha = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \partial_\alpha^{[k]} \otimes X^k t^k$ . 因为  $S'(\partial_\alpha^{(k)}) = (-1)^k \partial_{-\alpha}^{[k]}, S'X^k = (-1)^k X^k$ , 则  $\mathcal{U}_\alpha = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \partial_{-\alpha}^{[k]} X^k t^k, \mathcal{V}_\alpha = \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\alpha^{[k]} X^k t^k$ . 方便起见, 引入记号:  $\partial_0^{(n)} := \partial^{(n)}, \partial_0^{[n]} := \partial^{[n]}, D_0 := D, \mathcal{D}_0 := \mathcal{D}, \mathcal{U}_0 := \mathcal{U}, \mathcal{V}_0 := \mathcal{V}$ .

**定理 1.4**  $\mathbb{F}[[t]]$  上存在非交换且非余交换的 Hopf 代数结构  $(U(\mathcal{B})[[t]], \mu, \tau, \Delta, S, \epsilon)$ , 使得  $U(\mathcal{B})[[t]]/tU(\mathcal{B})[[t]] = U(\mathcal{B})$ , 其积和余单位保持不变, 余积和对极分别由下面式子给出:

$$\Delta(x^{a,i}) = x^{a,i} \otimes (1 - Xt)^\alpha + \sum_{k=0}^{\infty} (-1)^k b_k \partial^{(k)} \otimes (1 - Xt)^{-k} x^{a+ka_0, i+ki_0-k} t^k,$$

$$S(x^{a,i}) = -(1 - Xt)^{-\alpha} \sum_{k=0}^{\infty} b_k x^{a+ka_0, i+ki_0-k} \partial_k^{(k)} t^k,$$

$$\Delta(\hbar) = \hbar \otimes 1 + 1 \otimes \hbar + \hbar \otimes (1 - Xt)^{-1} Xt, \quad S(\hbar) = \hbar(1 - Xt)^{-1}(Xt - X^2 t^2) - \hbar,$$

其中  $\alpha = \frac{a}{a_0}, b_k = \frac{1}{k!} \prod_{n=0}^{k-1} ((a_0 - 1)(i + ni_0 - n) - (a + na_0 - 1)i_0), b_0 = 1$ .

**注记** 由于  $\mathcal{H}$  不能得到  $\mathcal{B}$  的所有双代数结构, 从而可能存在其它的量子化代数结构.

## 2 定理的证明

在给出主要定理的证明之前, 我们先给出几个引理.

**引理 2.1** 取  $\alpha = \frac{a}{a_0}$ , 且设  $m, k \in \mathbb{Z}_+$ . 对  $\forall \beta \in \mathbb{F}, x^{a,i}, x^{b,j} \in \mathcal{B}$ , 都有

$$x^{a,i} \partial_\beta^{[m]} = \partial_{\beta-\alpha}^{[m]} x^{a,i}, x^{a,i} \partial_\beta^{(m)} = \partial_{\beta-\alpha}^{(m)} x^{a,i}, X^k \partial_\beta^{[m]} = \partial_{\beta-k}^{[m]} X^k, X^k \partial_\beta^{(m)} = \partial_{\beta-k}^{(m)} X^k,$$

$$\hbar^k \partial_\beta^{[m]} = \partial_\beta^{[m]} \hbar^k, \hbar^k \partial_\beta^{(m)} = \partial_\beta^{(m)} \hbar^k, \hbar(x^{b,j})^m = mb(x^{b,j})^{m-1} \hbar + (x^{b,j})^m \hbar,$$

$$x^{a,i} (x^{b,j})^m = \sum_{k=0}^m (-1)^k \binom{m}{k} \prod_{p=0}^{k-1} ((b-1)(i+pj-p) - (a+pb-1)j) (x^{b,j})^{m-k} x^{a+kb, i+kj-k}.$$

**证** 根据  $[\partial, x^{a,i}] = \alpha x^{a,i}$ , 有  $x^{a,i} \partial = (\partial - \alpha)x^{a,i}$ . 参考 Grunspan<sup>[2]</sup>, 对  $m$  利用数学归纳法. 对  $m+1$ , 我们有

$$x^{a,i} \partial_\beta^{[m+1]} = x^{a,i} \partial_\beta^{[m]} (\partial + \beta - m) = \partial_{\beta-\alpha}^{[m]} x^{a,i} (\partial + \beta - m) = \partial_{\beta-\alpha}^{[m+1]} x^{a,i},$$

$$x^{a,i} \partial_\beta^{(m+1)} = x^{a,i} \partial_\beta^{(m)} (\partial + \beta + m) = \partial_{\beta-\alpha}^{(m)} x^{a,i} (\partial + \beta + m) = \partial_{\beta-\alpha}^{(m+1)} x^{a,i}.$$

