A METHOD FOR STABILITY ANALYSIS OF THE NON-LINEAR HEAT AND MASS TRANSFER PROCESSES

by

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Many systems with non-linear heat and mass transfer processes might be unstable at certain conditions. Small disturbances might bring out them of their equilibrium state, after which they achieve itself to a new stable state. The method developed here concerns a non-linear analysis of hydrodynamic stability of the systems with intensive heat and mass transfer. It allows the determination of the kinetic energy distribution between the main flow and the disturbance, when the equilibrium value of the disturbance amplitude is determined.

Key words: *non-linear stability, method of analysis, heat and mass transfer*

Introduction

Many systems with non-linear heat and mass transfer processes might be unstable at certain conditions [1]. Small disturbances bring them out of their equilibrium state, after which they achieve itself to a new stable state. As a result the disturbances amplitude of self-organized dissipative structure is a constant.

The linear theory of stability [2] allows the determination of parameters of the stable condition, but it could not determine the amplitude of dissipative structures.

The theoritical analisis of self-organized dissipative structures is possible only in the approximation of non-linear theory of stability [3], where the disturbances could be significant.

Mathematical model

As an example a gas (liquid) laminar boundary layer with intensive heat and mass transfer [1], leading to a change of velocity distribution in the laminar boundary layer is considered below. For this particular case the Prandtl equations are:

$$\tilde{u}\frac{\partial \tilde{u}}{\partial x} \quad \tilde{v}\frac{\partial \tilde{v}}{\partial y} \quad \upsilon\frac{\partial^2 \tilde{u}}{\partial y^2}, \quad \frac{\partial \tilde{u}}{\partial x} \quad \frac{\partial \tilde{v}}{\partial y} \quad 0$$
$$x \quad 0, \quad \tilde{u} \quad \bar{u}; \quad y \quad 0, \quad \tilde{u} \quad a_0, \quad \tilde{v} \quad b_0;$$
$$y \quad \infty, \quad \tilde{u} \quad \bar{u} \tag{1}$$

The last boundary condition for eqs. (1) is usually substituted by:

$$y \quad 0, \quad \frac{\partial \tilde{u}}{\partial y} \quad c_0 \tag{2}$$

and c_0 is determined in order to satisfy the last boundary condition of eqs. (1).

In eqs. (1) and (2) $\tilde{u}$ and $\tilde{v}$ are the components of the velocity, x and y are the co-ordinates, υ is the kinematic viscosity, and a_0, b_0, and c_0 are the boundary conditions, which might conform (correspond) to various effects at the phase boundary ($y = 0$) such as motion of the second phase, occurrence of secondary flows as a result of the non-linear heat and mass transfer.

The existence of disturbances in the system ($u \quad v \quad p$) leads to their interaction with the main flow ($\tilde{u}, \tilde{v}$), which creates a new main flow (u, v, p). The new flow (U, V, P) is non-stationary because of the non-stationary character of the disturbances:

$$U(x, y, t) \quad u(x, y, t) \quad u\,(x, y, t)$$
$$V(x, y, t) \quad v(x, y, t) \quad v\,(x, y, t)$$
$$P(x, y, t) \quad p(x, y, t) \quad p\,(x, y, t) \tag{3}$$

The new flow defined by eqs. (3) satisfies the full system of Navier-Stokes equations:

$$\frac{\partial U}{\partial t} \quad U\frac{\partial U}{\partial x} \quad V\frac{\partial U}{\partial y} \quad \frac{1}{\rho}\frac{\partial P}{\partial x} \quad \upsilon \quad \frac{\partial^2 U}{\partial x^2} \quad \frac{\partial^2 U}{\partial y^2}$$
$$\frac{\partial V}{\partial t} \quad U\frac{\partial V}{\partial x} \quad V\frac{\partial V}{\partial y} \quad \frac{1}{\rho}\frac{\partial P}{\partial y} \quad \upsilon \quad \frac{\partial^2 V}{\partial x^2} \quad \frac{\partial^2 V}{\partial y^2}$$
$$\frac{\partial U}{\partial x} \quad \frac{\partial V}{\partial y} \quad 0 \tag{4}$$

