IDENTIFICATION OF HEAT AND MASS TRANSFER PROCESSES IN BREAD DURING BAKING

by

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A mathematical model representing temperature and moisture content in bread during baking is developed. The model employs the coupled partial differential equations proposed by Luikov. Dependences of mass and thermal properties of dough on temperature and moisture content are included in the model. Resulting system of non-linear partial differential equations in time and one space dimension is reduced to algebraic system by applying a finite difference numerical method. A numerical solution of the model equations is obtained and simultaneous heat and moisture transfer in dough during baking is predicted. The changes of temperature and moisture content during the time of the process are graphically presented and commented.

Key words: *baking, bread, dough, heat and mass transfer, mathematical model, temperature moisture content*

Introduction

The mechanism of bread and pasta baking is complex, in which a chain of physical, chemical and biochemical changes occurs in the products. These changes are essentially the result of simultaneous heat and mass transfer (SHMT) within the products. So temperature and moisture content are dominating factors over these changes. During the baking of bread, in the dough, heat and water transport occurs mainly through the combination of conduction to the dough, convection from the surrounding hot air, radiation from the oven walls to the product surfaces, as well as evaporation of water and condensing steam in the gas cells of the dough. Since dough is a poor conductor of heat that limits heat transfer through conduction, the vapor and water diffusion mechanisms play a very important role in the process.

There has been a growing interest in the modeling of heat and mass transfer during baking and various models have been developed to analyze the multi-phase flow problem inside bread during baking. This has been done with the aim of understanding the processes more thoroughly. Also the setting up of mathematical models of the bread baking process is essential to control and optimize this process. There have been some works published in this area. Some of them will be mentioned here.

A mathematical model used to describe simultaneous heat and mass transfer during drying and baking was proposed by Luikov [5, 6]. The concepts of irreversible thermodynamics and the concept of moisture transfer potential for water movement in a capillary porous body are applied in the model. The model includes the coupling coefficients which gives the combined effects of temperature and pressure gradients on the transfer of moisture inside a porous material. Pressure gradients within the porous body are usually very small and are neglected. The Luikov's mathematical model employs the following assumptions: the solid body is represented structurally as a porous slab with capillaries; pressure gradients within the porous body are very small; external resistance to heat and mass transfer is negligible.

De Vries *et al.* [12] presented a mathematical model of simultaneous heat and mass transfer in dough and crumb (not in the bread crust) considering evaporation – condensation in the gaseous phase and conduction in the liquid phase. This model is based on a phenomenological hypothesis, which stress the effect of air bubbles, contained both in the dough and in the crumb, on heat and mass transfer. An one-dimensional finite difference method has been used to investigate heat transfer in the dough. Mass transfer is only determined by the evaporation-condensation mechanism.

Zanoni *et al.* [13] thought that the model of De Vries *et al*. [12] could not be used for bread crust. So they measured the internal temperature and water content with the time during bread baking at the temperature range of 150 to 300 °C using samples of different initial porosity. Experimental data indicated that the temperature of the bread center asymptotically trends to 100 °C while its surface temperature trends to the oven temperature. Results showed also that there was no constant drying rate period. Based on these experimental data Zanoni *et al*. [13] developed a phenomenological model of bread baking in a forced-convection electric oven. The results showed that the moisture and the temperature variations in bread during baking are determined by the formation of an evaporation front at 100 °C. This front moved inside the product during baking, forming two zones, the crust and the crumb. The temperature in the crust zone trends to the oven temperature, while the temperature of the crumb trends to 100 °C.

A mathematical model based on the Crank-Nicolson finite difference scheme was developed by Tong and Lund [10] for simultaneous heat and water transfer in baked dough products during microwave heating. The heat conduction and water diffusion are considered in one-dimension in Cartesian co-ordinate system. Diffusion together with evaporation and condensation has been assumed to be the mass transfer mechanisms inside the dough.

Thorvaldson and Janestad [9] have developed a model for simultaneous heat, water, and vapor diffusion inside foods during heat processing. Their model was evaluated for a drying process for bread crumb slabs. They have also measured local water content and temperature in several points inside the slab. They have proved that the simulated water content levels and temperatures conform well to the experimental values and show that the evaporation and condensation model describes well the diffusion mechanisms in a porous food.

