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Reliability Simulation of Scouring Downstream of Outlet Facilities

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Abstract

Alluvial streambeds downstream of an outlet facility may be seriously scoured under jet action. The degree of scour depends on the characteristics of the jet leaving the outlet facility, depth of tailwater, and properties of the bed material. Existing scour prediction equations reported in the literature are deterministic in nature, and do not account for the randomness of the parameters involved in the models. Therefore, the level of scouring risk is unknown. This paper is concerned with the investigation of the relationship between the reliability of scouring induced by vertical jets downstream of outlet facilities and safety factors. The scour equation proposed by Okyay has been used in the calibration of the simulation model based on Monte Carlo analysis. In reliability-based analysis, which can be carried out during the planning stage, the level of reliability for various foundation depths is estimated under design flow conditions. Furthermore, the relationship between the required size of armoring bed material and reliability under the desired foundation depth can be obtained. An example is presented to illustrate the use of the model.

Key words: Scour, Reliability, Outlet Facility, Monte Carlo Simulation, Safety Factor

Introduction

Scouring is a natural phenomenon caused by the erosive effects of flowing water in alluvial streams. Excessive scouring can progressively undermine the foundations of hydraulic structures, such as bridges, embankments, spurs, and the downstream of outlet facilities. The present study addresses the impacts of the problem and its stochastic treatment for the scouring of foundations downstream of outlet facilities. The large-scale scour downstream of outlet structures induced by jets of various configurations constitutes an important field of research because of the possibility of structural failure at the foundation level. The jet dissipates its excessive energy by excavating a large scour hole. Drastic examples of this phenomenon were experienced downstream of the Alder and Nacimiento Dams in the USA, Picote Dam in Portugal, Kariba Dam in Zambia, and Tarbela Dam in Pakistan (Mason, 1984).

The scouring mechanism is relatively complex because of difficulties arising from the modeling of sediment-laden flow in and around the scour hole induced by the jet action of the flow downstream of outlet facilities. Therefore, the experimental study of scouring has been limited to the consideration of governing variables involved in the phenomenon while keeping the other parameters of secondary importance constant. Simplified laboratory experiments may misrepresent the prototype conditions, which may be entirely different from those of a laboratory medium. This may pronounce the overall uncertainty of empirical models. Furthermore, relevant field data are normally limited in size and are of low precision, which restricts the calibration of the models with the actual field conditions. The scour prediction equations proposed to date are deterministic in that they are composed of parameters whose values are assumed to be known with certainty. As a result, they yield an unknown level of risk of foundation scouring. In fact, hydraulic variables involved in the scouring phenomenon, such as discharge, flow depth, and velocity, are stochastic in nature, which may be represented by relevant probability distributions. Therefore, the scouring phenomenon is also stochastic and needs the modeling of a random process.

This study is concerned with the reliability-based assessment of the scouring of foundations downstream of outlet facilities. Although considerable experimental data are available pertaining to the scour of cohesionless beds due to the impingement of jets at various angles to the bed, the present study is focused only on the investigation of scouring action induced by vertical jets. Reliability computations are carried out by simulating the safety margin using the Monte Carlo technique. The model presented herein was calibrated using the experimental values reported by Okyay (1973). The scour prediction equation proposed by Rajaratnam (1982) was also used to check the validity of the model. An example is presented to illustrate the use of the model. The outputs of the model may be accepted as key parameters in decision-making for the foundation design of outlet structures.

Local Scour Downstream of Outlet Facilities Induced by Vertical Jets

When a jet leaves a pressurized outlet work, it impinges on the bed downstream of the facility and excavates the streambed to a considerable depth. The flow pattern of water through an upper surface sluiceway may be approximated to the form of a vertical jet (See Figure 1). However, jet flow through relatively lower outlets is in the form of an inclined jet, which is outside the scope of the present study. Hydraulic parameters of primary importance in the scouring mechanism induced by vertical jets can be listed as the size and velocity of the jet, particle characteristics and fall velocity of the bed material, height of the jet above the tailwater elevation, depth of tailwater, and duration of the scouring action. In the early stages of erosion, the rate of scour development is extremely high. As the scour hole is enlarged progressively, the local velocity in the scour hole decreases to low values such that the wall shear stresses are not capable of eroding the bed to further elevations. Therefore, the depth of scour converges to a terminal value asymptotically. As jets of clear water characteristics leave a storage facility, the whole stream power is used to erode the bed. Bed materials eroded from the scour hole are then accumulated on the upstream and downstream sides of the scour hole. As time elapses the shape of the scour hole remains almost unchanged and the side slopes of the scour hole approximate the angle of repose of the bed material.

