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实 Clifford 分析中的拟 Bochner-Martinelli 型高阶奇异积分*

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摘要:该文借助于高阶奇异积分的 Hadmard 主值思想以及归纳法思想讨论了实 Clifford 分析中拟 Bochner-Martinelli 型高阶奇异积分 Hadmard 主值的存在性、递推公式、计算公式,以及在 Hadamard 主值意义下的微分公式.

关键词:实 Clifford 分析; 拟 Bochner-Martinelli 型高阶奇异积分; Hadamard 主值.

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1 引言

Clifford 分析是上世纪七十年代新兴起的一个数学分支. 研究的是从实向量空间映射到不可交换的实 Clifford 代数的函数理论. 它在数学,物理等方面有着重大的应用. 许多数学家对它给于很大关注. 1970 年以来,Robert P Gilbert[1],F Brack,R Relanghe,F Sommen[2],徐振远[3],闻国椿[4],Le HuangSong[5],黄沙[6−10],乔玉英[11,12] 等人将 Clifford 分析与单,多元复分析(广义)解析函数理论,边值问题以及多维奇异积分方程理论相对照,做了很多工作. 到上世纪末,Clifford 分析有了长足的发展.

根据 J Hadamard[13] 从发散积分引出积分有限部分的思想,人们在单复变中讨论了高阶奇异积分的 Hadamard 主值[14,15]. 在多元复分析中人们也讨论了高阶奇异积分的 Hadamard 主值[16]. 本文在文[17,18]的基础上,借助于高阶奇异积分的 Hadmard 主值思想以及归纳法思想,讨论了实 Clifford 分析中拟 Bochner-Martinelli 型高阶奇异积分 Hadmard 主值的存在性、递推公式、计算公式,以及在 Hadamard 主值意义下的微分公式.

2 预备知识

记以 $e_0,e_1,e_2,\cdots,e_n$ 为基底的实向量空间为 R^{n+1},e_0 为 R^{n+1} 的单位元素,R^{n+1} 的元素为

$$x=\sum_{k=0}^{n}x_ke_k,\quad \bar{x}=x_0e_0-\sum_{k=1}^{n}x_ke_k,$$

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设 $\mathscr{A}$ 是实 Clifford 代数,其基底由

$$e_\phi = e_0, e_A : A = (h_1, \cdots, h_r) \in \mathscr{P}\{1, \cdots, n\} : 1 \leqslant h_1 < h_2 < \cdots < h_r \leqslant n$$

给出. 且定义基底元素的乘积适合如下规则

$$e_0{}^2 = 1,\ e_i{}^2 = -1,\ i = 1, \cdots, n.$$

$$e_i e_j = -e_j e_i (i \neq j, i, j = 1, 2, \cdots, n).$$

将此乘积运算线性推广到 $\mathscr{A}$,显然 $\mathscr{A}$ 是不可交换的代数.

$\mathscr{A}$ 中的元素 u 可表示成 $u = \sum_A u_A e_A, u_A \in R$. 定义 $|u|^2 = \sum_A |u_A|^2$, 易证

$$|u+v| \leqslant |u| + |v|;\ |uv| \leqslant 2^{n-1} |u||v|;\ ||u| - |v|| \leqslant |u-v|.$$

设 D 为 R^{n+1} 中连通开集,我们研究 $C^m(m \geqslant 1)$ 类函数集合

$$F_D^{(m)} = \{f \mid f: D \to \mathscr{A},\ f(x) = \sum_A f_A(x) e_A, f_A \in C^m(D)\}.$$

定义 $F_D^{(m)}$ 算子

$$\overline{\partial}_x = e_0 \frac{\partial}{\partial x_0} + e_1 \frac{\partial}{\partial x_1} + \cdots + e_n \frac{\partial}{\partial x_n};$$

$$\partial_x = e_0 \frac{\partial}{\partial x_0} - e_1 \frac{\partial}{\partial x_1} - \cdots - e_n \frac{\partial}{\partial x_n}.$$

如果 $\overline{\partial}_x f = 0$,则称 f 为 D 内的左正则函数, 简称正则函数.

在下文中,设 Ω 为 D 边界, $x, y \in \Omega$, $\overline{\partial}_x f(x,y)$, $\partial_x f(x,y)$ 均为 Hölder 连续函数.