The aim of this paper is to obtain a nonlinear mathematical model for prediction of temperature and moisture content during baking of bread. The theoretical model developed by Luikov [5, 6] has been used as a basis of this study, but here without the assumption of constant thermophysical properties. It is well known that without the assumption of constant properties, the Luikov's model becomes nonlinear and obtaining an exact solution is not feasible. The assumption of constant thermophysical properties, however, may lead to erroneous prediction of temperature and moister content. Also it has been established that the properties such as thermal conductivity and mass diffusivity are functions of both moisture content and temperature. Therefore, the objective of this work is to develop a mathematical model of the processes during baking bread and to include in the model the variation of the dough properties with both temperature and moisture content. Also the aim of the present study is to solve the model system of nonlinear partial differential equations and to present some numerical and experimental results for distribution of temperature and moisture content within the bread during its baking.

Mathematical formulation of the problem

Heat and mass-transfer processes in the bread during its baking are studied in this paper. Heating of bread during baking is performed by convection, conduction and radiation mechanisms. The top surface of the bread is heated mainly by radiation and only a little part of the heat is transferred by convection (less than 10%) [5, 6, 9]. The bottom surface of the bread may be heated by the same mechanism or by conduction if the bread is put on a metal dish. Here the latter case is studied.

In this paragraph a mathematical model of the heat and mass transfer in baking bread sample is presented. The model includes a couple partial differential equations and boundary conditions for the top and the bottom surfaces of the sample. The thermophysical and mass parameters of the material are considered to be functions of temperature and moisture content.

Governing equations

A bread sample with diameter 200 mm and height 70 mm, placed on a metal dish in an oven, is considered (fig. 1). The top surface of the sample is exposed to heat transfer by radiation and natural convection. The bottom surface of the sample is exposed only to heat transfer by conduction because it is placed on a metal dish.

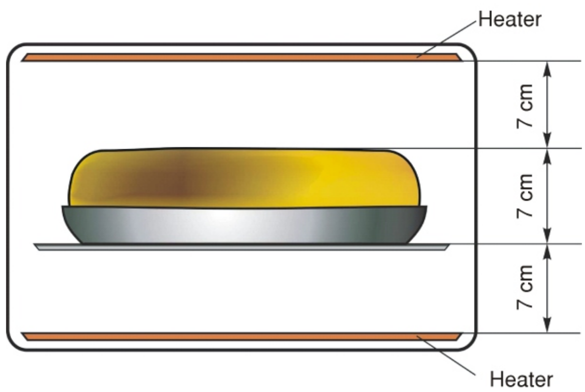


Figure 1. Scheme of the oven and the bread in it

Since the diameter of the bread sample is bigger than its height, the model sample is assumed as a slab and only one-dimensional heat and mass flow are modeled.

The mathematical model consists of a couple partial differential equations for SHMT. The model is based on the assumption that the material is structured as a porous slab. The first equation is derived from the energy balance over a reference element in the product, including a term for evaporation of the water. The second equation for diffusion of the water and of the vapor is derived from the Fick's law, including thermodiffusian mechanism. The model equations are [1, 4-6, 12]:

$$\frac{\partial T}{\partial \tau} \quad \frac{\partial}{\partial x} \quad a\frac{\partial T}{\partial x} \quad \varepsilon\frac{r}{c}\frac{\partial u}{\partial \tau} \tag{1}$$

$$\frac{\partial u}{\partial \tau} \quad \frac{\partial}{\partial x} \quad a_{\mathrm{m}}\frac{\partial u}{\partial x} \quad \frac{\partial}{\partial x} \quad a_{\mathrm{m}}\delta\frac{\partial}{\partial x} \tag{2}$$

The set of eqs. (1) and (2) is nonlinear since the diffusion coefficients depend on both the moisture content and temperature. The dimensionless form of eqs. (1) and (2), suitable for numerical simulations, is the following:

$$\frac{\partial \theta}{\partial t} \quad \frac{\partial}{\partial X} \quad \mathrm{Fo}\frac{\partial \theta}{\partial X} \quad \varepsilon\mathrm{Ko}\frac{\partial U}{\partial t} \tag{3}$$

$$\frac{\partial U}{\partial t} \quad \frac{\partial}{\partial} \quad \mathrm{FoLu}\frac{\partial U}{\partial X} \quad \frac{\partial}{\partial X} \quad \mathrm{FoLuPn}\frac{\partial \theta}{\partial X} \tag{4}$$

where the dimensionless variables are as follow: Fo – Fourier number, Ko – Kosovitch number, Lu – Luikov number, and Pn – Posnov number. This system describes the simultaneous heat and mass transfer during bread baking.

