EFFECT OF PRANDTL NUMBER ON HEAT TRANSFER CHARACTERISTICS IN AN AXISYMMETRIC SUDDEN EXPANSION: A NUMERICAL STUDY

by

Khalid N. Alammar

Original scientific paper
UDC: 532.517.4:536.255:66.011
BIBLID: 0354-9836, *11* (2007), 4, 171-178

Using the standard k-ε turbulence model, an incompressible, axisymmetric turbulent flow with a sudden expansion was simulated. Effect of Prandtl number on heat transfer characteristics downstream of the expansion was investigated. The simulation revealed circulation downstream of the expansion. A secondary circulation (corner eddy) was also predicted. Reattachment was predicted at approximately 10 step heights. Corresponding to Prandtl number of 7.0, a peak Nusselt number 13 times the fully-developed value was predicted. The ratio of peak to fully-developed Nusselt number was shown to decrease with decreasing Prandtl number. Location of maximum Nusselt number was insensitive to Prandtl number.

Key words: *turbulence, Prandtl number, heat transfer, expansion*

Introduction

The axisymmetric sudden expansion flow is encountered in numerous applications, including orifices, burners, and industrial duct and pipe systems. From a fundamental point of view, it is one the most popular flows involving separation and reattachment. Many studies have been conducted to examine physics of the flow. For example, it has been shown that with increasing expansion ratio, turbulence intensity downstream of the expansion increases and the reattachment length moves downstream of the expansion [1]. So [2] has reported that inlet Reynolds number has little or no effect on reattachment length. Shahnam and Morris [3] have reported that a decrease in the expansion ratio increases the circulation region. Laser-Doppler velocymeter (LDV) measurements of Durrett *et al*. [4] have clearly identified a weak secondary recirculation zone (corner eddy).

As pertaining to the heat transfer aspect of the flow, the experimental work of Baughn *et al*. [5] has shown that the local Nusselt number in the separated, reattached, and redevelopment regions were up to 9 times higher than those for fully-developed flows in pipes having the same diameter as the pipe downstream. Maximum and average heat transfer enhancements were shown to increase strongly with decreasing expansion ratio. Location of the maximum Nusselt number moved downstream as the expansion ra-

DOI:10.2298/TSCI0704171A

tio increased, and unlike maximum normalized turbulence intensity, which was insensitive to Reynolds number, the peak values of Nusselt number correlated fairly well with the Reynolds number based on the upstream pipe diameter irrespective of the expansion ratio. For a given expansion ratio, they reported that the ratio of peak to fully-developed Nusselt number decreases as the Reynolds number increases. They also reported that a minimum Nusselt number takes place around one step height downstream of the expansion for expansion ratios below 0.5, suggesting the presence of a secondary circulation.

Said *et al*. [6] have numerically investigated flow and heat transfer characteristics of pulsating flow downstream of an abrupt expansion for different frequencies (5 to 35 Hz), Reynolds numbers (5 and $10 \cdot 10^5$), expansion ratios (0.2 and 0.6), and Prandtl numbers (0.7 and 7.0). They reported that the influence of pulsation on the time-averaged Nusselt number is insignificant for fluids having a Prandtl number less than unity, while the increase is around 30% for fluids having a Prandtl number greater than unity. For all pulsating frequencies, the variation in the time-averaged Nusselt number, maximum Nusselt number, and its location with Reynolds number and diameter ratio exhibit similar characteristics to steady flows.

Evidently, the axisymmetric sudden expansion flow has been receiving much attention. Nonetheless, there still remain some aspects of the flow, in particular, the heat transfer aspect that needs to be investigated. For example, no work has been reported on the effect of Prandtl number on heat transfer characteristics downstream of the expansion over a wide range. In this work, therefore, experiment of Baughn *et al*. [5] was numerically extended to study such effect. An incompressible, axisymmetric, steady turbulent flow with a sudden expansion was simulated. Prandtl numbers in the range between 0.2 and 7.0 were investigated. The wall heat flux was fixed at 0.3 W/m^2, the expansion ratio (*d/D*) was 0.4, and the Reynolds number was 40,750. Fully-developed conditions were invoked prior to expansion. Schematic of the sudden expansion is shown in fig. 1.