由文[18]知, 几类拟 Bochner-Martinelli 型高阶奇异积分的定义如下

定义 1

$$\int_\Omega \frac{(\overline{x}-\overline{y}) f(x,y) \mathrm{d}\sigma_x}{|x-y|^{n+1+\alpha}} = \int_\Omega \frac{f(x,y)(\overline{x}-\overline{y}) \mathrm{d}\sigma_x}{|x-y|^{n+1+\alpha}} = \int_\Omega \frac{\partial_x f(x,y) \mathrm{d}\sigma_x}{(n-1+\alpha)|x-y|^{n-1+\alpha}},$$

$$\int_\Omega \frac{(x-y) f(x,y) \mathrm{d}\sigma_x}{|x-y|^{n+1+\alpha}} = \int_\Omega \frac{f(x,y)(x-y) \mathrm{d}\sigma_x}{|x-y|^{n+1+\alpha}} = \int_\Omega \frac{\overline{\partial}_x f(x,y) \mathrm{d}\sigma_x}{(n-1+\alpha)|x-y|^{n-1+\alpha}},$$

$$\int_\Omega \frac{f(x,y) \mathrm{d}\sigma_x}{|x-y|^{n+1+\alpha}} = \int_\Omega \frac{(\overline{\partial}_x f)(\overline{x}-\overline{y}) + (\partial_x f)(x-y)}{2\alpha |x-y|^{n+1+\alpha}} \mathrm{d}\sigma_x.$$

这里, $\alpha > 0, y \in \Omega$.

定义 2

$$\int_\Omega \left[\frac{(\overline{x}-\overline{y})^2}{|x-y|^{n+3+\alpha}} - \frac{(x-y)^2}{|x-y|^{n+3+\alpha}}\right] f(x,y) \mathrm{d}\sigma_x$$

$$= \frac{1}{n+1+\alpha} \int_\Omega \frac{(\overline{x}-\overline{y}) + (x-y)}{|x-y|^{n+1+\alpha}} [(\partial_x - \overline{\partial}_x) f(x,y)] \mathrm{d}\sigma_x,$$

这里, $\alpha > 0, y \in \Omega$.

定义 3

$$\int_\Omega \frac{(\overline{x}-\overline{y})^2 f(x,y)}{|x-y|^{n+3+\alpha}} \mathrm{d}\sigma_x = \frac{-(n-1+\alpha)}{n+1+\alpha} \int_\Omega \frac{f(x,y)}{|x-y|^{n+1+\alpha}} \mathrm{d}\sigma_x$$

$$+ \frac{1}{n+1+\alpha} \int_\Omega \frac{[(\overline{x}-\overline{y}) + (x-y)] \partial_x f(x,y)}{|x-y|^{n+1+\alpha}} \mathrm{d}\sigma_x,$$

这里, $\alpha > 0, y \in \Omega$.

定义 4

$$\int_\Omega \frac{(x-y)^2 f(x,y)}{|x-y|^{n+3+\alpha}} \mathrm{d}\sigma_x = \frac{-(n-1+\alpha)}{n+1+\alpha} \int_\Omega \frac{f(x,y)}{|x-y|^{n+1+\alpha}} \mathrm{d}\sigma_x$$

$$+\frac{1}{n+1+\alpha}\int_{\Omega}\frac{[(\bar{x}-\bar{y})+(x-y)]\bar{\partial}_x f(x,y)}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x,$$

这里,$\alpha>0, y\in\Omega$.

3　拟 Bochner-Martinelli 型高阶奇异积分的递推公式

定义 5　设 $f(x,y)$ 在 $D\times D$ 内具有各阶连续偏导数,连续到边界,且经过 $\bar{\partial}_x, \partial_x$ 运算 p 次($p\leqslant m$)仍是 Hölder 连续的,并以 $0<\beta<1$ 为 Hölder 指数,则称 $f(x,y)$属于 $H_x^{(m)}(\beta)$ 类,记作 $f\in H_x^{(m)}(\beta)$.

由文[17]知

引理 1　设 $n>0,\alpha>2m>0, f(x,y)\in H_x^{2m+2}(\beta), y\in\Omega, 1\leqslant l\leqslant m, l\in N$,则

$$\int_{\Omega}\frac{f(x,y)(\bar{x}-\bar{y})}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x=\mu\int_{\Omega}\frac{\Delta_x^l(\partial_x f(x,y))}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x, \tag{1}$$

$$\int_{\Omega}\frac{f(x,y)(x-y)}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x=\mu\int_{\Omega}\frac{\Delta_x^l(\bar{\partial}_x f(x,y))}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x, \tag{2}$$

$$\int_{\Omega}\frac{f(x,y)}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x=\frac{\mu}{\alpha}\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x, \tag{3}$$

这里,算子 $\Delta_x=\partial_x\bar{\partial}_x=\bar{\partial}_x\partial_x$, $\mu=\frac{(\alpha-2l-2)!!(n+\alpha-2l-3)!!}{(\alpha-2)!!(n+\alpha-1)!!}$,记号 $r!!$ 表示从 r 开始递减(每次减 2)直到最小整数为止所得到数的乘积.

