INSTABILITY OF TWO ROTATING VISCOELASTIC (WALTERS B′) SUPERPOSED FLUIDS WITH SUSPENDED PARTICLES IN POROUS MEDIUM

by

Pardeep KUMAR and Mahinder SINGH

Original scientific paper
UDC: 532.546/.584
BIBLID: 0354-9836, *11* (2007), 1, 93-102

The instability of the plane interface between two Walters B′ viscoelastic superposed fluids permeated with suspended particles and uniform rotation in porous medium is considered following the linearized perturbation theory and normal mode analysis. For the stable configuration the system is found to be stable or unstable if $\nu' < or > k_1/\varepsilon$*, depending on kinematic viscoelasticity, permeability of the medium and porosity of the medium. However, the system is found to be unstable for the potentially unstable configuration.*

Key words: *Rayleigh-Taylor instability, Walters B′ viscoelastic fluids, suspended particles, uniform rotation, porous medium*

Introduction

The instability of the plane interface separating two Newtonian fluids when one is accelerated towards the other or when one is superposed over the other has been studied by several authors, and Chandrasekhar [1] has given a detailed account of these investigations. The influence of viscosity on the stability of the plane interface separating two electrically conducting, incompressible superposed fluids of uniform densities, when the whole system is immersed in a uniform horizontal magnetic field, has been studied by Bhatia [2]. Chandra [3] observed a contradiction between the theory for the onset of convection in fluids heated from below and his experiment. He performed the experiment in an air layer and found that the instability depended on the depth of layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection that was different in character from that in deeper layers occurred at much lower gradients than predicted, if the layer depth was less than 7 mm and called columnar instability. He added an aerosol to mark the flow pattern. Motivated by interest in fluid-particle mixtures and columnar instability, Scanlon and Segel [4] studied the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The suspended particles were thus found to destabilize the layer. Palaniswamy and Purushotham [5] have studied the stability of shear flow of stratified fluids with fine dust and found the effects of fine dust to increase the region of instability.

The thermal instability of Maxwellian viscoelastic fluid in the presence of a uniform rotation has been considered by Bhatia and Steiner [6], where rotation is found to have a destabilizing effect. This is in contrast to the thermal instability of a Newtonian fluid where rotation has a stabilizing effect. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation has been studied by Sharma [7]. An experimental demonstration by Toms and Strawbridge [8] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd [9]. There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such fluid is Walters B' elastico-viscous fluid having relevance and importance in geophysical fluid dynamics, chemical technology, and petroleum industry. Walters [10] reported that the mixture of polymethyl methacrylate and pyridine at 25 ºC containing 30.5 g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters B' elastico-viscous fluid. Polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastics engineering equipments, contact lens, *etc*. Walters B' elastico-viscous fluid forms the basis for the manufacture of many such important and useful products.

In recent years, the investigations of flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in the books by Phillips [11], Ingham and Pop [12], and Nield and Bejan [13]. When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result this macroscopic law, the usual viscous term in the equation of Walters B' fluid motion is replaced by the resistance term $(1/k_1)(\mu - \mu' \partial/\partial t)\vec{u}$ where μ and μ' are the viscosity and viscoelasticity of the Walters B' fluid, k_1 is the medium permeability and $\vec{u}$ is the Darcian (filter) velocity of the fluid. The Rayleigh instability of a thermal boundary layer in flow through porous medium has been considered by Wooding [14]. Kumar [15] has studied the stability of two superposed Walters B' viscoelastic fluid-particle mixtures in porous medium. The stability of two superposed Walters B' viscoelastic fluids in the presence of suspended particles and variable magnetic field in porous medium has been studied by Sharma and Kango [16].

The present paper attempts to study the instability of two rotating viscoelastic (Walters B') superposed fluids permeated with suspended particles in porous medium. The knowledge regarding viscoelastic fluid-particle mixtures is not commensurate with their scientific and industrial importance. The analysis would be relevant to the stability of some polymer solutions and the problem finds its usefulness in several geophysical situations and in chemical technology. These aspects form the motivation for the present study.