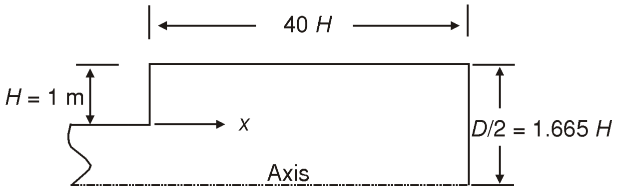

Figure 1. Schematic of the axisymmetric sudden expanison

The mathematical model

The mathematical model consisted of the following steady Reynolds-averaged conservation equations in Cartesian tensor form.

Continuity equation

$$\frac{\partial (\rho U_i)}{\partial x_i} \quad 0 \tag{1}$$

Momentum equations

$$\frac{\partial(\rho U_i U_j)}{\partial x_j} \qquad \frac{\partial p}{\partial x_i} \quad \frac{\partial}{\partial x_j} \mu \; \frac{\partial U_i}{\partial x_j} \quad \frac{\partial U_j}{\partial x_i} \quad \frac{2}{3}\delta_{ij} \frac{\partial U_i}{\partial x_i} \qquad \frac{\partial}{\partial x_j}(\quad \rho\overline{u_i u_j}) \tag{2}$$

For the Reynolds stresses, the Boussinesq hypothesis [7] was invoked:

$$\rho\overline{u_i u_j} \quad \mu_{\mathrm{t}} \; \frac{\partial U_i}{\partial x_j} \quad \frac{\partial U_j}{\partial x_i} \quad \frac{2}{3} \; \rho k \quad \mu_{\mathrm{t}} \frac{\partial U_i}{\partial x_i} \; \delta_{ij} \tag{3}$$

Energy equation

$$\frac{\partial[U_i(\rho E \quad p)]}{\partial x_i} \quad \frac{\partial}{\partial x_j} \quad k \quad \frac{c_{\mathrm{p}}\mu_{\mathrm{t}}}{\mathrm{Pr}_{\mathrm{t}}} \quad \frac{\partial T}{\partial x_j} \tag{4}$$

Turbulence model

The standard *k*-*ε* turbulence model [8] was used. The model is well suited for high Reynolds number internal flows and has been validated for many industrial flows.

$$\frac{\partial(\rho k U_i)}{\partial x_i} \quad \frac{\partial}{\partial x_j} \quad \mu \quad \frac{\mu_{\mathrm{t}}}{\sigma_{\mathrm{k}}} \; \frac{\partial k}{\partial x_j} \qquad G_{\mathrm{k}} \quad \rho\varepsilon \tag{5}$$

$$\frac{\partial(\rho\varepsilon U_i)}{\partial x_i} \quad \frac{\partial}{\partial x_j} \quad \mu \quad \frac{\mu_t}{\sigma_\varepsilon} \; \frac{\partial \varepsilon}{\partial x_j} \qquad C_{1\varepsilon}\frac{\varepsilon}{k}G_k \quad C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} \tag{6}$$

Production of turbulence kinetic energy is given by:

$$G_{\mathrm{k}} \qquad \rho\overline{u_i u_j}\,\frac{\partial U_j}{\partial x_i} \tag{7}$$

And the Eddy viscosity is:

$$\mu_{\mathrm{t}} \quad \rho C_\mu \frac{k^2}{\varepsilon} \tag{8}$$

Wall treatment

The non-equilibrium wall function by Kim and Choudhury [9] was implemented. For more details, the reader is referred to Fluent 6.2 user guide.

The numerical procedure

Fluent 6.2 was used as the solver. The structured grid was built using Gambit 2.0. The mesh consisted of approximately 30,000 structured cells. The simulation was carried out using SIMPLE [10] and second-order schemes. The linearized equations were solved using Gauss-Seidel method, in conjunction with an algebraic Multigrid scheme [11]. Due to small overall temperature changes in the fluid, properties were assumed constant. The heat transfer coefficient was calculated using the bulk temperature as defined in experiment of Baughn *et al.* [5].

Uncertainty analysis

There are mainly two sources of uncertainty in Computer fluid dinamics, namely modeling and numerical [12]. Modeling uncertainty is approximated through experimental validation while numerical uncertainty can be approximated through grid independence. Numerical uncertainty has two main sources, namely truncation and round-off errors. Higher order schemes have less truncation error, and as was outlined earlier, the discretization schemes invoked were second-order. In explicit schemes, round-off error increases with increasing iterations, and is reduced by increasing significant digits (machine precision). However, having used Gauss-Seidel iterative procedure in a steady-state simulation renders the calculation insensitive to round-off error.