定理 1　设 $n>0,\alpha>2m>0, f(x,y)\in H_x^{2m+2}(\beta)$, $y\in\Omega, 0<l\leqslant m, l\in N$, 则

$$\int_{\Omega}\frac{(\bar{x}-\bar{y})^2 f(x,y)}{|x-y|^{n+3+\alpha}}\mathrm{d}\sigma_x=\frac{1-n}{\alpha}\upsilon\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x+\upsilon\int_{\Omega}\frac{\Delta_x^l\partial_x^2 f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x, \tag{4}$$

$$\int_{\Omega}\frac{(x-y)^2 f(x,y)}{|x-y|^{n+3+\alpha}}\mathrm{d}\sigma_x=\frac{1-n}{\alpha}\upsilon\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x+\upsilon\int_{\Omega}\frac{\Delta_x^l\bar{\partial}_x^2 f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x, \tag{5}$$

$$\int_{\Omega}\left[\frac{(\bar{x}-\bar{y})^2}{|x-y|^{n+3+\alpha}}-\frac{(x-y)^2}{|x-y|^{n+3+\alpha}}\right]f(x,y)\mathrm{d}\sigma_x=\upsilon\int_{\Omega}\frac{\Delta_x^l(\partial_x^2-\partial_x^2)f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x, \tag{6}$$

这里,$\upsilon=\frac{\mu}{n+1+\alpha}$,$\mu$ 如引理 1 所述.

证　由定义 3,引理 1 得

$$\begin{aligned}
&\int_{\Omega}\frac{(\bar{x}-\bar{y})^2 f(x,y)}{|x-y|^{n+3+\alpha}}\mathrm{d}\sigma_x\\
&=\frac{-(n-1+\alpha)}{n+1+\alpha}\int_{\Omega}\frac{f(x,y)}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x+\frac{1}{n+1+\alpha}\int_{\Omega}\frac{(\bar{x}-\bar{y})\partial_x f(x,y)}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x\\
&\quad+\frac{1}{n+1+\alpha}\int_{\Omega}\frac{(x-y)\partial_x f(x,y)}{|x-y|^{n+1+\alpha}}\mathrm{d}\sigma_x\\
&=\frac{-(n-1+\alpha)}{n+1+\alpha}\frac{\mu}{\alpha}\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x+\frac{\mu}{n+1+\alpha}\int_{\Omega}\frac{\Delta_x^l(\partial_x^2 f(x,y))}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x\\
&\quad+\frac{\mu}{n+1+\alpha}\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x\\
&=\frac{-(n-1)}{\alpha}\frac{\mu}{(n+1+\alpha)}\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x+\frac{\mu}{n+1+\alpha}\int_{\Omega}\frac{\Delta_x^l(\partial_x^2 f(x,y))}{|x-y|^{n+\alpha-2l-1}}\mathrm{d}\sigma_x
\end{aligned}$$

$$=\frac{1-n}{\alpha}\upsilon\int_{\Omega}\frac{\Delta_x^{l+1}f(x,y)}{|x-y|^{n+\alpha-2l-1}}d\sigma_x+\upsilon\int_{\Omega}\frac{\Delta_x^{l}(\partial_x^2 f(x,y))}{|x-y|^{n+\alpha-2l-1}}d\sigma_x.$$

这里，$\upsilon=\frac{\mu}{n+1+\alpha}$.(4)式得证.类似地，由定义4，引理1可得(5)式.由(4)式与(5)式可得(6)式.定理1证毕. ▮

4 拟 Bochner-Martinelli 型高阶奇异积分的计算公式

由文[17]知

引理 2 设

$$f(x,y)\in H_x^{(2k+2)}(\beta),\ y\in\Omega, 0\leqslant r<2,\quad \lambda_1=\frac{(n-2-r)!!}{(n+2k-r)!!(2k-1-r)!!},$$

则下列高阶奇异积分存在并且可以用以下公式计算

$$\int_{\Omega}\frac{f(x,y)(\bar{x}-\bar{y})d\sigma_x}{|x-y|^{n+2k+2-r}}=\lambda_1\int_{\Omega}\frac{(\Delta_x^k\partial_x f(x,y))d\sigma_x}{|x-y|^{n-r}},\tag{7}$$

$$\int_{\Omega}\frac{f(x,y)(x-y)d\sigma_x}{|x-y|^{n+2k+2-r}}=\lambda_1\int_{\Omega}\frac{(\Delta_x^k\bar{\partial}_x f(x,y))d\sigma_x}{|x-y|^{n-r}},\tag{8}$$

$$\int_{\Omega}\frac{f(x,y)d\sigma_x}{|x-y|^{n+2k+2-r}}=\frac{\lambda_1}{2k+1-r}\int_{\Omega}\frac{(\Delta_x^{k+1}f(x,y))d\sigma_x}{|x-y|^{n-r}}.\tag{9}$$

定理 2 设

$$f(x,y)\in H_x^{(2k+2)}(\beta),\ y\in\Omega, 0\leqslant r<2,\quad \lambda_2=\frac{(n-2-r)!!}{(n+2k+2-r)!!(2k-1-r)!!},$$