第 3-6 个式子可先后对  $m$  和  $k$  采用数学归纳法得出. 注意到

$$(\text{adx}^{b,j})^k(x^{a,i}) = \prod_{p=0}^{k-1} ((b-1)(i+pj-p) - (a+pb-1)j)x^{a+kb,i+kj-k},$$

我们可推出最后一个式子. 故本引理成立. |

Grunspan<sup>[2]</sup> 给出了如下的引理.

**引理 2.2** (1)  $\mathcal{D}_\alpha D_\beta = 1 \otimes (1 - Xt)^{\alpha-\beta}$ ,  $\mathcal{V}_\alpha \mathcal{U}_\beta = (1 - Xt)^{-\alpha-\beta}$ ,  $\forall \alpha, \beta \in \mathbb{F}$ .

(2)  $\Delta' \partial^{[m]} = \sum_{k=0}^m \binom{m}{k} \partial_{-\alpha}^{[k]} \otimes \partial_\alpha^{[m-k]}$ ,  $\forall \alpha \in \mathbb{F}$ . 特别,  $\Delta' \partial^{[m]} = \sum_{k=0}^m \binom{m}{k} \partial^{[k]} \otimes \partial^{[m-k]}$ .

(3)  $\mathcal{D} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \partial^{[k]} \otimes X^k t^k$  是  $U(\mathcal{B})[[t]]$  中的一 Drinfeld 扭.

**引理 2.3** 设  $\alpha, b_k$  如定理 1.1 所述, 则对  $\forall \beta \in \mathbb{F}$ ,  $x^{a,i} \in \mathcal{B}$ , 都有

$$(x^{a,i} \otimes 1)D_\beta = D_{\beta-\alpha}(x^{a,i} \otimes 1), \quad (2.1)$$

$$(1 \otimes x^{a,i})D_\beta = \sum_{k=0}^{\infty} (-1)^k b_k D_{\beta+k} (\partial_\beta^{(k)} \otimes x^{a+ka_0,i+ki_0-k} t^k), \quad (2.2)$$

$$x^{a,i} \mathcal{U}_\beta = \mathcal{U}_{\beta+\alpha} \sum_{k=0}^{\infty} b_k x^{a+ka_0,i+ki_0-k} \partial_{-\beta+k}^{(k)} t^k, \quad X \mathcal{U}_\beta = \mathcal{U}_{\beta+1} X, \quad (2.3)$$

$$\hbar \mathcal{U}_\beta = -a_0 \partial_{-\beta}^{[1]} \mathcal{U}_{\beta+1} X t + \mathcal{U}_\beta \hbar, \quad \mathcal{V}_\beta \partial_{-\beta}^{[1]} = \partial_{-\beta}^{[1]} \mathcal{V}_\beta - \partial_\beta^{[1]} \mathcal{V}_{\beta-1} X t, \quad (2.4)$$

$$(\hbar \otimes 1)D_\beta = D_\beta(\hbar \otimes 1), \quad (1 \otimes \hbar)D_\beta = D_{\beta+1}(\partial_\beta^{(1)} \otimes a_0 X t) + D_\beta(1 \otimes \hbar). \quad (2.5)$$

**证** 由等式

$$(x^{a,i} \otimes 1)D_\beta = \sum_{k=0}^{\infty} \frac{1}{k!} x^{a,i} \partial_\beta^{(k)} \otimes X^k t^k = \sum_{k=0}^{\infty} \frac{1}{k!} \partial_{\beta-\alpha}^{(k)} x^{a,i} \otimes X^k t^k = D_{\beta-\alpha}(x^{a,i} \otimes 1)$$