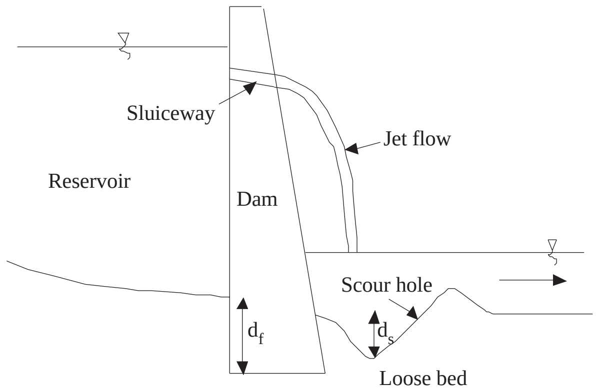

Figure 1. Definition sketch for scouring parameters.

Experimental studies on scouring induced by vertical jets have been carried out by several investigators, including Doddiah (1953), Clarke (1962), Westrich and Kobus (1973), Okyay (1973), Kobus (1979), and Rajaratnam (1982). This study is mainly based on the model calibration using Okyay's (1973) equation, which is based on a semi-empirical investigation of the phenomenon. The results obtained by this equation are also compared using a recent equation proposed by Rajaratnam (1982).

Okyay (1973) carried out laboratory experiments as outlined in Table 1. Using the conservation of mass principle, i.e. sediment continuity equation, he developed a semi-empirical method for determining the temporal variation of the scour depth induced by round vertical jet action. In modeling the sediment transport rate out of the scour hole, he used a sediment pickup function. A similar approach has also been used for bridge pier scours (See Yanmaz, 1989; Yanmaz and Altınbilek, 1991). Since the maximum possible scour depth is of importance in decision-making for foundation design, the temporal variation of scouring will not be considered in this study. As a result, the probabilistic analysis of the terminal scour depth will be the main concern of this study. Okyay (1973) proposed the following relationship for the dimensionless terminal scour depth:

$$\frac{d_s}{b} = \frac{30.67\left(\frac{u}{W_f}\right)^{2.01}\left(\frac{D_g}{b}\right)^{1.128}}{F_r^{1.119}\left(\frac{y}{b}\right)^{0.431}} \tag{1}$$

where d_s is the terminal scour depth, b is the thickness of the round jet, u is the velocity of the jet leaving a pressurized nozzle, W_f is the fall velocity of the bed material, D_g is the geometric mean particle size of the bed material, F_r is the Froude number of the jet, which is given by $u/(gb)^{0.5}$, g is gravitational acceleration, and y is tailwater depth. Equation (1) was obtained by regression analysis of the relevant variables listed in Table 1 to yield a relatively high multiple linear correlation coefficient of 0.974. Therefore, it can be assumed that the use of Equation (1) in reliability computations would reduce model uncertainty.

Among the aforementioned works, Rajaratnam's (1982) scour equation, which was derived using experimental data similar to those of Okyay (1973), can also be considered to check the predictive ability of the model calibrated using Okyay's (1973) data. Rajaratnam's (1982) empirical equation is as follows:

$$\frac{d_s}{b} = 0.13\left(\frac{u^2}{\Delta g D} + \frac{2H}{\Delta D}\right)^{0.5} \tag{2}$$

where Δ is the relative density of sediment, D is median sediment size, H is the height of the jet above the tailwater stage, and the other terms are as previously defined.

Reliability Simulation

Conventional hydraulic design is deterministic as it is based on the selection of an appropriate safety factor on the basis of judgment and experience. This approach does not account for possible variations associated with the variables involved in the phenomenon concerned. With the application of reliability theory to hydraulic engineering practices, probabilistic design approaches have been proposed that enable the assessment of various reliability levels under different combinations of design parameters. In these applications, probability distributions of the parameters having specified coefficients of variations were considered. Although there have been several examples based on the reliability of conveyance structures reported in the literature, there are few studies on the reliability of scouring around hydraulic structures. A significant contribution to the reliability-based assessment of bridge pier scouring can be attributed to the model proposed by Yanmaz and Çiçekdağ (2001), which was derived using resistance-loading interference. This model provided generalized dimensionless reliability expressions.