则下列高阶奇异积分存在并且可以用以下公式计算

$$\int_{\Omega}\frac{(\bar{x}-\bar{y})^2 f(x,y)d\sigma_x}{|x-y|^{n+2k+4-r}}=\frac{1-n}{2k+1-r}\lambda_2\int_{\Omega}\frac{\Delta_x^{k+1}f(x,y)d\sigma_x}{|x-y|^{n-r}}+\lambda_2\int_{\Omega}\frac{\Delta_x^k\partial_x^2 f(x,y)d\sigma_x}{|x-y|^{n-r}},\tag{10}$$

$$\int_{\Omega}\frac{(x-y)^2 f(x,y)d\sigma_x}{|x-y|^{n+2k+4-r}}=\frac{1-n}{2k+1-r}\lambda_2\int_{\Omega}\frac{\Delta_x^{k+1}f(x,y)d\sigma_x}{|x-y|^{n-r}}+\lambda_2\int_{\Omega}\frac{\Delta_x^k\bar{\partial}_x^2 f(x,y)d\sigma_x}{|x-y|^{n-r}},\tag{11}$$

$$\int_{\Omega}\left[\frac{(\bar{x}-\bar{y})^2}{|x-y|^{n+2k+4-r}}-\frac{(x-y)^2}{|x-y|^{n+2k+4-r}}\right]f(x,y)d\sigma_x=\lambda_2\int_{\Omega}\frac{\Delta_x^k(\partial_x^2-\bar{\partial}_x^2)f(x,y)}{|x-y|^{n-r}}d\sigma_x.\tag{12}$$

证 在(4)式中,令 $\alpha=2k+1-r$, $l=k$, 则

$$\begin{aligned}\upsilon&=\frac{(\alpha-2l-2)!!(n+\alpha-2l-3)!!}{(\alpha-2)!!(n+1+\alpha)!!}\\&=\frac{(n+\alpha-2l-3)!!}{(n+1+\alpha)!!(\alpha-2)(\alpha-4)\cdots(\alpha-2l)}\\&=\frac{(n+2k+1-r-2k-3)!!}{(n+2k+2-r)!!(2k+1-r-2)(2k+1-r-4)\cdots(2k+1-r-2k)}\\&=\frac{(n-2-r)!!}{(n+2k+2-r)!!(2k-1-r)(2k-3-r)\cdots(1-r)}\\&=\frac{(n-2-r)!!}{(n+2k+2-r)!!(2k-1-r)!!}\\&=\lambda_2.\end{aligned}$$

那么

$$\int_\Omega \frac{(\bar{x}-\bar{y})^2 f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}} = \frac{1-n}{2k+1-r}\lambda_2\int_\Omega \frac{\Delta_x^{k+1} f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n-r}} + \lambda_2\int_\Omega \frac{\Delta_x^k \partial_x^2 f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n-r}},$$

(10)式得证. 类似可得(11)式. 由(10)式与(11)式可得(12)式. 定理 2 证毕. ▌

5　拟 Bochner-Martinelli 型高阶奇异积分的微分公式

定义 6　设 $f(x,y)$在 $D\times D$ 内具有各阶连续的偏导数,连续到边界. 且 $f\in H_x^m(\beta_1)$, $f\in H_y^p(\beta_2)$, $0<\beta_i<1$, $1\leqslant i\leqslant 2$, 则称 $f(x,y)$属于 $H_{x,y}^{(m,p)}(\beta_1,\beta_2)$,记作 $f(x,y)\in H_{x,y}^{(m,p)}(\beta_1,\beta_2)$.

由文[17]知

引理 3　设 $f(x,y)\in H_{x,y}^{(m+2k+2,m)}(\beta_1,\beta_2)$, $0<\beta_i<1$, $i=1,2$. $0\leqslant r<2$, λ_1 如引理 2 所述,则

$$\partial_y^m\int_\Omega \frac{f(x,y)(\bar{x}-\bar{y})\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} = \lambda_1\int_\Omega \frac{[(\partial_y+\partial_x)^m\Delta_x^k\partial_x f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}, \tag{13}$$

$$\partial_y^m\int_\Omega \frac{f(x,y)(x-y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} = \lambda_1\int_\Omega \frac{[(\partial_y+\partial_x)^m\Delta_x^k\bar{\partial}_x f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}, \tag{14}$$

$$\partial_y^m\int_\Omega \frac{f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} = \frac{\lambda_1}{2k+1-r}\int_\Omega \frac{[(\partial_y+\partial_x)^m\Delta_x^{k+1} f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}. \tag{15}$$