Formulation of the problem and perturbation equations

Let T_{ij}, τ_{ij}, e_{ij}, δ_{ij}, v_i, x_i, p, μ, and μ' denote the stress tensor, shear stress tensor, rate-of-strain tensor, Kronecker delta, velocity vector, position vector, isotropic pressure,

viscosity, and viscoelasticity, respectively. The constitutive relations for the Walters B' viscoelastic fluid are:

$$\begin{aligned} T_{ij} &= -p\delta_{ij} + \tau_{ij} \\ \tau_{ij} &= 2\mu' - \mu\frac{\partial}{\partial t}e_{ij} \\ e_{ij} &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \end{aligned} \tag{1}$$

Consider a static state in which an incompressible Walters B' viscoelastic fluid permeated with suspended particles is arranged in horizontal strata in a porous medium and the pressure p and density ρ are functions of the vertical coordinate z only. A uniform rotation $\vec{\Omega}(0, 0, \Omega)$ pervades the whole system. The character of the equilibrium of this initial static state is determined, as usual, by supposing that the system is slightly disturbed and then by following its further evolution.

Let ρ, p, $\vec{u}(u, v, w)$, and $\vec{\Omega}(0, 0, \Omega)$ denote, respectively, the density, the pressure, the velocity of pure fluid, and the uniform rotation; $\vec{v}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of the particles, respectively. $K = 6\pi\mu\eta$, where η is the particle radius, is the Stokes drag coefficient, $\vec{v} = (l, r, s)$, $\bar{x} = (x, y, z)$, and $\vec{\lambda} = (0, 0, 1)$. Let ε, k_1, and g stand for medium porosity, medium permeability, and acceleration due to gravity, respectively. Then the equations of motion and continuity for the rotating Walters B' viscoelastic fluid containing suspended particles in a porous medium are:

$$\begin{aligned} &\frac{\rho}{\varepsilon}\frac{\partial\vec{u}}{\partial t} + \frac{1}{\varepsilon}(\vec{u}\cdot\nabla)\vec{u} = -\nabla p - \frac{\rho}{2}\nabla\left|\vec{\Omega}\times\vec{x}\right|^2 + \rho g\vec{\lambda} \\ &\quad + \frac{KN}{\varepsilon}(\vec{v}-\vec{u}) + \frac{2\rho}{\varepsilon}(\vec{u}\times\vec{\Omega}) - \frac{1}{k_1}\mu\vec{u} - \mu'\frac{\partial}{\partial t}\vec{u} \end{aligned} \tag{2}$$

$$\nabla\cdot\vec{u} = 0 \tag{3}$$

Since the density of every fluid particle remains unchanged as we follow it with its motion, we have:

$$\varepsilon\frac{\partial\rho}{\partial t} + (\vec{u}\cdot\nabla)\rho = 0 \tag{4}$$

In the equations of motion (2), by assuming a uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term proportional to the velocity difference between the particles and the fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion of the particles. The distances between particles are assumed quite large compared with their diameter so that interparticle

reactions are ignored. The effects of pressure, gravity, and Darcian force on the suspended particles are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions are:

$$mN \frac{\partial \vec{v}}{\partial t} \quad \frac{1}{\varepsilon}(\vec{v} \quad)\vec{v} \quad KN(\vec{u} \quad \vec{v}) \tag{5}$$

$$\varepsilon \frac{\partial N}{\partial t} \quad (N\vec{v}) \quad 0 \tag{6}$$

Initially, density = $\rho(z)$, pressure = $p(z)$, fluid velocity = (0, 0, 0) and particle velocity = (0, 0, 0).

Let $\delta\rho, \delta p, \vec{u}(u, v, w)$, and $\vec{v}(l, r, s)$ denote, respectively, the perturbations in fluid density ρ, fluid pressure p, fluid velocity (0, 0, 0), and particle velocity (0, 0, 0).