We can introduce eqs. (3) into eqs. (4) and eliminate the pressure using the differentiation and subsequent subtracting of the first two equations in eqs. (4). The following result is obtained:

$$\frac{\partial \psi}{\partial t} \quad (u \quad u\,)\frac{\partial \psi}{\partial x} \quad (v \quad v\,)\frac{\partial \psi}{\partial y} \quad \upsilon \quad \frac{\partial^2 \psi}{\partial x^2} \quad \frac{\partial^2 \psi}{\partial y^2}$$
$$\frac{\partial \psi}{\partial t} \quad (u \quad u\,)\frac{\partial \psi}{\partial x} \quad (v \quad v\,)\frac{\partial \psi}{\partial y} \quad \upsilon \quad \frac{\partial^2 \psi}{\partial x^2} \quad \frac{\partial^2 \psi}{\partial y^2}$$
$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0, \quad \frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{5}$$

where

$$\psi \quad \frac{\partial u}{\partial y} \quad \frac{\partial v}{\partial x} \quad \text{and} \quad \psi \quad \frac{\partial u}{\partial y} \quad \frac{\partial v}{\partial x} \tag{6}$$

Thus, eqs. (5) stresses on the segregation of the effects:
– the first equation renders the influence of disturbance over the main flow,
– the second equation renders the influence of main flow over the disturbance.
However the influences in both equations have non-linear character.

Form of the disturbances

The particular solutions of eqs. (5) in the form of "normal" disturbances, *i. e.* periodic disturbances which amplitude depends exponentially of time will be commented further:

$$u(x,y,t) \quad \exp(\ \omega t) u_0(x,y)$$

$$v(x,y,t) \quad \exp(\ \omega t) \ \frac{\partial u_0}{\partial x} dy$$

$$u\ (x,y,t) \quad \exp(\ \omega t)[v_0(x,y) \quad u_1(x,y)\sin nx \quad v_1(x,y)\cos nx] \tag{7}$$

$$v\ (x,y,t) \quad \exp(\ \omega t) \quad \frac{\partial v_0}{\partial x} \quad \frac{\partial u_1}{\partial x} \quad n v_1 \quad \sin nx \quad \frac{\partial v_1}{\partial x} \quad n u_1 \quad \cos nx \quad dy$$

The substitution of the eqs. (7) into eqs. (5) allows the determination of a stable dissipative structure at $\omega = 0$. This partial solution depends on the eigenvalues, values of the wave number $n = 2\pi/\lambda$, where λ is the wave length of disturbance.

As a result we have two equations for u_0,v_0, u_1, and v_1, corresponding to the eqs. (5), where $\cos^2 nx = 1 - \sin^2 nx$. From these equations it is possible to obtain a set of equations, if we put their apperiodical parts and all parts containing $\sin nx$, $\cos nx$, $\sin^2 nx$, and $\sin nx \cos nx$ to be equal to zero.

Set of equations

From apperiodical parts we can obtain directly two equations in order to determine u_0 and v_0:

$$(u_0 \quad v_0) \ \frac{\partial^2 u_0}{\partial x \partial y} \quad \frac{\partial^3 u_0}{\partial x^3} dy \quad \frac{\partial u_0}{\partial x} dy \quad \frac{\partial v_0}{\partial x} dy \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^2 u_0}{\partial y^2}$$

$$\upsilon \quad \frac{\partial^4 u_0}{\partial x^4} dy \quad 2\frac{\partial^3 u_0}{\partial x^2 \partial y} \quad \frac{\partial^3 u_0}{\partial y^3} \tag{8a}$$

$$(u_0 \quad v_0) \frac{\partial^2 v_0}{\partial x \partial y} \quad \frac{\partial^3 v_0}{\partial x^3} dy \quad \frac{\partial u_0}{\partial x} dy \quad \frac{\partial^2 v_0}{\partial x^2} \quad \frac{\partial^2 v_0}{\partial y^2}$$

$$\upsilon \quad \frac{\partial^4 v_0}{\partial x^4} dy \quad 2\frac{\partial^3 v_0}{\partial x^2 \partial y} \quad \frac{\partial^3 v_0}{\partial y^3}$$