定理 3　设 $f(x,y)\in H_{x,y}^{(m+2k+2,m)}(\beta_1,\beta_2)$, $0<\beta_i<1$, $i=1,2$. $0\leqslant r<2$, λ_1 如引理 2 所述,则

$$\begin{aligned}&\partial_y^m\int_\Omega \frac{(\bar{x}-\bar{y})^2 f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}\\ &= \frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1\int_\Omega \frac{(\partial_y+\partial_x)^m\Delta_x^{k+1} f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x\\ &\quad+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega \frac{(\partial_y+\partial_x)^m\Delta_x^k\partial_x^2 f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x,\end{aligned} \tag{16}$$

$$\begin{aligned}&\partial_y^m\int_\Omega \frac{(x-y)^2 f\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}\\ &= \frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1\int_\Omega \frac{(\partial_y+\partial_x)^m\Delta_x^{k+1} f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x\\ &\quad+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega \frac{(\partial_y+\partial_x)^m\Delta_x^k\bar{\partial}_x^2 f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x,\end{aligned} \tag{17}$$

$$\begin{aligned}&\partial_y^m\int_\Omega \left[\frac{(\bar{x}-\bar{y})^2}{|x-y|^{n+2k+4-r}} - \frac{(x-y)^2}{|x-y|^{n+2k+4-r}}\right] f(x,y)\mathrm{d}\sigma_x\\ &= \frac{1}{n+2k+2-r}\lambda_1\int_\Omega \frac{(\partial_y+\partial_x)^m\Delta_x^k(\partial_x^2-\bar{\partial}_x^2) f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x.\end{aligned} \tag{18}$$

证　由定义 3,引理 3 得

$$\begin{aligned}&\partial_y^m\int_\Omega \frac{(\bar{x}-\bar{y})^2 f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}\\ &= \frac{-(n+2k-r)}{n+2k+2-r}\partial_y^m\int_\Omega \frac{f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} + \frac{1}{n+2k+2-r}\partial_y^m\int_\Omega \frac{(\bar{x}-\bar{y})\partial_x f(x,y)}{|x-y|^{n+2k+2-r}}\mathrm{d}\sigma_x\end{aligned}$$

$$+\frac{1}{n+2k+2-r}\partial_y^m\int_\Omega\frac{(x-y)\partial_x f(x,y)}{|x-y|^{n+2k+2-r}}\mathrm{d}\sigma_x$$

$$=\frac{-(n+2k-r)}{n+2k+2-r}\frac{\lambda_1}{2k+1-r}\int_\Omega\frac{[(\partial_y+\partial_x)^m\Delta_x^{k+1}f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega\frac{[(\partial_y+\partial_x)^m\Delta_x^k\partial_x^2 f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega\frac{[(\partial_y+\partial_x)^m\Delta_x^{k+1}f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$=\frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1\int_\Omega\frac{[(\partial_y+\partial_x)^m\Delta_x^{k+1}f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega\frac{[(\partial_y+\partial_x)^m\Delta_x^k\partial_x^2 f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}},$$

(16)式得证. 类似地，由定义 4,引理 3,可证得(17)式. 由(16)式与(17)式可得(18)式. 定理 3 证毕. ▍

由文[17]知

引理 4 设 $f(x,y)\in H_{x,y}^{(m+2k+2,m)}(\beta_1,\beta_2)$, $0<\beta_i<1$, $i=1,2$. $0\leqslant r<2$, λ_1 如引理 2 所述,则

$$\bar\partial_y^m\int_\Omega\frac{f(x,y)(\bar x-\bar y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}}=\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^k\partial_x f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}},\tag{19}$$

$$\bar\partial_y^m\int_\Omega\frac{f(x,y)(x-y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}}=\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^k\bar\partial_x f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}},\tag{20}$$

$$\bar\partial_y^m\int_\Omega\frac{f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}}=\frac{\lambda_1}{2k+1-r}\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^{k+1}f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}.\tag{21}$$

定理 4 设 $f(x,y)\in H_{x,y}^{(m+2k+2,m)}(\beta_1,\beta_2)$, $0<\beta_i<1$, $i=1,2$. $0\leqslant r<2$, λ_1 如引理 2 所述,则

$$\bar\partial_y^m\int_\Omega\frac{(\bar x-\bar y)^2f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}$$

$$=\frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^{k+1}f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^k\partial_x^2 f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}},\tag{22}$$

$$\bar\partial_y^m\int_\Omega\frac{(x-y)^2f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}$$

$$=\frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^{k+1}f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+\frac{1}{n+2k+2-r}\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^k\bar\partial_x^2 f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}},\tag{23}$$

$$\bar\partial_y^m\int_\Omega\Big[\frac{(\bar x-\bar y)^2}{|x-y|^{n+2k+4-r}}-\frac{(x-y)^2}{|x-y|^{n+2k+4-r}}\Big]f(x,y)\mathrm{d}\sigma_x$$

$$=\frac{1}{n+2k+2-r}\lambda_1\int_\Omega\frac{[(\bar\partial_y+\bar\partial_x)^m\Delta_x^k(\partial_x^2-\bar\partial_x^2)f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}.\tag{24}$$