Therefore, after perturbations, we have:

$$\rho(z) \quad \delta\rho(x, y, z, t), \quad p(z) \quad \delta p(x, y, z, t),$$
$$(0,0,0) \quad \vec{u}[u(x, y, z, t), v(x, y, z, t), w(x, y, z, t)] \text{ and}$$
$$(0,0,0) \quad \vec{v}[l(x, y, z, t), r(x, y, z, t), s(x, y, z, t)]$$

Substituting these values into eqs. (2)-(6), using initial values, we get the linearized perturbation equations of the fluid-particle layer as:

$$\frac{\rho}{\varepsilon}\frac{\partial \vec{u}}{\partial t} \quad \delta p \quad \vec{g}\,\delta\rho \quad \frac{KN}{\varepsilon}(\vec{v} \quad \vec{u}) \quad \frac{2\rho}{\varepsilon}(\vec{u} \quad \vec{\Omega}) \quad \frac{1}{k_1} \quad \mu \quad \mu \frac{\partial}{\partial t} \quad \vec{u} \tag{7}$$

$$\vec{u} \quad 0 \tag{8}$$

$$\varepsilon \frac{\partial}{\partial t}(\delta\rho) \quad w(D\rho) \tag{9}$$

$$\frac{m}{K}\frac{\partial}{\partial t} \quad 1 \ \vec{v} \quad \vec{u} \tag{10}$$

and

$$\frac{\partial M}{\partial t} \quad \vec{v} \quad 0 \tag{11}$$

where $M = \varepsilon N N$, N_0 and N stand for initial uniform number density perturbation in number density, respectively, and $D = \mathrm{d}/\mathrm{d}z$.

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, and t is given by:

$$e^{(ik_x x \ \ ik_y y \ \ nt)} \tag{12}$$

where k_x and k_y are wave numbers along x and y-directions, $k^2 \quad k_x^2 \quad k_y^2$, and n is, in general, a complex constant.

For perturbations of the form (12), eqs. (7)-(10) after eliminating $\vec{v}$ give:

$$\frac{1}{\varepsilon}\ \rho\quad \frac{mN}{\tau n\quad 1}\ nu\qquad ik_x\delta p\quad \frac{1}{k_1}(\mu\quad \mu\ n)u\quad \frac{2\rho\Omega}{\varepsilon}v \tag{13}$$

$$\frac{1}{\varepsilon}\ \rho\quad \frac{mN}{\tau n\quad 1}\ nv\qquad ik_y\delta p\quad \frac{1}{k_1}(\mu\quad \mu\ n)v\quad \frac{2\rho\Omega}{\varepsilon}u \tag{14}$$

$$\frac{1}{\varepsilon}\ \rho\quad \frac{mN}{\tau n\quad 1}\ nw\qquad D\delta p\quad g\,\delta\rho\quad \frac{1}{k_1}(\mu\quad \mu\ n)w \tag{15}$$

$$ik_x u + ik_y v + Dw = 0 \tag{16}$$

and

$$\varepsilon n\delta\rho = -wD\rho \tag{17}$$

where τ= m/K.

Eliminating δp between eqs. (13)-(15) and using eqs. (16) and (17), we obtain:

$$(\tau n\quad 1)n\quad \frac{\varepsilon}{k_1}(\tau n\quad 1)(\nu\quad \nu\ n)\ [D(\rho Dw)\quad k^2\rho w]\quad n[D(mNDw)\quad mNk^2 w]$$

$$\frac{gk^2(\tau n\quad 1)}{n}\ (D\rho)w\quad 4(\tau n\quad 1)^2\Omega^2\quad D\ \frac{\rho Dw}{(\tau n\quad 1)n\quad \frac{mnN}{\rho}\quad \frac{\varepsilon}{k_1}(\tau n\quad 1)(\nu\quad \nu\ n)}\qquad 0 \tag{18}$$

where $\nu = \mu/\rho$ and $\nu' = \mu'/\rho$ stand for kinematic viscosity and kinematic viscoelasticity.

Two uniform viscoelastic (Walters B') fluids separated by a horizontal boundary

Consider the case of two uniform Walters B' viscoelastic fluids of densities, viscosities, viscoelasticities, suspended particles number densities as $\rho_2, \mu_2, \mu_2, N_2$, and ρ_1, μ, μ_1, N_1 separated by a horizontal boundary at $z = 0$. The subscripts 1 and 2 distinguish the lower and the upper fluids, respectively.