Boundary conditions

The system of eqs. (3) and (4) has to be closed with boundary and initial conditions for temperature and moisture content.

The heat – transfer realized by convectional and radiation mechanisms defines the boundary condition for the temperature at the top surface of the dough ($X = 0$). The mass transfer with the air in the oven is also set in the boundary condition for the moisture content [4-6].

For $X = 0$ and $\tau > 0$:

$$\lambda\frac{\partial T}{\partial X} \quad \alpha(T_{\mathrm{a}} \quad T) \quad q \tag{5}$$

$$a_{\mathrm{m}} \frac{\partial u}{\partial X} \quad \alpha_{\mathrm{m}} (u_{\mathrm{a}} \quad u) \tag{6}$$

The boundary condition for the temperature at the bottom surface of the dough ($X = 1$) takes into consideration that the dough is put on a metal dish, and its temperature is measured. There isn't a moisture exchange between the dough and the metal dish. So the boundary conditions are as follows.

For $X = 1$ and $\tau > 0$:

$$T = T_1(\tau) \tag{7}$$

where $T_1(\tau)$ is a known function of time by the experimental data, and

$$\frac{\partial u}{\partial x} \quad 0 \tag{8}$$

The eqs. (5)-(8) are written in nondimensional form as follows.

For $X = 0$ and $\tau > 0$:

$$\frac{\partial \theta}{\partial X} \quad \mathrm{Bi}(\theta_{\mathrm{a}} \quad \theta) \quad \mathrm{Ki} \tag{9}$$

$$\frac{\partial U}{\partial X} \quad \frac{\mathrm{Bi}_{\mathrm{m}}}{\rho} (U_{\mathrm{a}} \quad U) \tag{10}$$

For $X = 1$ and $\tau > 0$:

$$\theta = \theta_1(t) \tag{11}$$

$$\frac{\partial U}{\partial X} \quad 0 \tag{12}$$

where Bi is Biot number, Bi_{m} – Biot number for mass transfer, and Ki – Kirpitchov number = $qL/\lambda T_0$.

Since it is assumed that initially the temperature and the moisture content are given constant, the initial conditions are at $\tau = 0$, $T = T_0$; $u = u_0$.

Method of numerical solution

The resulting system of non-linear partial differential equations versus time and one space dimension is reduced to algebraic system by applying a finite difference method. A 4-point approximation was used for the numerical solution, which approximates the space derivatives with central differences and the time derivative with one side approximation "forward" [8]. The space and time steps size are 7 mm and 5 s, respectively. A change in the time step 2 s gives a medium deviation in temperature and moisture content of approximately 1.5%. Halving of the space step gives a deviation 2%. This corresponds to an uncertainly in the estimated material parameters in tab. 1 of approximately 5-10%. The values of temperature and moisture content are calculated in each node at given time. On the base of this scheme is developed a numerical algorithm given in fig. 2. The source code was written in Matlab.

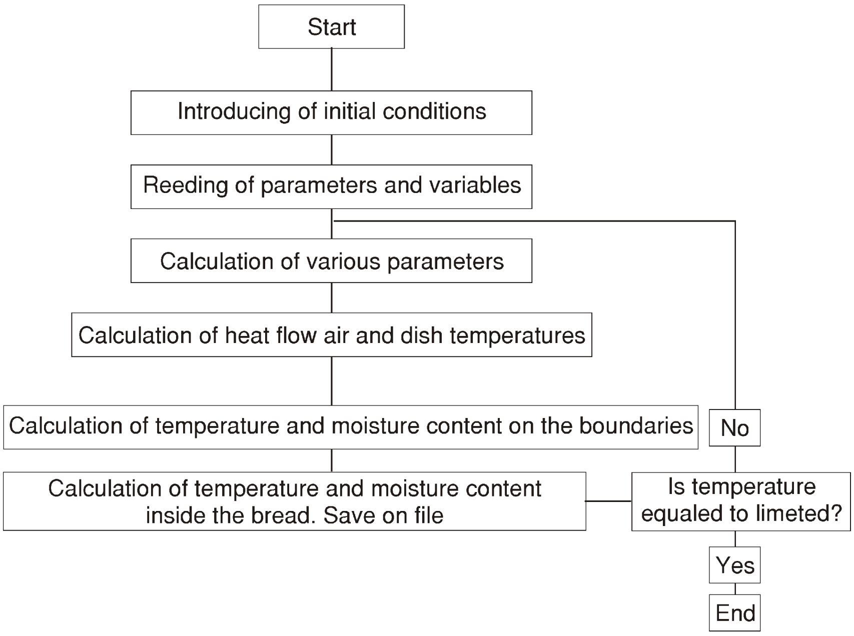


Figure 2. Block-scheme of the algorithm

Materials and equipment

In this part are considered preparation of the sample, thermo-physical properties of the dough and construction of the oven.