证 由定义 3,引理 4 可得

$$\overline{\partial}_y^m \int_\Omega \frac{(\overline{x}-\overline{y})^2 f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}$$

$$= \frac{-(n+2k-r)}{n+2k+2-r}\overline{\partial}_y^m \int_\Omega \frac{f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} + \frac{1}{n+2k+2-r}\overline{\partial}_y^m \int_\Omega \frac{(\overline{x}-\overline{y})\partial_x f(x,y)}{|x-y|^{n+2k+2-r}}\mathrm{d}\sigma_x$$

$$+ \frac{1}{n+2k+2-r}\overline{\partial}_y^m \int_\Omega \frac{(x-y)\partial_x f(x,y)}{|x-y|^{n+2k+2-r}}\mathrm{d}\sigma_x$$

$$= \frac{-(n+2k-r)}{n+2k+2-r}\frac{\lambda_1}{2k+1-r}\int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m \Delta_x^{k+1} f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+ \frac{1}{n+2k+2-r}\lambda_1 \int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m \Delta_x^k \partial_x^2 f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+ \frac{1}{n+2k+2-r}\lambda_1 \int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m \Delta_x^{k+1} f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$= \frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1 \int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m \Delta_x^{k+1} f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+ \frac{1}{n+2k+2-r}\lambda_1 \int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m \Delta_x^k \partial_x^2 f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}},$$

(22)式得证.类似地，由定义 4,引理 4 可得(23)式.再由(22)式与(23)式可得(24)式.定理 4 证毕. ▌

引理 5　设 $f(x,y)\in H_{x,y}^{(m+2k+2+p,m+p)}(\beta_1,\beta_2)$，$0<\beta_i<1$，$i=1,2$. $0\leqslant r<2$，λ_1 如引理 2 所述，则

$$\overline{\partial}_y^m \partial_y^p \int_\Omega \frac{f(x,y)(\overline{x}-\overline{y})\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} = \lambda_1 \int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^k \partial_x f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}, \tag{25}$$

$$\overline{\partial}_y^m \partial_y^p \int_\Omega \frac{f(x,y)(x-y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} = \lambda_1 \int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^k \overline{\partial}_x f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}, \tag{26}$$

$$\overline{\partial}_y^m \partial_y^p \int_\Omega \frac{f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+2-r}} = \frac{\lambda_1}{2k+1-r}\int_\Omega \frac{[(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^{k+1} f(x,y)]\mathrm{d}\sigma_x}{|x-y|^{n-r}}. \tag{27}$$

定理 5　设 $f(x,y)\in H_{x,y}^{(m+2k+2,m)}(\beta_1,\beta_2)$，$0<\beta_i<1$，$i=1,2$. $0\leqslant r<2$，λ_1 如引理 2 所述，则

$$\overline{\partial}_y^m \partial_y^p \int_\Omega \frac{(\overline{x}-\overline{y})^2 f(x,y)\mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}$$

$$= \frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1 \int_\Omega \frac{(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^{k+1} f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x$$

$$+ \frac{1}{n+2k+2-r}\lambda_1 \int_\Omega \frac{(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^k \partial_x^2 f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x, \tag{28}$$

$$\overline{\partial}_y^m \partial_y^p \int_\Omega \frac{(x-y)^2 f(x,y)}{|x-y|^{n+2k+4-r}}\mathrm{d}\sigma_x$$

$$= \frac{1-n}{(n+2k+2-r)(2k+1-r)}\lambda_1 \int_\Omega \frac{(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^{k+1} f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x$$

$$+ \frac{1}{n+2k+2-r}\lambda_1 \int_\Omega \frac{(\overline{\partial}_y+\overline{\partial}_x)^m(\partial_y+\partial_x)^p \Delta_x^k \overline{\partial}_x^2 f(x,y)}{|x-y|^{n-r}}\mathrm{d}\sigma_x, \tag{29}$$

$$\overline{\partial}_y^m \partial_y^p \int_\Omega [\frac{(\overline{x}-\overline{y})^2}{|x-y|^{n+2k+4-r}} - \frac{(x-y)^2}{|x-y|^{n+2k+4-r}}] f(x,y) \mathrm{d}\sigma_x$$

$$= \frac{1}{n+2k+2-r} \lambda_1 \int_\Omega \frac{(\overline{\partial}_y + \overline{\partial}_x)^m (\partial_y + \partial_x)^p \Delta_x^k (\partial_x^2 - \overline{\partial}_x^2) f(x,y)}{|x-y|^{n-r}} \mathrm{d}\sigma_x. \tag{30}$$