$$v_1 \quad \frac{\partial^2 v_1}{\partial x \partial y} \quad n\frac{\partial u_1}{\partial y} \quad \frac{\partial^3 v_1}{\partial x^3} \quad 3n\frac{\partial^2 u_1}{\partial x^2} \quad 3n^2 \frac{\partial v_1}{\partial x} \quad n^3 u_1 \quad dy$$

$$\frac{\partial^2 v_1}{\partial x^2} \quad \frac{\partial^2 v_1}{\partial y^2} \quad 2n\frac{\partial u_1}{\partial x} \quad n^2 v_1 \quad \frac{\partial v_1}{\partial x} \quad nu_1 \quad dy \tag{8b}$$

The periodical part in the equation for u' contains terms with sin nx and cos nx (after the summing up separately the parts containing sin x and cos x and equalization of these sums in separate to zero):

$$u_1 \quad \frac{\partial^2 u_0}{\partial x \partial y} \quad \frac{\partial^3 u_0}{\partial x^3} dy \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^2 u_0}{\partial y^2} \quad nv_1 \quad \frac{\partial u_1}{\partial x} \quad dy \quad 0$$

$$v_1 \quad \frac{\partial^2 u_0}{\partial x \partial y} \quad \frac{\partial^3 u_0}{\partial x^3} dy \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^2 u_0}{\partial y^2} \quad nu_1 \quad \frac{\partial v_1}{\partial x} \quad dy \quad 0 \tag{9}$$

By analogy, from the terms with sin nx and cos nx in the equation for v we have two equations for determination of u_1 and v_1:

$$u_0 \quad \frac{\partial^2 u_1}{\partial x \partial y} \quad n\frac{\partial v_1}{\partial y} \quad \frac{\partial^3 u_1}{\partial x^3} \quad 3n\frac{\partial^2 v_1}{\partial x^2} \quad 3n^2 \frac{\partial u_1}{\partial x} \quad n^3 v_1 \quad dy$$

$$\frac{\partial u_0}{\partial x} dy \quad \frac{\partial^2 u_1}{\partial y^2} \quad \frac{\partial^2 u_1}{\partial x^2} \quad 2n\frac{\partial v_1}{\partial x} \quad n^2 u_1$$

$$\upsilon \quad 2\frac{\partial^3 u_1}{\partial x^2 \partial y} \quad 4n\frac{\partial^2 v_1}{\partial x \partial y} \quad 2n^2 \frac{\partial u_1}{\partial y}$$

$$\frac{\partial^4 u_1}{\partial x^4} \quad 4n\frac{\partial^3 v_1}{\partial x^3} \quad 6n^2 \frac{\partial^2 u_1}{\partial x^2} \quad 4n^3 \frac{\partial v_1}{\partial x} \quad n^4 u_1 \quad dy \quad \frac{\partial^3 u_1}{\partial y^3}$$

$$u_1 \quad \frac{\partial^2 v_0}{\partial x \partial y} \quad \frac{\partial^3 v_0}{\partial x^3} dy \quad v_0 \quad \frac{\partial^2 u_1}{\partial x \partial y} \quad n\frac{\partial v_1}{\partial y}$$

$$v_0 \quad \frac{\partial^3 u_1}{\partial x^3} \quad 3n\frac{\partial^2 v_1}{\partial x^2} \quad 3n^2 \frac{\partial u_1}{\partial x} \quad n^3 v_1 \quad dy$$

$$\frac{\partial^2 u_1}{\partial x^2} \quad \frac{\partial^2 u_1}{\partial y^2} \quad 2n\frac{\partial v_1}{\partial x} \quad n^2 u_1 \qquad \frac{\partial v_0}{\partial x} dy \qquad \frac{\partial^2 v_0}{\partial x^2} \quad \frac{\partial^2 v_0}{\partial y^2} \qquad \frac{\partial u_1}{\partial x} \quad n v_1 \quad dy \tag{10a}$$

$$u_0 \quad \frac{\partial^2 v_1}{\partial x \partial y} \quad n\frac{\partial u_1}{\partial y} \qquad \frac{\partial^3 v_1}{\partial x^3} \quad 3n\frac{\partial^2 u_1}{\partial x^2} \quad 3n^2 \frac{\partial v_1}{\partial x} \quad n^3 u_1 \quad dy$$