The present study is based on the simulation of reliability using the Monte Carlo technique. Scouring reliability can be determined from randomly generated numbers giving a safety margin greater than or equal to zero. The safety margin, SM, can be defined as the difference between the depth of foundation, d_f, and maximum depth of scour, d_s, i.e. SM = d_f-d_s. Reliability is then defined as the probability that the scour depth is less than or equal to the depth of foundation, i.e. $\alpha = P(SM \geq 0)$, where P is the probability. Simplifying Equation (1) and substituting it into the safety margin yields the following in SI units:

$$\begin{aligned} \alpha &= P(SM > 0) \\ &= P\left(d_f - \lambda_m \frac{110 b^{0.862} u^{0.891} D_g^{1.128}}{y^{0.431} W_f^{2.01}} > 0\right) \end{aligned} \tag{3}$$

where λ_m is the model correction factor, which is the ratio of the observed scour depth to the computed scour depth obtained from Equation (1). Since Equation (1) is a best-fit equation obtained through a regression analysis, the mean value of the model correction factor can be taken as unity. The coefficient of variation of the model correction factor is determined from the calibration data presented in Table 1 to be 0.19. Since the model correction factor was determined using a dimensionally homogeneous equation, i.e. Equation (1), it is assumed that it is also applicable to field conditions.

Reliability using Rajaratnam's (1982) equation can also be computed in a similar manner. In Rajaratnam's (1982) equation, the relative density, which is approximately 1.65 for quartz sand, is considered to be a deterministic variable. Simplifying Equation (2), one can obtain the following relationship in SI units:

$$\begin{aligned} \alpha &= P(SM > 0) \\ &= P\left(d_f - 0.13 b \lambda_m \sqrt{\frac{u^2}{16.2D} + \frac{1.21H}{D}} > 0\right) \end{aligned} \tag{4}$$

Based on the previous information obtained from Okyay's (1973) data, the coefficient of variation of the model correction factor in Equation (4) was also taken as 0.19. Equations (3) and (4) are nonlinear; therefore, a simulation technique, such as Monte Carlo analysis can be used to determine reliability.

In Monte Carlo analysis, random numbers between 0 and 1 are generated for the variables having uniform distribution (Ang and Tang, 1984). However, these random numbers are transformed to the desired distribution through an inverse transform method using a computer algorithm.

Application

The following information has been obtained for the design flow conditions of an upper surface sluiceway of a gravity dam. The sluiceway is considered to be located at a relatively higher elevation such that the discharging jet is assumed to be in the form of a vertical jet. The thickness of the round jet and its leaving velocity are 0.30 m and 7 m/s, respectively. The elevation difference between the exit of the outlet and the tailwater stage is H = 30 m. The geometric mean size of the bed material is 5 mm. The tailwater depth is 4 m. Fall velocity of the bed material is 0.3 m/s. Decision-making is required for the depth of foundation that can withstand the erosive effects of flow due to jet action based on the simulation of the safety margin using the Monte Carlo technique. In the analysis, Okyay's (1973) and Rajaratnam's (1982) equations were used. It should be noted that the other governing factors that may be involved in structural and geotechnical aspects were not considered in this example. Proper probability density functions (PDF) and coefficients of variations (Ω) are assigned to the variables involved in Equations (3) and (4) with reference to previous work carried out by Johnson (1999), Yanmaz (2000), and Yanmaz and Çiçekdağ (2001) (See Table 2). In the solution, various foundation depths have been considered, i.e. d_f = 6, 8, 12, and 15 m, to assess the level of reliability that can be achieved under the design conditions.

In Monte Carlo analysis, the number of simulation cycles, i.e. the number of trials to generate random numbers, influences the level of reliability. The number of cycles required in a Monte Carlo simulation to determine the exact reliability must be large in order to obtain a significant sampling of simulation events. The accuracy of the mean reliability under a particular simulation cycle may be estimated by the coefficient of variation of reliability, Ω_r, which decreases with increasing sample size. Therefore, simulations should be carried out several times for large cycles such that the corresponding value of Ω_r is relatively small. According to Johnson (1999), it is desirable to have $\Omega_r < 0.1$. Variations of Ω_r against number of simulation cycles are shown in Figure 2 for Okyay's (1973) equation. It is observed that as the number of simulation cycles increases, Ω_r approaches a constant value, which is approximately 0.002. Therefore, it can be considered that further increases in number of simulation cycles would not lead to significant accuracy in the computations. To this end, a 5000-cycle is brought into the analysis. The solution technique is repeated for the aforementioned foundation depths. Variations of reliability with respect to safety factor, SF = d_f/d_s, are shown in Figure 3. In the computation of the safety factor, the scour depths are computed using the mean values of the parameters involved in Equations (1) and (2), whereas the randomness of scouring parameters are taken into account in reliability simulations. The results of both approaches are close to each other. As can be seen from this figure, reliability increases with increasing foundation depth. It should be emphasized, however, that a final design might be achieved by modifying the present results with special reference to structural requirements as well as local geologic and topographic conditions.