A comparison between the current simulation and experiment of Baughn *et al.* [5] is depicted in fig. 2. Here, the Nusselt number has been normalized by the fully-developed value obtained using Dittus-Boelter correlation [13]. This normalization was used in the experiment of Baughn *et al.*, and is intended to compare the local Nusselt number in presence of expansion with that of a fully-developed straight pipe. The numerical prediction with 30,000 cells is in good agreement with experimental data which was reported with an error of 5%. Therefore, we assume the modeling uncertainty to be 5%. The numerical error is approximated by the difference between 20,000 and 30,000 cells, which is about 2%. Hence, we conclude that the overall uncertainty is determined by the modeling uncertainty of 5%.

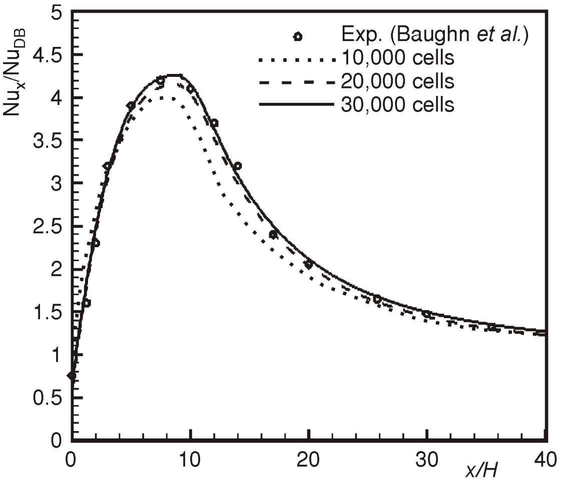

Figure 2. Normalized Nusselt number

Results and discussion

Distribution of the normalized turbulence kinetic energy near the wall, along with the friction coefficient is depicted in fig. 3. For incompressible flows, flow parameters are independent of the Prandtl number. Hence, only one case is shown here, namely Pr = = 0.7. Maximum turbulence kinetic energy is shown to occur at 7 *H* (step height) downstream of the expansion, whereas reattachment is around 10 *H*. Maximum friction coefficient is shown to take place at 4 *H*, which is before maximum turbulence, suggesting that turbulence is not the major contributor to shear stress in the circulation region.

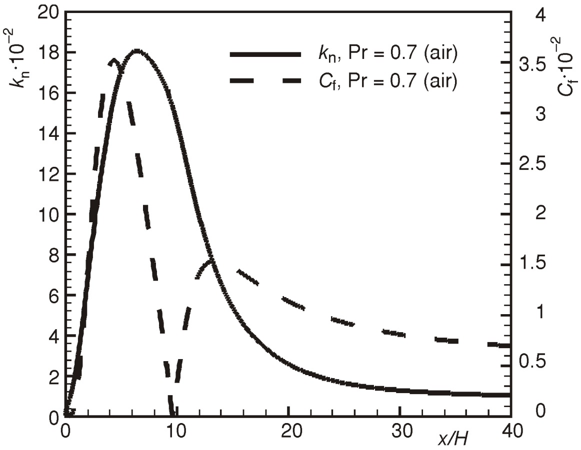

Figure 3. Distribution of the normalized turbulence kinetic energy near the wall, along with the friction coefficient downstream of the expansion

Nusselt number distribution downstream of the expansion is depicted in fig. 4. Including the experimental data of Baughn *et al.* [5] for air, the profiles cover Prandtl number range from 0.2 to 7.0. Nusselt number profiles are shown to increase with increasing Prandtl number. For Pr = 0.7, maximum Nusselt number takes place around 7.0 *H*, which is the same location for maximum turbulence, suggesting that turbulence plays a major role in enhancing heat transfer. The ratio of peak to fully-developed Nusselt number is predicted to decrease with decreasing Prandtl number. This is due to increasing conduction role relative to turbulence and mean flow convection.