Sample preparation

A standard recipe for white bread [2, 3, 10] was used in the experiment. The following products were used for one loaf of bread: 640 g wheat flour, 380 g water, 6.5 g yeast, and 9 g salt. The dough was kneaded by hand and was left to rise for 40 min. at the temperature of 29-30 °C and kneaded again. Next the dough was allowed to rise for 60 min. in 30-33 °C and then was put on a metal dish and then into the oven. The thickness of the dough, measured 2 minutes after the beginning of the baking, was 70 mm and it didn't change during the baking process.

Thermophysical properties of the tested bread sample

The thermophysical and mass properties of the dough for this study were taken from the suitable literature [2, 3]. The thermophysical and mass properties of white wheat flour dough, which gradients are as in the sample using in this investigation were taken from the experimental data published by Ginzburg [2, 3]. The dependences of thermophysical and mass properties of the dough on temperature and moisture contents are summarized in tab. 1.

Table 1. Thermophysical and mass properties of dough from white wheat flour

Parameters	Value
Thermal diffusivity	$a = (13.4 + 0.37u + 0.027T)10^{-8}$ [m^2/s]
Specific heat	$c = 2890 - 1.7T$ [J/kgK]
Thermal conductivity	$\lambda = 0.152 + 0.007u + 0.027T$ [W/mK]
Specific heat of evaporation of water	$r = 24.31 \cdot 10^5$ [J/kg]
Heat moisture conductivity coefficient	$\delta = 4 \cdot 10^{-4}$ [%/K]
Mass diffusivity for water	$a_m = (-9.75 + 0.15u + 0.14T)10^{-11}$ [m^2/s]

The surface heat transfer coefficient α and the surface mass transfer coefficient α_m are calculated by formulas for free moving of fluids.

Construction of the oven

The oven used in this study was with infrared heaters [7]. The power of the upper heater is 57% of the entire power and the power of the bottom heater – 43%, respectively. The oven was constructed on the base of a conventional oven without forced convection. The bread sample was placed in the oven on a metal dish. The distance between the top sample surface and the top heater was 70 mm. The distance between the metal dish and the bottom heater was 70 mm, too (see fig. 1).

The temperature of the air in the oven and the temperature of the metal dish increased during the baking. Both temperatures were measured with iron-constantan thermocouples. The temperature was read on a digital thermometer (0.1 °C). Experimental data for both temperatures given in fig. 3 were approximated with second order polynomials as follows:

Temperature of the air: $T_a = 30 + 19.65\tau - 0.455\tau^2$

Temperature of the metal dish: $T_1 = 34 + 8.28\tau - 0.168\tau^2$

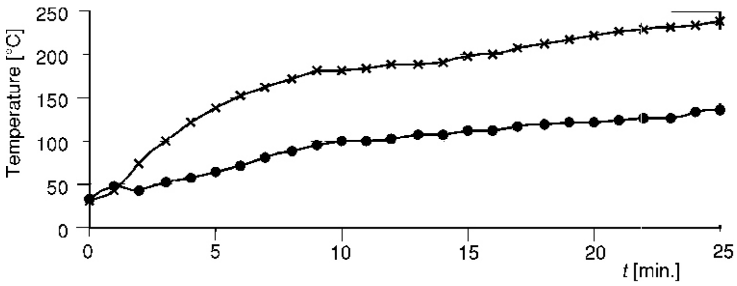


Figure 3. Experimental data for temperatures of the air in the oven ( ) and the temperature of the metal dish versus time ( )

The temperature of the upper heater was measured with the same technique. The heat flow was calculated on the base of experimental data and the Stefan-Boltzmann low. The emissivities used in calculations are: for the heater 0.88 and for the sample 0.90 [9], respectively. The obtained results were approximated with a second order polynomial, too:

$$q = 583.8 + 22.61\tau + 0.106\tau^2$$

The obtained approximations of the temperatures and the heat flow were used as boundary conditions in eqs. (5) and (7).