可推出 (2.1). 根据引理 2.1 的最后一个等式,

$$\begin{aligned} (1 \otimes x^{a,i})D_\beta &= \sum_{m=0}^{\infty} \frac{1}{m!} \partial_\beta^{(m)} \otimes x^{a,i} X^m t^m \\ &= \sum_{m=0}^{\infty} \sum_{k=0}^m \frac{(-1)^k}{k!(m-k)!} \partial_\beta^{(m)} \\ &\quad \otimes \prod_{p=0}^{k-1} ((a_0-1)(i+pi_0-p) - (a+pa_0-1)i_0) X^{m-k} x^{a+ka_0,i+ki_0-k} t^m \\ &= \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!m!} \partial_\beta^{(m+k)} \\ &\quad \otimes \prod_{p=0}^{k-1} ((a_0-1)(i+pi_0-p) - (a+pa_0-1)i_0) X^m x^{a+ka_0,i+ki_0-k} t^{m+k} \\ &= \sum_{k=0}^{\infty} (-1)^k b_k \sum_{m=0}^{\infty} \left( \frac{1}{m!} \partial_{\beta+k}^{(m)} \otimes X^m t^m \right) (\partial_\beta^{(k)} \otimes x^{a+ka_0,i+ki_0-k} t^k) \\ &= \sum_{k=0}^{\infty} (-1)^k b_k D_{\beta+k} (\partial_\beta^{(k)} \otimes x^{a+ka_0,i+ki_0-k} t^k). \end{aligned}$$

因此 (2.2) 式成立. 利用 Grunspan<sup>[2]</sup> 中的技巧和引理 2.1, (2.5) 式可如下证明:

$$\begin{aligned} (\hbar \otimes 1)D_\beta &= (\hbar \otimes 1) \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\beta^{(k)} \otimes X^k t^k = \sum_{k=0}^{\infty} \frac{1}{k!} \hbar \partial_\beta^{(k)} \otimes X^k t^k = D_\beta (\hbar \otimes 1), \\ (1 \otimes \hbar)D_\beta &= \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\beta^{(k)} \otimes \hbar X^k t^k = \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \partial_\beta^{(k)} \otimes a_0 X^k t^k + \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\beta^{(k)} \otimes X^k \hbar t^k. \end{aligned}$$

关于 (2.3) 式中的两个等式, 我们有

$$\begin{aligned} x^{a,i} \mathcal{U}_\beta &= \sum_{p=0}^{\infty} \frac{(-1)^p}{p!} x^{a,i} \partial_{-\beta}^{[p]} X^p t^p = \sum_{p=0}^{\infty} \frac{(-1)^p}{p!} \partial_{-\alpha-\beta}^{[p]} x^{a,i} X^p t^p \\ &= \sum_{p=k}^{\infty} \frac{(-1)^p}{(p-k)!} \partial_{-\alpha-\beta}^{[p]} \sum_{k=0}^p (-1)^k b_k X^{p-k} x^{a+ka_0, i+ki_0-k} t^p \\ &= \sum_{k=0}^{\infty} \sum_{p=0}^{\infty} \frac{(-1)^p}{p!} \partial_{-\alpha-\beta}^{[p]} X^p t^p \partial_{-\alpha-\beta}^{[k]} b_k x^{a+ka_0, i+ki_0-k} t^k \\ &= \mathcal{U}_{\alpha+\beta} \sum_{k=0}^{\infty} b_k x^{a+ka_0, i+ki_0-k} \partial_{k-\beta}^{[k]} t^k, \\ X \mathcal{U}_\beta &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} X \partial_{-\beta}^{[k]} X^k t^k = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \partial_{-\beta-1}^{[k]} X^{k+1} t^k = \mathcal{U}_{\beta+1} X. \end{aligned}$$

下面的计算可得到 (2.4) 式中的两个等式.

$$\begin{aligned} \hbar \mathcal{U}_\beta &= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \hbar \partial_{-\beta}^{[k]} X^k t^k = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \partial_{-\beta}^{[k]} (ka_0 X^k + X^k \hbar) t^k \\ &= -a_0 \sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{(k-1)!} \partial_{-\beta}^{[k]} X^{k-1} t^{k-1} X t + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \partial_{-\beta}^{[k]} X^k t^k \hbar \\ &= -a_0 \partial_{-\beta}^{[1]} \mathcal{U}_{\beta+1} X t + \mathcal{U}_\beta \hbar, \\ \mathcal{V}_\beta \partial_{-\beta}^{[1]} &= \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\beta^{[k]} X^k \partial_{-\beta}^{[1]} t^k = \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\beta^{[k]} (\partial - \beta - k) X^k t^k \\ &= \sum_{k=0}^{\infty} \frac{1}{k!} \partial_\beta^{[k]} (\partial - \beta) X^k t^k - \sum_{k=0}^{\infty} \frac{1}{(k-1)!} \partial_\beta^{[k]} X^{k-1} t^{k-1} X t \\ &= \partial_{-\beta}^{[1]} \mathcal{V}_\beta - \partial_\beta^{[1]} \mathcal{V}_{\beta-1} X t. \end{aligned}$$