$$\frac{\partial u_0}{\partial x} dy \quad \frac{\partial^2 v_1}{\partial x^2} \quad \frac{\partial^2 v_1}{\partial y^2} \quad 2n\frac{\partial u_1}{\partial x} \quad n^2 v_1$$

$$\upsilon \quad 2\frac{\partial^3 v_1}{\partial x^2 \partial y} \quad 4n\frac{\partial^2 u_1}{\partial x \partial y} \quad 2n^2 \frac{\partial v_1}{\partial y}$$

$$\frac{\partial^4 v_1}{\partial x^4} \quad 4n\frac{\partial^3 u_1}{\partial x^3} \quad 6n^2 \frac{\partial^2 v_1}{\partial x^2} \quad 4n^3 \frac{\partial u_1}{\partial x} \quad n^4 v_1 \quad dy \quad \frac{\partial^3 v_1}{\partial y^3}$$

$$v_1 \quad \frac{\partial^2 v_0}{\partial x \partial y} \qquad \frac{\partial^3 v_0}{\partial x^3} dx \quad v_0 \quad \frac{\partial^2 v_1}{\partial x \partial y} \quad n\frac{\partial u_1}{\partial y}$$

$$v_0 \qquad \frac{\partial^3 v_1}{\partial x^3} \quad 3n\frac{\partial^2 u_1}{\partial x^2} \quad 3n^2 \frac{\partial v_1}{\partial x} \quad n^3 u_1 \quad dy$$

$$\frac{\partial^2 v_1}{\partial x^2} \quad \frac{\partial^2 v_1}{\partial y^2} \quad 2n\frac{\partial u_1}{\partial x} \quad n^2 v_1 \qquad \frac{\partial v_0}{\partial x} dy$$

$$\frac{\partial^2 v_0}{\partial x^2} \quad \frac{\partial^2 v_0}{\partial y^2} \qquad \frac{\partial v_1}{\partial x} \quad n u_1 \quad dy \tag{10b}$$

From the terms with $\sin^2 nx$ and $\sin nx \cos nx$ we have:

$$u_1 \quad \frac{\partial^2 u_1}{\partial x \partial y} \quad n\frac{\partial v_1}{\partial y} \qquad \frac{\partial^3 u_1}{\partial x^3} \quad 3n\frac{\partial^2 v_1}{\partial x^2} \quad 3n^2 \frac{\partial u_1}{\partial x} \quad n^3 v_1 \quad dy$$

$$\frac{\partial^2 u_1}{\partial x^2} \quad \frac{\partial^2 u_1}{\partial y^2} \quad 2n\frac{\partial v_1}{\partial x} \quad n^2 u_1 \qquad \frac{\partial u_1}{\partial x} \quad n v_1 \quad dy$$

$$v_1 \quad \frac{\partial^2 v_1}{\partial x \partial y} \quad n\frac{\partial u_1}{\partial y} \qquad \frac{\partial^3 v_1}{\partial x^3} \quad 3n\frac{\partial^2 u_1}{dx^2} \quad 3n^2 \frac{\partial v_1}{\partial x} \quad n^3 u_1 \quad dy$$

$$\frac{\partial^2 v_1}{\partial x^2} \quad \frac{\partial^2 v_1}{\partial y^2} \quad 2n\frac{\partial u_1}{\partial x} \quad n^2 v_1 \qquad \frac{\partial v_1}{\partial x} \quad n u_1 \quad dy \quad 0 \tag{11a}$$

$$u_1 \quad \frac{\partial^2 v_1}{\partial x\,\partial y} \quad n\frac{\partial u_1}{\partial y} \qquad \frac{\partial^3 v_1}{\partial x^3} \quad 3n\frac{\partial^2 u_1}{\partial x^2} \quad 3n^2\frac{\partial v_1}{\partial x} \quad n^3 u_1 \quad dy$$