Temperature measurement

The bread sample temperatures during baking were measured with iron-constantan thermocouples. They were situated on the top surface, on the bottom surface, and in the sample from the top surface on a distance as it follows: 7, 14, 21, 27, 35, 42, 49, 56, and 63 mm. The temperature was read on a digital thermometer (0.1 °C).

Results and discussions

In this paragraph are given experimental data, calculated results and is made their comparison.

Numerical results

The present mathematical model, which incorporates variables mass and thermal diffusivities, gives numerical results for the temperature and moisture distribution during baking of the bread sample.

The initial parameters of the process are as follow. The temperature of the air in the oven is 30 °C; the moisture content of the air is 11%; the temperature of the dough is 30 °C and its moisture content is 40%. The thickness of the dough is 70 mm and the duration of the baking is 25 minutes.

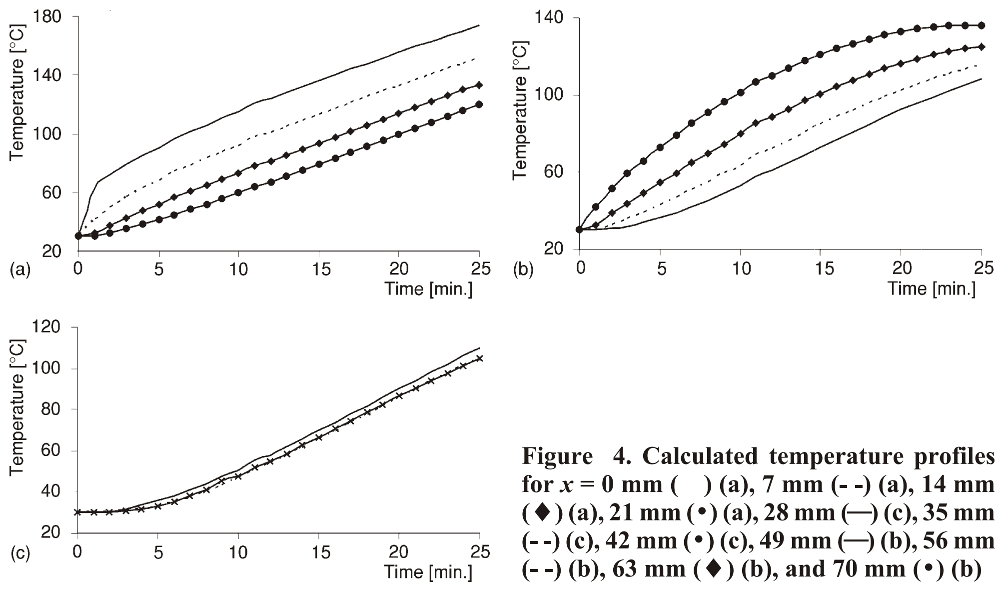


Figure 4. Calculated temperature profiles for x = 0 mm ( ) (a), 7 mm (- -) (a), 14 mm (♦) (a), 21 mm (•) (a), 28 mm (—) (c), 35 mm (- -) (c), 42 mm (•) (c), 49 mm (—) (b), 56 mm (- -) (b), 63 mm (♦) (b), and 70 mm (•) (b)

The calculated temperature profiles of the dough in x-direction for different time values are shown in fig. 4. When the bread sample is placed in the oven, the surface temperatures rise quickly as: the top surface temperature increases from 30 to 173.4 °C (with 143.4 °C) and the bottom surface temperature increases from 30 to 136 °C (with 106 °C), respectively. The calculated top surface temperature is higher than the bottom surface temperature during the whole process of baking because the top surface is heating by radiation and convection mechanisms and the bottom surface only by conduction mechanism. The temperature gradient in the depth of the dough is different. It is most considerable near the surfaces of the dough and it is negligible small in the middle of the dough. The temperature gradient on the upper part of the bread is bigger than the one on the lower part during the whole process of baking.

The temperature profiles for $x = 35$ mm and $x = 42$ mm are practically identical (fig. 4c). The comparison between the temperatures of the bread near the crumbs for $x = 7$ mm and $x = 63$ mm shows that the temperature of the layer, situated near the top surface is 12-15 °C higher than the temperature of the layer near the bottom surface at the time of the baking process, but at the end of the process the difference reaches 25 °C.