证 由定义 3，引理 5 可得

$$\overline{\partial}_y^m \partial_y^p \int_\Omega \frac{(\overline{x}-\overline{y})^2 f(x,y) \mathrm{d}\sigma_x}{|x-y|^{n+2k+4-r}}$$

$$= \frac{-(n+2k-r)}{n+2k+2-r} \overline{\partial}_y^m \partial_y^p \int_\Omega \frac{f(x,y)}{|x-y|^{n+2k+2-r}} \mathrm{d}\sigma_x$$

$$+ \frac{1}{n+2k+2-r} \overline{\partial}_y^m \partial_y^p \int_\Omega \frac{(\overline{x}-\overline{y}) \partial_x f(x,y)}{|x-y|^{n+2k+2-r}} \mathrm{d}\sigma_x$$

$$+ \frac{1}{n+2k+2-r} \overline{\partial}_y^m \partial_y^p \int_\Omega \frac{(x-y) \partial_x f(x,y)}{|x-y|^{n+2k+2-r}} \mathrm{d}\sigma_x$$

$$= \frac{-(n+2k-r)}{n+2k+2-r} \frac{\lambda_1}{2k+1-r} \int_\Omega \frac{[(\overline{\partial}_y + \overline{\partial}_x)^m (\partial_y + \partial_x)^p \Delta_x^{k+1} f(x,y)] \mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+ \frac{1}{n+2k+2-r} \lambda_1 \int_\Omega \frac{[(\overline{\partial}_y + \overline{\partial}_x)^m (\partial_y + \partial_x)^p \Delta_x^k \partial_x^2 f(x,y)] \mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$+ \frac{1}{n+2k+2-r} \lambda_1 \int_\Omega \frac{[(\overline{\partial}_y + \overline{\partial}_x)^m (\partial_y + \partial_x)^p \Delta_x^{k+1} f(x,y)] \mathrm{d}\sigma_x}{|x-y|^{n-r}}$$

$$= \frac{1-n}{(n+2k+2-r)(2k+1-r)} \lambda_1 \int_\Omega \frac{(\overline{\partial}_y + \overline{\partial}_x)^m (\partial_y + \partial_x)^p \Delta_x^{k+1} f(x,y)}{|x-y|^{n-r}} \mathrm{d}\sigma_x$$

$$+ \frac{1}{n+2k+2-r} \lambda_1 \int_\Omega \frac{(\overline{\partial}_y + \overline{\partial}_x)^m (\partial_y + \partial_x)^p \Delta_x^k \partial_x^2 f(x,y)}{|x-y|^{n-r}} \mathrm{d}\sigma_x,$$

(28)式得证. 类似地，由定义 4，引理 5，可得(29)式. 再由(28)式与(29)式可得(30)式. 定理 5 证毕. ▮

由文[17]知

引理 6 设 $u = u_0 e_0, v(x) = \sum_{i=0}^n v_i(x) e_i \in C^m(R^{n+1})$，$m \geqslant 1, x \in R^{n+1}$，则

$$\overline{\partial}_x (uv) = (\overline{\partial}_x u) v + u (\overline{\partial}_x v), \tag{31}$$

$$\partial_x (uv) = (\partial_x u) v + u (\partial_x v), \tag{32}$$

$$u(\overline{\partial}_x \overline{v} + \partial_x v) = -(\overline{\partial}_x u) \overline{v} - (\partial_x u) v + \partial_x (uv) + \overline{\partial}_x (u\overline{v}). \tag{33}$$

引理 7 设 $x, y \in R^{n+1}, x \neq y$，则

$$\overline{\partial}_x (\overline{x} - \overline{y}) = n+1,\ \partial_x (x-y) = n+1, \tag{34}$$

$$\overline{\partial}_x |x-y|^\sigma = \sigma |x-y|^{\sigma-2} (x-y),\ \partial_x |x-y|^\sigma = \sigma |x-y|^{\sigma-2} (\overline{x}-\overline{y}), \tag{35}$$

$$\overline{\partial}_x [\frac{\overline{x}-\overline{y}}{(-\alpha)|x-y|^{n+1+\alpha}}] = \partial_x [\frac{x-y}{(-\alpha)|x-y|^{n+1+\alpha}}] = \frac{1}{|x-y|^{n+1+\alpha}}, \tag{36}$$

$$\overline{\partial}_x [\frac{1}{(1-n-\alpha)|x-y|^{n-1+\alpha}}] = \frac{x-y}{|x-y|^{n+1+\alpha}},$$

$$\partial_x [\frac{1}{(1-n-\alpha)|x-y|^{n-1+\alpha}}] = \frac{\overline{x}-\overline{y}}{|x-y|^{n+1+\alpha}}. \tag{37}$$

定理 6 设 $f(x,y) \in H_{x,t_2}^{(m+2k+2,p)}(\beta_1, \beta_2)$，$0 < \beta_i < 1, i = 1,2, t_1, t_2 \in \Omega$. $0 \leqslant r < 2$，λ_2 如定