$$v_1 \quad \frac{\partial^2 u_1}{\partial x\,\partial y} \quad n\frac{\partial v_1}{\partial y} \qquad \frac{\partial^3 u_1}{\partial x^3} \quad 3n\frac{\partial^2 v_1}{\partial x^2} \quad 3n^2\frac{\partial u_1}{\partial x} \quad n^3 v_1 \quad dy$$

$$\frac{\partial^2 v_1}{\partial x^2} \quad \frac{\partial^2 v_1}{\partial y^2} \quad 2n\frac{\partial u_1}{\partial x} \quad n^2 v_1 \qquad \frac{\partial u_1}{\partial x} \quad nv_1 \quad dy$$

$$\frac{\partial^2 u_1}{\partial x^2} \quad \frac{\partial^2 u_1}{\partial y^2} \quad 2n\frac{\partial v_1}{\partial x} \quad n^2 u_1 \qquad \frac{\partial v_1}{\partial x} \quad nu_1 \quad dy \quad 0 \tag{11b}$$

Here must be marked that the right hand side of eq. (8b) is different if we put $\sin^2 nx = 1 - \cos^2 nx$. On the other hand from the eq. (11a) can be seen that for these two cases the right hand sides are equal.

Similarity variables

The solution of eqs. (1) can be expressed in similarity variables:

$$\tilde{u} \quad \bar{u}F\,(\xi), \quad \tilde{v} \quad \sqrt{\frac{\upsilon\bar{u}}{4x}}(\xi F \quad F), \quad \xi \quad y\sqrt{\frac{\bar{u}}{\upsilon x}} \tag{12}$$

Introducing of variables (12) into eqs. (1) leads to the following:

$$\begin{gathered} 2F \quad FF \quad 0 \\ \xi \quad 0, \quad F \quad a, \quad F \quad b, \quad F \quad c \end{gathered} \tag{13}$$

where a, b, and c are determined in advance taking into account the movement of the second phase, the secondary flows, *etc*. (see, for example, 1).

Those similarity variables can be also introduced into eqs. (8)-(11):

$$u_0 \quad \bar{u}f_0(\xi), \quad v_0 \quad \bar{u}\beta\varphi_0(\xi), \quad u_1 \quad \bar{u}\beta f\,(\xi), \quad v_1 \quad \bar{u}\beta\varphi\,(\xi), \quad \beta \quad \frac{\bar{u}_1}{\bar{u}} \tag{14}$$

where $\bar{u}$ and $\bar{u}_1$ are characteristic velocities of the main flow and disturbances and β is the dimensionless amplitude of the disturbances.

The problem will be solved in approximation of the laminar boundary layer theory, *i. e*. in zero approximation of the small parameter γ^2:

$$\gamma^2 = 0 \tag{15}$$

where

$$\gamma^2 \quad \frac{\delta}{x}^{\ 2} \quad \text{Re}^{\ 1/2}, \quad \delta \quad \sqrt{\frac{\upsilon x}{\bar{u}}}, \quad \text{Re} \quad \frac{\bar{u}x}{\upsilon} \tag{16}$$

In this approximations ($\gamma^2 = 0$) A_0, β, and Re are parameters:

$$A \quad n\delta \quad \frac{2\pi x}{\lambda}\text{Re}^{\ 1/2} \quad A_0\gamma, \quad A_0 \quad \frac{2\pi x}{\lambda} \tag{17}$$

because the dependence of parameters on x is very weak.

In similarity variables (14) β is small parameter and problem will be solved in zero approximation of the small parameter $\beta^2, \beta^2 = 0$.

The introduction of similarity variables (14) and approximation (15) in eqs. (8a) and (8b) leads to the equations for f_0 and φ_0 :

$$2f_0 \quad (f_0 \quad \beta\varphi_0)f_0 \quad 0 \tag{18a}$$

$$2\varphi_0 \quad (f_0 \quad \beta\varphi_0)\varphi_0 \quad 0 \tag{18b}$$

The introduction of similarity variables (14) and approximation (15) in eqs. (9a) and (9b) leads to the conditions:

$$2f_0 \quad \frac{2f}{f \quad 2A_0\varphi} f_0 \quad 0$$

$$2f_0 \quad \frac{2\varphi}{\varphi \quad 2A_0 f} f_0 \quad 0 \tag{19}$$

From eqs. (19) and the eq. (18a) are obtained the conditions:

$$f_0 \quad \beta\varphi_0 \quad \frac{2f}{f \quad 2A_0\varphi} \quad \frac{2\varphi}{\varphi \quad 2A_0 f} \tag{20}$$