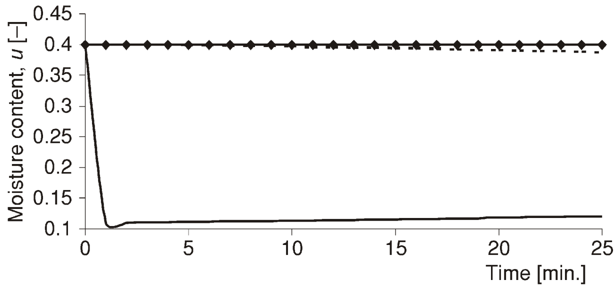


Figure 5. Calculated moisture content profiles for $x = 0$ mm (—), 7 mm (- -), 14 mm (♦)

The moisture content profiles for the same simulation are given in fig. 5. They show the moisture content profiles on the top surface, $x = 7$ mm and $x = 14$ mm. It is well seen that the moisture content on the top surface decrease very fast in the first minutes of baking. After that it increases slightly. The moisture content for $x = 7$ mm is change very slightly and it doesn't change for $x = 14$ mm. The similar information for changes of moisture content is described in [10].

Experimental data

Experimental temperature profiles versus time on the top surface, for $x = 14$ mm, 27 mm, 35 mm, and 49 mm, and the bottom surface are given in fig. 6. The top surface temperature is higher than the bottom surface temperature during the whole baking process. The temperatures within the sample don't rise so quickly that those in the surfaces. The lowest temperatures are measured in the center of the sample. It is well seen that the surfaces temperature profiles are drown as irregular curves and the temperature profiles

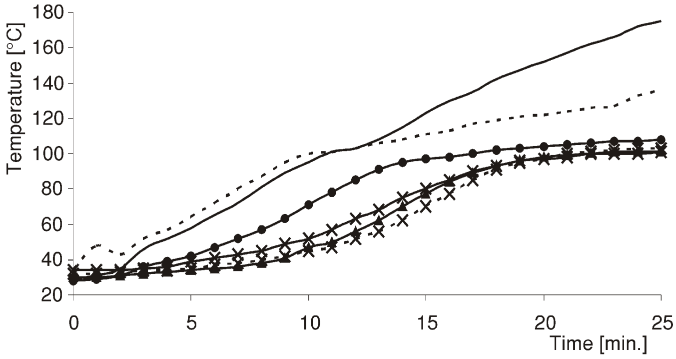


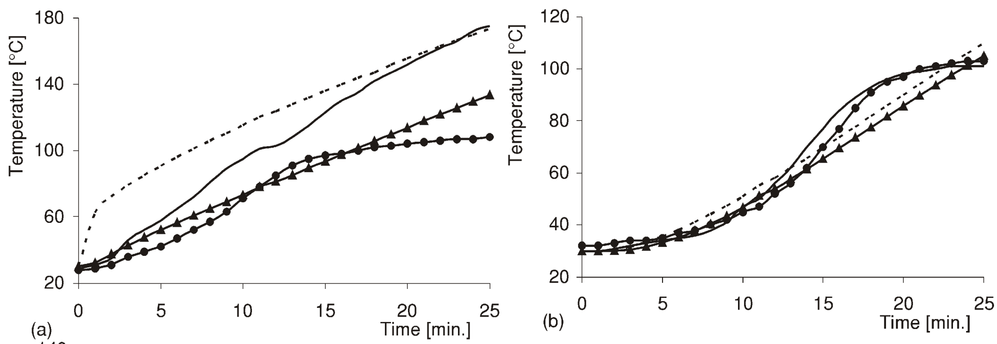
Figure 6. Measured temperatures of dough for top surface (—), 14 mm (•), 27 mm (▲), 35 mm (- -), 49 mm ( ), and bottom surface (- -)

within the sample are presented as exponential curves with a constant temperatures at 20-25 min. The temperature in the sample center in the end of the baking process is reached 101 ºC.

Comparison between calculated and measured temperatures

In fig. 7a the calculated and the measured temperature profiles for $x = 0$ mm (the top surface) and $x = 14$ mm are given. The comparison between both temperatures on the top surface of the bread-dough shows that in the time period from the beginning of the process till the 20th minute, the calculated temperature is higher than the measured one and in the time period from the 20th to the 25th minutes both temperatures are close each to other. The difference between them is the most significant at the beginning of the process – from the 1st to the 9th minutes and after that it decreases gradually. May be the calculated temperature is higher than measured temperature in the beginning of the baking because the walls of the oven are cold and a part of the heat flow is used to heat them. The measured and calculated temperatures in the depth 14 mm from the top surface are considerably closer than those on the top surface.