Then in each region of constantρ, constant μ, constant μ', and constant mN, eq. (18) reduces to:

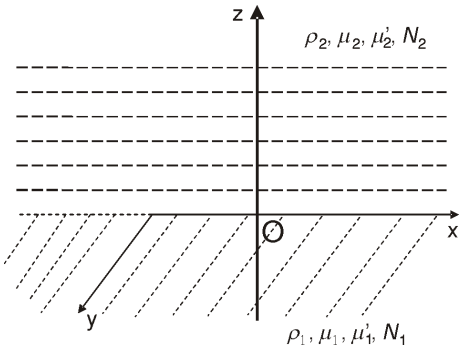
$$(D^2 - \kappa^2)w = 0 \tag{19}$$

where

$$\kappa\quad \frac{k}{\sqrt{1\quad \frac{2\Omega^2(\tau n\quad 1)^2}{n(\tau n\quad 1)\quad \frac{mnN}{\rho}\quad \frac{\varepsilon}{k_1}(\tau n\quad 1)(\nu\quad \nu\ n}}^{\ 2}}$$

and in case of highly viscous and viscoelastic fluid:

$$\kappa \quad \frac{k}{1 \quad \dfrac{2\Omega^2(\tau n \quad 1)^2}{n(\tau n \quad 1) \quad \dfrac{mnN}{\rho} \quad \dfrac{\varepsilon}{k_1}(\tau n \quad 1)(\nu \quad \nu n}} \qquad (20)$$



The general solution of eq. (20) is:

$$w = Ae^{+\kappa z} + Be^{-\kappa z} \qquad (21)$$

where A and B are arbitrary constants. The boundary conditions to be satisfied in the present problem are:

(1) the velocity w should vanish when $z \quad +\infty$ (for the upper fluid) and $z \quad -\infty$ (for the lower fluid),
(2) $w(z)$ is continuous at $z = 0$, and
(3) the pressure should be continuous across the interface.

Applying the boundary conditions (1) and (2), we have:

$$w_1 = Ae^{+\kappa z} \qquad (z < 0) \qquad (22)$$

$$w_2 = Ae^{-\kappa z} \qquad (z > 0) \qquad (23)$$

where the same constant A has been chosen to ensure the continuity of w at $z = 0$.

Here we assumed the kinematic viscosities and kinematic viscoelasticities of both fluids to be equal *i. e.* $\nu_1 = \nu_2 = \nu$ (Chandrasekhar [1], p. 443), $\nu_1 \quad \nu_2 \quad \nu$ and $mN/\rho = mN_1/\rho_1 = mN_2/\rho_2 \ (= M)$ as these simplifying assumptions do not obscure any of the essential features of the problem.

Integrating eq. (18) across the interface $z = 0$, we obtain the boundary condition:

$$n(\tau n \quad 1) \quad \frac{\varepsilon}{k_1}(\tau n \quad 1)(\nu \quad \nu n)\ \Delta_0(\rho Dw) \quad n\Delta_0(mNDw) \quad (\tau n \quad 1)\frac{g\,k^2}{n}\Delta_0(\rho)w_0$$
$$\frac{4(\tau n \quad 1)^2\Omega^2}{n(\tau n \quad 1) \quad \dfrac{mnN}{\rho} \quad \dfrac{\varepsilon}{k_1}(\tau n \quad 1)(\nu \quad \nu n)}\Delta_0(\rho Dw) \quad 0 \qquad (24)$$

where w_0 is the common value of w at $z = 0$ and $\ _0(f)$ is the jump which a quantity f experiences at the interface $z = 0$.