The introduction of eq. (20) in eqs. (18) leads to:

$$2f_0 \quad \frac{2f}{f \quad 2A_0\varphi} f_0 \quad 0$$

$$2\varphi_0 \quad \frac{2\varphi}{\varphi \quad 2A_0 f} \varphi_0 \quad 0 \tag{21}$$

The introduction of similarity variables (14) and approximation (15) in eqs. (10a) and (10b) leads to the equations for f and φ:

$$2f^{IV} \quad f_0 f \quad f_0 f \quad 2A_0 f_0 \varphi \quad \beta(f \varphi_0 \quad f \varphi_0 \quad f \varphi_0 \quad f\varphi_0) \; 2\beta A_0(\varphi_0 \varphi \quad \varphi\varphi_0) \quad 0$$

$$2\varphi^{IV} \quad f_0 \varphi \quad f_0 \varphi \quad 2A_0 f_0 f \quad \beta(\varphi \varphi_0 \quad \varphi \varphi_0 \quad \varphi \varphi_0 \quad \varphi\varphi_0) \tag{22}$$

$$2\beta A_0(\varphi_0 f \quad f\varphi_0) \quad 0$$

With the approximations ($\beta^2 = 0$, $\gamma^2 = 0$) used, the eqs. (11) are eliminated.

The boundary conditions are obtained using two considerations:

(1) boundary conditions for F and f_0 are equal, and
(2) the dependences of the disturbances from the main flow are identical in the volume and at the interface.

From eq. (13) follows:

$$F(0) \quad a, \quad F\,(0) \quad b, \quad F\,(0) \quad c$$

$$F\,(0) \quad \frac{F\,(0)F(0)}{2} \quad \frac{ac}{2}$$

$$F^{IV}(0) \quad \frac{bc}{2} \quad \frac{a^2 c}{4} \tag{23}$$

$$F^{V}(0) \quad \frac{c^2}{2} \quad \frac{3abc}{4} \quad \frac{a^3 c}{8}$$

where $F^{IV}(0)$ and $F^{V}(0)$ were obtained after double differentiation of eq. (13).

From eq. (18a) follows:

$$\varphi_0 \quad \frac{2f_0}{\beta f_0} \quad \frac{f_0}{\beta} \tag{24}$$

By double differentiation of eq. (24) and introduction of $\xi = 0$ in φ_0, φ_0, and φ_0 and using eq. (24) allows the determination of boundary conditions in f_0 and φ_0:

$$f_0(0) \quad a, \quad f_0(0) \quad b, \quad f_0 \quad c$$

$$\varphi_0(0) \quad 0, \quad \varphi_0(0) \quad 0, \quad \varphi_0 \quad 0 \tag{25}$$

From the conditions (20) is directly obtained:

$$2f \quad (f_0 \quad \beta\varphi_0)(f \quad 2A_0 \varphi)$$

$$2\varphi \quad (f_0 \quad \beta\varphi_0)(\varphi \quad 2A_0 f) \tag{26}$$

If we note $\alpha_1 = f(0)$ and $\alpha_2 = \varphi(0)$, we could determine from eqs. (26) boundary conditions for eq. (22) after differentiation of eqs. (26) and use of eq. (25):

$$f(0) \quad \alpha_1, \quad f\,(0) \quad \frac{a}{2}(\alpha_1 \quad 2A_0\alpha_2)$$

$$f\;(0) \quad \frac{\alpha_1}{2} \quad b \quad \frac{a^2}{2} \quad 2A_0^2 a^2 \quad A_0\alpha_2(a^2 \quad b) \tag{27a}$$

$$f\;\;(0) \quad \frac{\alpha_1}{2} \quad c \quad \frac{3ab}{2} \quad 6abA_0^2 \quad \frac{1}{4}a^3 \quad 3a^3A_0^2 \quad \alpha_2 A_0(c \quad 3ab \quad \frac{4}{4}a^3 \quad a^3A_0^2)$$