理 2 所述，则

$$\overline{\partial}_{t_1}^{m}\partial_{t_2}^{p}\int_{\Omega}\frac{(\overline{x}-\overline{t}_1)^2 f(x,t_2)}{|x-t_1|^{n+2k+4-r}}\mathrm{d}\sigma_x$$

$$=\frac{\lambda_2(1-n)}{2k+1-r}\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k+1}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x+\lambda_2\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k}\partial_x^{2}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x, \tag{38}$$

$$\overline{\partial}_{t_1}^{m}\partial_{t_2}^{p}\int_{\Omega}\frac{(x-t_1)^2 f(x,t_2)}{|x-t_1|^{n+2k+4-r}}\mathrm{d}\sigma_x$$

$$=\frac{\lambda_2(1-n)}{2k+1-r}\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k+1}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x+\lambda_2\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k}\,\overline{\partial}_x^{2}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x, \tag{39}$$

$$\overline{\partial}_{t_1}^{m}\partial_{t_2}^{p}\int_{\Omega}\left[\frac{(\overline{x}-\overline{t}_1)^2}{|x-t_1|^{n+2k+4-r}}-\frac{(x-t_1)^2}{|x-t_1|^{n+2k+4-r}}\right]f(x,t_2)\mathrm{d}\sigma_x$$

$$=\lambda_2\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k}(\partial_x^{2}-\overline{\partial}_x^{2})f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x. \tag{40}$$

证 由定理 2，引理 6，引理 7，定义 1 可得

$$\overline{\partial}_{t_1}^{m}\partial_{t_2}^{p}\int_{\Omega}\frac{(\overline{x}-\overline{t}_1)^2 f(x,t_2)}{|x-t_1|^{n+2k+4-r}}\mathrm{d}\sigma_x$$

$$=\overline{\partial}_{t_1}^{m}\partial_{t_2}^{p}\left(\frac{1-n}{2k+1-r}\lambda_2\int_{\Omega}\frac{\Delta_x^{k+1}f(x,t_2)\mathrm{d}\sigma_x}{|x-t_1|^{n-r}}+\lambda_2\int_{\Omega}\frac{\Delta_x^{k}\partial_x^{2}f(x,t_2)\mathrm{d}\sigma_x}{|x-t_1|^{n-r}}\right)$$

$$=\frac{\lambda_2(1-n)}{2k+1-r}\overline{\partial}_{t_1}^{m}\int_{\Omega}\frac{\partial_{t_2}^{p}\Delta_x^{k+1}f(x,t_2)\mathrm{d}\sigma_x}{|x-t_1|^{n-r}}+\lambda_2\overline{\partial}_{t_1}^{m}\int_{\Omega}\frac{\partial_{t_2}^{p}\Delta_x^{k}\partial_x^{2}f(x,t_2)\mathrm{d}\sigma_x}{|x-t_1|^{n-r}}$$

$$=\frac{\lambda_2(1-n)}{2k+1-r}\overline{\partial}_{t_1}^{m-1}\int_{\Omega}\frac{[\partial_{t_2}^{p}\Delta_x^{k+1}f(x,t_2)](x-t_1)(n-r)}{|x-t_1|^{n+2-r}}\mathrm{d}\sigma_x$$

$$+\lambda_2\overline{\partial}_{t_1}^{m-1}\int_{\Omega}\frac{[\partial_{t_2}^{p}\Delta_x^{k}\partial_x^{2}f(x,t_2)](x-t_1)(n-r)}{|x-t_1|^{n+2-r}}\mathrm{d}\sigma_x$$

$$=\frac{\lambda_2(1-n)}{2k+1-r}\overline{\partial}_{t_1}^{m-1}\int_{\Omega}\frac{\overline{\partial}_x\partial_{t_2}^{p}\Delta_x^{k+1}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x+\lambda_2\overline{\partial}_{t_1}^{m-1}\int_{\Omega}\frac{\overline{\partial}_x\partial_{t_2}^{p}\Delta_x^{k}\partial_x^{2}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x$$

$$=\frac{\lambda_2(1-n)}{2k+1-r}\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k+1}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x+\lambda_2\int_{\Omega}\frac{\overline{\partial}_x^{m}\partial_{t_2}^{p}\Delta_x^{k}\partial_x^{2}f(x,t_2)}{|x-t_1|^{n-r}}\mathrm{d}\sigma_x,$$

(38)式得证. 类似地，由定理 2，引理 6，引理 7，定义 1 可证得(39)式. 再由(38)式与(39)式可证得(40)式. 定理 6 证毕. ▌

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Quasi-Bochner-Martinelli-Type High Order Singular Integral In Real Clifford Analysis

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Abstract: According to the idea of Hadamard principle value for high order singular integral and the idea of induction, the authors discuss the existence of Hadamard principle value, recursive formula, computation formula and differential formula on the sense of Hadamard principle value for Quasi-Bochner-Martinelli-type high order singular integral in real Clifford analysis.

Key words: Real Clifford analysis; Quasi-Bochner-Martinelli-type high order singular integral; Hadamard principle value.

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