Applying the boundary condition (24) to the solutions (22) and (23), we obtain:

$$1 \quad \frac{mn(N_1 \quad N_2)}{(\rho_1 \quad \rho_2)\, n(\tau n \quad 1) \quad \frac{}{k_1}(\tau n \quad 1)(\nu \quad \nu n)} \quad \frac{(\tau n \quad 1)\mathrm{g}\, k^2(\alpha_2 \quad \alpha_1)}{n\kappa\, n(\tau n \quad 1) \quad \frac{}{k_1}(\tau n \quad 1)(\nu \quad \nu n)}$$

$$\frac{4(\tau n \quad 1)^2 \Omega^2}{n(\tau n \quad 1) \quad Mn \quad \frac{\varepsilon}{k_1}(\tau n \quad 1)(\nu \quad \nu n) \quad n(\tau n \quad 1) \quad \frac{\varepsilon}{k_1}(\tau n \quad 1)(\nu \quad \nu n)} \quad 0 \qquad (25)$$

where $\alpha_1 = \rho_1/(\rho_1 + \rho_2)$ and $\alpha_2 = \rho_2/(\rho_1 + \rho_2)$

Equation (25), after substituting the value of κ from eq. (20) and simplification, yields:

$$A_7 n^7 \quad A_6 n^6 \quad A_5 n^5 \quad \ldots\ldots \quad A_2 n^2 \quad A_1 n \quad A_0 \quad 0 \qquad (26)$$

where

$$A_7 \quad \tau^3 \quad 1 \quad \frac{\varepsilon\nu}{k_1}^3$$

$$A_6 \quad 2M \quad \frac{3\varepsilon\nu\tau}{k_1} \quad 3 \quad 1 \quad \frac{\varepsilon\nu}{k_1} \quad \frac{m(N_1 \quad N_2}{\rho_1 \quad \rho_2}\, \tau^2 \quad 1 \quad \frac{\varepsilon\nu}{k_1}^2$$

$$A_5 \quad 1 \quad \frac{\varepsilon\nu}{k_1} \quad 4M \quad \frac{9\varepsilon\nu\tau}{k_1} \quad \tau^2 \mathrm{g}\, k(\alpha_2 \quad \alpha_1) \quad \frac{2m(N_1 \quad N_2)}{\rho_1 \quad \rho_2} \quad M \quad \frac{\varepsilon\nu\tau}{k_1}$$

$$1 \quad \frac{\varepsilon\nu}{k_1} \quad 3 \quad 1 \quad \frac{\varepsilon\nu}{k_1}^2 \quad M \quad \frac{\varepsilon\nu\tau}{k_1} \quad M \quad \frac{3\varepsilon\nu\tau}{k_1} \quad \tau \quad 1 \quad \frac{\varepsilon\nu}{k_1}$$

$$A_4 \quad 1 \quad \frac{\varepsilon\nu}{k_1} \quad M \quad \frac{\varepsilon\nu\tau}{k_1} \quad \frac{5\varepsilon\nu\tau}{k_1} \quad M \quad 2\tau^2 \mathrm{g}\, k(\alpha_2 \quad \alpha_1) \quad 2 \quad M \quad \frac{2\varepsilon\nu\tau}{k_1}$$

$$\frac{\varepsilon\nu\tau}{k_1} \quad \frac{m(N_1 \quad N_2)}{\rho_1 \quad \rho_2} \quad 1 \quad \frac{\varepsilon\nu}{k_1}^2 \quad \frac{9\varepsilon\nu\tau}{k_1} \quad 2M \quad \frac{m(N_1 \quad N_2)}{\rho_1 \quad \rho_2} \quad 3\tau^2 \mathrm{g}\, k(\alpha_2 \quad \alpha_1)$$

$$1 \quad \frac{\varepsilon\nu}{k_1}^3 \quad M \quad \frac{\varepsilon\nu\tau}{k_1}^2 \quad \frac{\varepsilon\nu\tau}{k_1} \quad \frac{m(N_1 \quad N_2)}{\rho_1 \quad \rho_2}$$

$$A_3 \quad 1 \quad \frac{\varepsilon\nu}{k_1} \quad \frac{\varepsilon\nu}{k_1} \quad \frac{9\varepsilon\nu\tau}{k_1} \quad \frac{4M}{k_1} \quad \frac{2m(N_1 \quad N_2)}{\rho_1 \quad \rho_2} \quad 6\tau^2 \mathrm{g}\, k(\alpha_2 \quad \alpha_1)$$