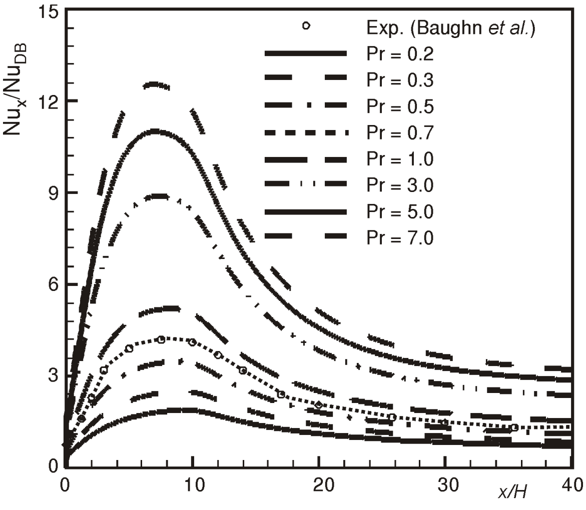

Figure 4. Nusselt number distribution downstream of the expanison

The streamlines downstream of the expansion are depicted in fig. 5. Manifested in closer streamline spacing, velocities are relatively higher at the beginning of expansion, while they decrease toward the separated region where the streamlines are far apart. Circulation is shown to expand well into the large pipe. By entraining mainstream into the boundary layer, circulation enhances mixing, and hence heat transfer downstream of the expansion. While a secondary circulation (corner eddy) has been predicted in this simulation, the minimum Nusselt number at the corner reported by Baughn *et al.* [5] has not been resolved. This suggests that a finer mesh maybe needed to resolve the secondary circulation more accurately, for which one would need to resort to higher-order turbulence models.

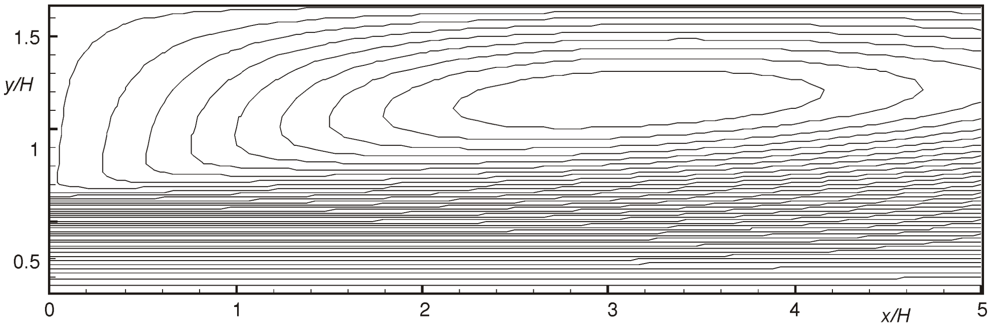

Figure 5. Streamlines downstream of the expansion

Conclusions

Using the standard k-ε turbulence model with a non-equilibrium wall function, an incompressible, axisymmetric turbulent flow with a sudden expansion was simulated. Effect of Prandtl number on heat transfer characteristics downstream of the expansion was investigated. The simulation revealed circulation downstream of the expansion. A secondary circulation (corner eddy) was also predicted. Reattachment was predicted around 10 step heights. The ratio of peak to fully-developed Nusselt number was predicted to decrease with decreasing Prandtl number. This simulation shows that the standard k-ε turbulence model with a non-equilibrium wall function can predict average flow and heat transfer charactersitics in axisymmetric turbulent flows with sudden expansions.

Nomenclature

C_f – friction coefficient (= $\tau_w/0.5\,\rho U_m^2$), [–]
C_μ – empirical constant = 0.09
$C_{1\varepsilon}$ – empirical constant = 1.44
$C_{2\varepsilon}$ – empirical constant = 1.92
c_p – specific heat at constant pressure, [kJkg^{-1}K^{-1}]

D – pipe diameter downstream of expansion, [m]
d – pipe diameter upstream of expansion, [m]
E – specific energy, [$\mathrm{Jkg^{-1}}$]
H – step height, [m]
h – heat transfer coefficient (= $q/(T_\mathrm{w} - T_\mathrm{bulk})$, [$\mathrm{Wm^{-2}K^{-1}}$]
K – turbulence kinetic energy, [$\mathrm{m^2s^{-2}}$]
k – fluid thermal conductivity, [$\mathrm{Wm^{-1}K^{-1}}$]
k_n – normalized turbulence energy (= k/U^2m), [–]
$\mathrm{Nu_{DB}}$ – Nusselt number obtained by the Dittus-Boelter correlation (= $0.023\ \mathrm{Re}^{4/5}\mathrm{Pr}^{0.4}$), [–]
$\mathrm{Nu_x}$ – local Nusselt number (= hD/k), [–]
Pr – fluid Prandtl number, [–]
$\mathrm{Pr_t}$ – turbulent Prandtl number (= 0.85), [–]
p – pressure of the fluid, [Pa]
q – heat flux through the wall, [$\mathrm{Wm^{-2}}$]
Re – Reynolds number (= $\rho U_\mathrm{m} D/\mu$), [–]
T – temperature, [K]
T_bulk – (=$4xq/\mathrm{Re}\mu c_\mathrm{p} + T_\mathrm{in}$), [K]
T_in – inlet temperature, [K]
T_w – wall temperature, [K]
U_i, U_j – velocity of the fluid, [$\mathrm{ms^{-1}}$]
U_m – area-averaged velocity downstream of expansion, [$\mathrm{ms^{-1}}$]
x_i, x_j – Cartesian coordinates, [–]