证毕. |

**定理 1.4 的证明** 对  $\forall x^{a,i} \in \mathcal{B}$  和  $\hbar \in \mathcal{B}$ , 我们有下面的推理:

$$\begin{aligned} \Delta(x^{a,i}) &= \mathcal{D}(x^{a,i} \otimes 1) \mathcal{D}^{-1} + \mathcal{D}(1 \otimes x^{a,i}) \mathcal{D}^{-1} = \mathcal{D}(x^{a,i} \otimes 1) \mathcal{D} + \mathcal{D}(1 \otimes x^{a,i}) \mathcal{D} \\ &= \mathcal{D} \mathcal{D}_{-\alpha}(x^{a,i} \otimes 1) + \mathcal{D} \sum_{k=0}^{\infty} (-1)^k b_k \mathcal{D}_k (\partial^{(k)} \otimes x^{a+ka_0, j+ki_0-k} t^k) \\ &= (1 \otimes (1 - Xt)^\alpha) (x^{a,i} \otimes 1) + \sum_{k=0}^{\infty} (-1)^k b_k (1 \otimes (1 - Xt)^{-k}) (\partial^{(k)} \otimes x^{a+ka_0, j+ki_0-k} t^k) \\ &= x^{a,i} \otimes (1 - Xt)^\alpha + \sum_{k=0}^{\infty} (-1)^k b_k \partial^{(k)} \otimes (1 - Xt)^{-k} x^{a+ka_0, j+ki_0-k} t^k, \end{aligned}$$

$$\Delta(\hbar) = \mathcal{D}D(\hbar \otimes 1) + \mathcal{D}(D_1(\partial^{(1)} \otimes a_0 X t) + D(1 \otimes \hbar)) = \hbar \otimes 1 + 1 \otimes \hbar + \hbar \otimes (1 - X t)^{-1} X t,$$

$$S(x^{a,i}) = -\mathcal{V}U_\alpha \sum_{k=0}^{\infty} b_k x^{a+ka_0, i+ki_0-k} \partial_k^{(k)} t^k = -(1 - X t)^{-\alpha} \sum_{k=0}^{\infty} b_k x^{a+ka_0, i+ki_0-k} \partial_k^{(k)} t^k,$$

$$S(\hbar) = U^{-1} S'(\hbar) U = a_0 (\partial \mathcal{V} - \partial \mathcal{V}_{-1} X t) U_1 X t - \hbar = a_0 \partial (1 - X t)^{-1} (X t - X^2 t^2) - \hbar.$$

证毕. |

### 参 考 文 献

- [1] Li J, Su Y, Xin B. Lie bialgebras of Block type Lie algebras. Chin Math Annl, 2008, **29B**: 487–500
- [2] Grunspan C. Quantizations of Witt algebra and of simple Lie algebras in char  $p$ . J Alg, 2004, **280**: 145–161
- [3] Hu N, Wang X. Quantizations of generalized-Witt algebra and of Jacobson -Witt algebra in the modular case. J Algebra, 2007, **312**(2): 902–929
- [4] Song G, Su Y, Wu Y. Quantization of generalized Virasoro-like algebras. L Alg Appl, 2008, **428**: 2888–2899
- [5] Ng H, Taft J. Classification of the Lie bialgebra structures on the Witt and Virasoro algebras. J Pure Appl alg, 2000, **151**: 67–88
- [6] Su Y. Quasifinite representations of a family of Lie algebras of Block type. J Pure Alg, 2004, **192**: 293–305
- [7] Xu X. Generalizations of the Block algebras. Manuscripta Math, 1999, **100**: 489–518
- [8] Drinfel'd V. Quantum groups. Providence: Amer Math Soc, 1987. 789–820

## Quantization of Block Type Algebras

<sup>1,2</sup>Li Junbo   <sup>2</sup>Su Yucai

<sup>(1)</sup>Department of Mathematics, Changshu Institute of Technology, Changshu 2155008;

<sup>(2)</sup>Department of Mathematics, University of Science and Technology of China, Hefei 230026)

**Abstract:** We quantize a family of Block type algebras, whose Lie bialgebra structures were investigated in a recent paper [1] by the authors.

**Key words:** Quantizations; Block type algebras; Quantum groups; Lie bialgebras.

**MR(2000) Subject Classification:** 17B05; 17B56; 17B65