$$4\tau \mathrm{g}\, k(\alpha_2 \quad \alpha_1) M \quad 3 \quad 1 \quad \frac{\varepsilon\nu}{k_1}^2 \quad \frac{\varepsilon\nu}{k_1} \quad \tau \mathrm{g}\, k(\alpha_2 \quad \alpha_1) \quad M \quad \frac{\varepsilon\nu\tau}{k_1} \quad \frac{\varepsilon\nu}{k_1} \quad \frac{3\varepsilon\nu\tau}{k_1}$$

$$\frac{M}{k_1} \quad \frac{2m(N_1 \quad N_2)}{\rho_1 \quad \rho_2} \quad \tau \mathrm{g}\, k(\alpha_2 \quad \alpha_1) \quad M \quad \frac{\varepsilon\nu\tau}{k_1} \quad 2\Omega^2 \tau^3 \mathrm{g}\, k(\alpha_2 \quad \alpha_1)$$

$$A_2 \quad 1 \quad \frac{\varepsilon\nu}{k_1} \quad \frac{3\varepsilon^2\nu^2}{k_1^2} \quad 2\mathrm{g}k(\alpha_2 \quad \alpha_1)\,\frac{3\varepsilon\nu\tau}{k_1} \quad M \quad \mathrm{g}k(\alpha_2 \quad \alpha_1)\,1 \quad \frac{\varepsilon\nu}{k_1}^2$$

$$\frac{\varepsilon^2\nu^2}{k_1^2} \quad \frac{3\varepsilon\nu\tau}{k_1} \quad 2M \quad \frac{m(N_1 \quad N_2)}{\rho_1 \quad \rho_2} \quad \mathrm{g}k(\alpha_2 \quad \alpha_1)\,M \quad \frac{\varepsilon\nu\tau}{k_1}$$

$$\frac{3\varepsilon\nu\tau}{k_1} \quad M \quad 6\tau^2\mathrm{g}k(\alpha_2 \quad \alpha_1)\Omega^2$$

$$A_1 \quad \frac{\varepsilon^2\nu^2}{k_1^2} \quad \frac{\varepsilon\nu}{k_1} \quad \tau\mathrm{g}\,k(\alpha_2 \quad \alpha_1) \quad 6\tau\mathrm{g}\,k(\alpha_2 \quad \alpha_1)\Omega^2$$

$$\frac{2\varepsilon\nu\mathrm{g}k(\alpha_2 \quad \alpha_1)}{k_1} \quad 1 \quad \frac{\varepsilon\nu}{k_1} \quad M \quad \frac{\varepsilon\nu\tau}{k_1}$$

$$A_0 \quad \mathrm{g}k(\alpha_2 \quad \alpha_1)\,\frac{\varepsilon^2\nu^2}{k_1^2} \quad 2\Omega^2 \tag{27}$$

Discussion

(a) *Stable case* ($\alpha_2 < \alpha_1$)

For the potentially stable case ($\alpha_2 < \alpha_1$), if:

$$1 \quad \frac{\varepsilon\nu}{k_1} \quad i.\ e. \quad \nu \quad \frac{k_1}{\varepsilon} \tag{28}$$

eq. (26) does not allow any change of sign and so has no positive root. The system is therefore stable. But if:

$$\nu \quad \frac{k_1}{\varepsilon} \tag{29}$$

the coefficient of n^7 *i. e.* A_7 in eq. (26) is negative. Equation (26), therefore, allows at least one change of sign and hence one positive root. The occurrence of a positive root implies that the system is unstable.

(b) *Unstable case* ($\alpha_2 > \alpha_1$)

For the potentially unstable case ($\alpha_2 > \alpha_1$), the constant term A_0 in eq. (26) is negative. Equation (26), therefore, allows one change of sign and so has one positive root and hence the system is unstable.

Thus for the stable case ($\alpha_2 < \alpha_1$), the system is unstable or stable depending on kinematic viscoelasticity (assumed equal for both fluids) whether it is greater than or smaller than the medium permeability divided by medium porosity. However, the system

is unstable for unstable configuration. Also, it is clear from eq. (26) that suspended particles and rotation effects do not affect the stability or instability of the system.