$$\varphi(0) \quad \alpha_2, \quad \varphi\,(0) \quad \frac{a}{2}(\alpha_2 \quad 2A_0\alpha_1)$$

$$\varphi\;(0) \quad A_0\alpha_1(a^2 \quad b) \quad \frac{\alpha_2}{2} \quad b \quad \frac{1}{2}a^2 \quad 2a^2A_0^2 \tag{27b}$$

$$\varphi\;\;(0) \quad \frac{\alpha_2}{2} \quad c \quad \frac{3ab}{2} \quad 6abA_0^2 \quad \frac{1}{4}a^3 \quad 3a^3A_0^2 \quad \alpha_1 A_0 \quad c \quad 3ab \quad \frac{3}{4}a^3 \quad a^3A_0^3$$

where α_1 and α_2 are eigenvalues of the problem.

Determination of β

The parameter β might be determined from the condition that the kinetic energy E of the main flow is distributed between the energy of new flow E_0 and the energy of disturbance E_1:

$$E = E_0 + E_1 \tag{28}$$

If we assume that in all three cases kinetic energies are proportional to the velocity square summed in the area of the boundary layer (s):

$$E \sim \int_{(s)} \tilde{u}^2 dxdy, \quad E_0 \quad \int_{(s)} u^2 dxdy, \quad E_1 \quad \int_{(s)} u^{\,2} dxdy \tag{29}$$

we obtain ($\omega = 0$):

$$E \sim \int_0^{\lambda}\int_0^{\delta} \tilde{u}^2 dydx \quad \bar{u}^2\delta\lambda \int_0^6 F^{\;2} d\xi$$

$$E_0 \sim \int_0^{\lambda}\int_0^{\delta} u_0^2 dydx \quad \bar{u}^2\delta\lambda \int_0^6 f_0^{\;2} d\xi \tag{30}$$

$$E_1 \sim \int_0^{\lambda}\int_0^{\delta} (v_0 \quad u_1 \sin nx \quad v_2 \cos nx)^2 dydx$$

$$\bar{u}^2\delta\lambda\beta^2 \int_0^6 \varphi_0^{\;2} d\xi \quad \frac{1}{2}\int_0^6 f^{\;2} d\xi \quad \frac{1}{2}\int_0^6 \varphi^{\;2} d\xi$$

The introduction of the expressions (30) in eq. (28) allows the determination of β:

$$\beta \quad \sqrt{\frac{\int_0^6 F^2 d\xi \quad \int_0^6 f_0^2 d\xi}{\int_0^6 \varphi_0^2 d\xi \quad \frac{1}{2}\int_0^6 f^2 d\xi \quad \frac{1}{2}\int_0^6 \varphi^2 d\xi}} \tag{31}$$

Method of solution

The eqs. (21) and (22) with boundary conditions (25) and (27) present an eigenvalues problem where α_1, α_2, β, and A_0 are eigenvalues. They may be determined from condition for minimum of the least square function Q:

$$Q(\alpha_1, \alpha_2, \beta, A_0) = (\beta_{i-1} - \beta_i)^2 \tag{32}$$

where $i = 0, 1, 2, ...$ is the iteration number in the minimum search procedure of the function Q. The problem is reduced to the combined (joint) solving of eq. (21) and (22) with the corresponding boundary conditions (25) and (27). The first step in the solving is to assign initial values of α_1, α_2, β, and A_0, determine β and then search for which values of α_1, α_2, and A_0 the function Q has a minimum.

Conclusions

The proposed method for non-linear analysis of hydrodynamic stability in systems with intensive heat and mass transfer use the kinetic energy distribution between the main flow and the disturbances for determination of the equilibrium amplitude value of the disturbances.

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Nomenclature

a_0, b_0, c_0 – boundary conditions
n – wave number
$\tilde{u}$ – x component of the velocity, [m/s]
$\tilde{v}$ – y component of the velocity, [m/s
x, y – coordinates, m

Greek letters

β – amplitude of the disturbance, –
γ – small parameter
λ – wave-length of disturbance
υ – kinematic viscosity, m^2/s

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