The calculated and measured temperatures for $x = 27$ mm, and $x = 35$ mm are shown in fig. 7b. It is apparent that for $x = 27$ mm both temperatures are close for the time from $\tau = 0$ min. to 14 min.



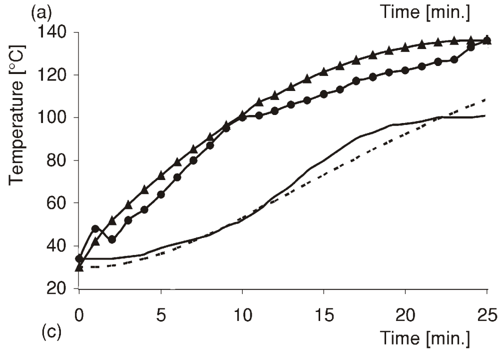


Figure 7. Measured and calculated temperatures of dough
(a) for x = 0 mm ( ) – measured, (- -) – calculated; 14 mm ( ) – measured, (▲) – calculated; (b) 27 mm ( ) – measured, (- -) – calculated; 35 mm ( ) – measured, (▲) – calculated; (c) 49 mm ( ) – measured, (- -) – calculated; 70 mm ( ) – measured, (▲) – calculated

The maximum temperature differences between them during the time is obtained for τ = 16 to 19 min. when the calculated temperature is lower than the measured one. The situation for x = 35 mm is the same. In fig. 7c are given the measured and calculated temperature for x = 49 mm and x = 70 mm (the bottom surface). The analyses of the temperature profiles for x = 49 mm show that the same tendency is observed. The differences between calculated and measured temperatures at the bottom surface (x = 70 mm) are probably due to the deviation caused by the approximation of the measured temperature with the second-order polynomial. It can be observed that the model simulates better temperature profiles of bread crumb than of bread crust.

Conclusions

A nonlinear mathematical model for predicting the temperature and moisture content in baking bread is presented. The model describes well the mechanisms of simultaneous heat and mass transfer in an one-dimensional slab that is heated by radiation and convection from the top surface and with conduction from the bottom surface. The numerical results for the temperature and moisture content distributions in the dough for given thermal conditions are presented.

The simulation results obtained from the Matlab programming code conform well to the experimental results. The model explains well the heat and mass transfer in bread during baking.

In the further work the investigation has to be continued with a development of a model of vaporization, a research on the two-dimensional case, and a determination of thermo physical properties of the sample using for experiments.

Nomenclature

a – thermal diffusivity, [m^2/s]
a_m – mass diffusivity, [m^2/s]
Bi – Biot number, ($= \alpha L/\lambda$), [–]
$\mathrm{Bi_m}$ – Biot number for mass transfer, ($= \alpha_m L/a_m$), [–]
c – specific heat capacity, [J/kgK]
Fo – Fourier number, ($= a\tau_0/L^2$), [–]
Ki – Kirpitchov number, ($= qL/\lambda T_0$), [–]
Ko – Kosovitch number, [$=(r/c)(u_a - u_0)/(T_a - T_0)$], [–]
L – initial thickness of the dough, [m]
Lu – Luikov number, ($= a_m/a$), [–]
Pn – Posnov number, [$= \delta(T_a - T_0)/(u_a - u_0)$], [–]
q – radiation heat flow, [W/m^3]
r – specific heat of evaporation of water, [J/kg]
T – temperature, [ºC]
t – dimensionless time, ($=\tau/\tau_0$)
U – dimensionless moisture content, ($= u/u_0$)
u – moisture content, [–]

X – dimensionless linear coordinate in the depth of the dough, (= x/L)
x – linear coordinate in the depth of the dough, [m]

Greec letters

α – surface heat transfer coefficient, [W/m^{2}K]
α_m – surface mass transfer coefficient, [s/m]
δ – heat moisture conductivity coefficient, [%/K]
ε – phase change criterion, [–]
θ – dimensionless temperature, (= T/T_0)
λ – thermal conductivity, [W/mK]
ρ – density of bread, [kg/m^3]
τ – time, [s]

Subscripts

a – parameters of the air in the oven
0 – initial
1 – metal dish

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