Nomenclature

g – acceleration due to gravity, [ms^{-2}]
$\vec{g}$ – gravity field, [ms^{-2}]
K – Stokes' drag coefficient, [kgs^{-1}]
k – wave-number, [m^{-1}]
k_x, k_y – horizontal wave-numbers, [m^{-1}]
k_1 – medium permeability, [m^2]
m – mass of single particle, [g]
N – suspended particle number density, [m^{-3}]
n – growth rate, [s^{-1}]
p – fluid pressure, [Pa]
t – time, [s]
$\vec{u}$ – fluid velocity, [ms^{-1}]
$\vec{v}$ – suspended particle velocity, [ms^{-1}]

Greek letters

ε – medium porosity , [–]
μ – dynamic viscosity, [kgm^{-1}s^{-1}]
ν – kinematic viscosity, [m^{2}s^{-1}]
ν' – kinematic viscoelasticity, [m^{2}s^{-1}]
ρ – density, [kgm^{-3}]

Acknowledgments

The authors are grateful to the referee for his critical comments, which led to a significant improvement of the paper.

References

[1] Chandrasekhar, S., Hydrodynamic and Hydromagnetic Stability, Dover Publications, New York, USA, 1981
[2] Bhatia, P. K., Rayleigh-Taylor Instability of Two Viscous Superposed Conducting Fluids, *Nuov. Cim., 19 B* (1974), pp. 161-168
[3] Chandra , K., Instability of Fluids Heated from Below, *Proc. Roy. Soc. London, A164* (1938), pp. 231-242
[4] Scanlon, J. W., Segel, L. A., Some Effects of Suspended Particles on the onset of Be'nard Convection, *Phys. Fluids, 16* (1973), 10, pp. 1573-1578
[5] Palaniswamy, V. I., Purushotham, C. M., Stability of Shear Flow of Stratified Fluids with Fine Dust, *Phys. Fluids, 24* (1981), pp. 1224-1229

[6] Bhatia, P. K., Steiner, J. M., Convective Instability in a Rotating Viscoelastic Fluid Layer, *Z. Angew Math. Mech., 52* (1972), pp. 321-324
[7] Sharma, R. C., Effect of Rotation on Thermal Instability of a Viscoelastic Fluid, *Acta Physica Hung., 40* (1976), 1, pp. 11-17
[8] Toms, B. A., Strawbridge, D. J., Elastic and Viscous Properties of Dilute Solutions of Polymethyl Methacrylate in Organic Liquids, *Trans. Faraday Soc., 49* (1953), pp. 1225-1232
[9] Oldroyd, J. G., Non-Newtonian Effects in Steady Motion of Some Idealized Elastico-Viscous Liquids, *Proc. Roy. Soc. London, A 245* (1958), pp. 278-297
[10] Walters, K., The Solution of Flow Problems in Case of Materials with Memory, *J. Mecanique, 1* (1962), pp. 469-479
[11] Phillips, O. M., Flow and Reaction in Permeable Rocks, Cambridge University Press, Cambridge, UK, 1991
[12] Ingham, D. B., Pop, I., Transport Phenomena in Porous Medium, Pergamon Press, Oxford, UK, 1998
[13] Nield, D. A., Bejan, A., Convection in Porous Medium (2nd ed.), Springer Verlag, New York, USA, 1999
[14] Wooding, R. A., Rayleigh Instability of a Thermal Boundary Layer in Flow through a Porous Medium, *J. Fluid Mech., 9* (1960), pp. 183-192
[15] Kumar, P., Stability of Two Superposed Viscoelastic (Walters B') Fluid-Particle Mixture in Porous Medium, *Z. Naturforsch., 54a* (1998), pp. 343-347
[16] Sharma, R. C., Kango, S. K., Stability of Two Superposed Walters B' Viscoelastic Fluids in the Presence of Suspended Particles and Variable Magnetic Field in Porous Medium, *Appl. Mech. Engng., 4* (1999), pp. 255-269

Authors' address:

P. Kumar, M. Singh
Department of Mathematics, ICDEOL
Himachal Pradesh University
Summer Hill, SHIMLA-5, India

Corresponding author P. Kumar
E-mail: drpardeep@sancharnet.in or pkdureja@gmail.com

Paper submitted: August 3, 2006
Paper revised: March 6, 2007
Paper accepted: March 10, 2007