Greek symbols

δ_{ij} – Kronecker delta
ε – turbulence dissipation rate, [$\mathrm{m^2s^{-3}}$]
μ – dynamic viscosity of the fluid, [$\mathrm{kgm^{-1}s^{-1}}$]
μ_t – eddy viscosity, [$\mathrm{kgm^{-1}s^{-1}}$]
σ_k, σ_ε – empirical constants in turbulent model, [–]
ρ – density of the fluid, [$\mathrm{kgm^{-3}}$]
$\rho\overline{u_i u_j}$ – Reynolds stresses, [Pa]
τ_w – wall shear stress, [Pa]

References

[1] Eaton, J. K., Johnston, J. P., A Review of Research on Subsonic Turbulent Flow Reattachment, *AIAA Journal, 19* (1981), 9, pp. 1093-1100

[2] So, R., Inlet Centerline Turbulence Effects on Reattachment Length in Axisymmetric Sudden-Expansion Flows, *Experiments in Fluids,* 5 (1987), 6, pp. 424-426

[3] Shahnam, M., Morris, G. J., Turbulent Flow Measurements in an Axisymmetric Sudden Expansion, *Proceedings*, 4th International Conference Laser Anemometry – Advanced and Application, Cleveland, Oh., USA, 1991, Vol. 3, pp. 63-69

[4] Durrett, R. P., Stevenson, W. H., Thompson, H. D., LDV Measurements Near the Step in an Axisymmetric Sudden Expansion Air Flow, *Proceedings*, 2nd International Symposium on Applications of Laser Anemometry to Fluid Mechanics, Lisbon, 1985, A86-16351 05-34

[5] Baughn, J. W., Hoffman, M. A., Takahashi, R. K., Launder, B. E., Local Heat Transfer Downstream of an Abrupt Expansion in a Circular Channel with Constant Wall Heat Flux, *Journal of Heat Transfer, 106* (1984), 4, pp. 789-796

[6] Said, S., Habib, M. A., Iqbal, M. O., Heat Transfer to Pulsating Turbulent Flow in an Abrupt Pipe Expansion, *International Journal of Numerical Methods for Heat & Fluid Flow, 13* (2003), 3, pp. 286-308
[7] Hinze, J. O., Turbulence, McGraw-Hill Publishing Co., New York, USA, 1950
[8] Launder, B. E., Spalding, D. B., Lectures in Mathematical Models of Turbulence, Academic Press, London, 1972, pp. 90-110
[9] Kim, S., Choudhury, D., Near-Wall Treatment Using Wall Functions Sensitized to Pressure Gradient, ASME FED, 1995, Vol. 217, Separated and Complex Flows
[10] Patankar, S. V., Spalding, D. B., A Calculation Procedure for Heat, Mass and Momentum Transfer in Three-Dimensional Parabolic Flows, *International Journal of Heat and Mass Transfer, 15* (1972), 10, pp. 1787-1806
[11] Sharov, D., Nakahashi, K., Hybrid Prismatic/Tetrahedral Grid Generation for Viscous Flow Applications, *AIAA Journal, 36* (1998), 2, pp. 157-162
[12] Stern, F., Wilson, R., Coleman, H., Paterson, E., Verification and Validation of CFD Simulations, Iowa Institute of Hydraulic Research, IIHR Report 407, College of Engineering, University of Iowa, Iowa City, Ia., USA, 1999
[13] Kakac, S., Yener, Y., Convective Heat Transfer, 2nd ed., CRC Press, Boca Raton, Fl., USA, 1995

Author's address:

K. N. Alammar
Department of Mechanical Engineering, King Saud University
P. O. Box 800, Riyadh 11421, Saudi Arabia

E-mail: alammar@ksu.edu.sa

Paper submitted: March 3, 2007
Paper revised: October 16, 2007
Paper accepted: November 10, 2007