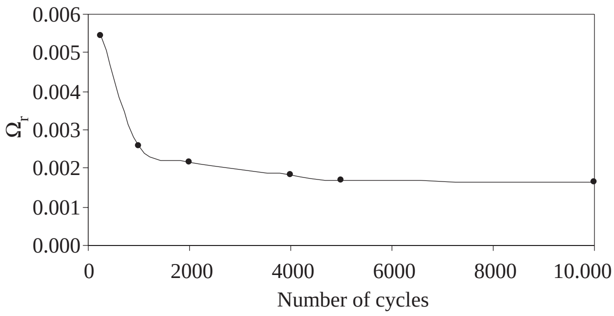

Figure 2. Variation of Ω_r against number of simulation cycles.

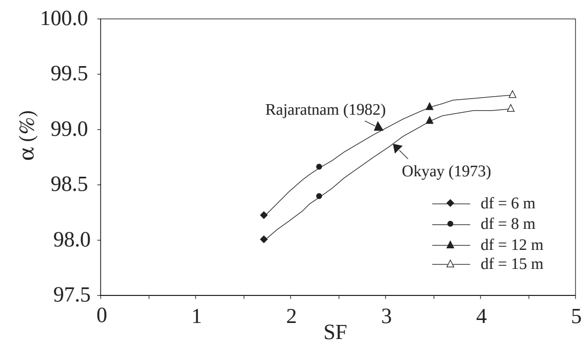

Figure 3. Variation of reliability with the safety factor.

Table 1. Calibration data (Okyay, 1973).

Run No (1)	Sand Label (2)	b (cm) (3)	u (m/s) (4)	D_g (mm) (5)	y (cm) (6)	W_f (cm/s) (7)	F_r (8)	d_s (cm) (9)
12		2	1	3.37	20	40	2.26	6.5
13		2	2	3.37	20	40	4.51	17
14		2	3	3.37	20	40	6.77	24
17		2	2	3.37	36	40	4.51	8
18		2	2	3.37	4	40	4.51	28
19		2	3	3.37	4.6	40	6.77	36
20		2	1	3.37	4.	40	2.26	16
21	A	2	3	3.37	36	40	6.77	16
22		1	1	3.37	20	40	3.19	4
23		1	1	3.37	4	40	3.19	10
24		1	2	3.37	4	40	6.38	16
26		1	2	3.37	20	40	6.38	7
27		1	3	3.37	20	40	9.58	12
28		1	3	3.37	36	40	9.58	4
29		1	3	3.37	4	40	9.58	20
30		1	3	1.72	36	25	9.58	5
31		1	3	1.72	20	25	9.58	13
32		1	2	1.72	20	25	6.38	8
33		1	3	1.72	4	25	9.58	28
34		1	1	1.72	20	25	3.19	6
35		1	2	1.72	4	25	6.38	20
36		1	1	1.72	4	25	3.19	11
37	B	2	2	1.72	20	25	4.51	18
38		2	3	1.72	20	25	6.77	28
39		2	1	1.72	20	25	2.26	8
40		2	2	1.72	36	25	4.51	9.5
41		2	3	1.72	36	25	6.77	20
42		2	2	1.72	4	25	4.51	32
43		2	1	1.72	4	25	2.26	19
44		2	3	1.72	4.6	25	6.77	44

Table 2. Statistical information on the parameters used in the application.

Parameter	Mean	Ω	PDF
b	0.3 m	0.01	Normal
u	7.0 m/s	0.2	Symmetrical triangular
H	30.0 m	0.2	Symmetrical triangular
D_g	0.005 m	0.05	Uniform
y	4.0 m	0.2	Symmetrical triangular
W_f	0.30 m/s	0.2	Symmetrical triangular
λ_m	1.0	0.19	Normal

As a further analysis, the effect of mean particle size of the bed material on reliability under a given foundation depth can be studied. To this end, variations of reliability against mean particle size have been examined using Okyay's (1973) equation for the stated design conditions under the foundation depth of d_f = 12 m. The results of this analysis are presented in Figure 4. Any increase in mean particle size would result in a reduction in the scour depth; therefore, reliability increases with increasing bed material size in a nonlinear manner, as shown in Figure 4. This information may provide a means for assessing the suitability of possible armoring countermeasures downstream of an outlet facility. The placement of gravel of $D_g = 25$ mm at the base of an outlet facility would result in an increase of reliability relative to the case having $D_g = 5$ mm. A final decision on this issue can be made by considering the mutual interference between the desired level of reliability, which may change according to the specific site and economic conditions.

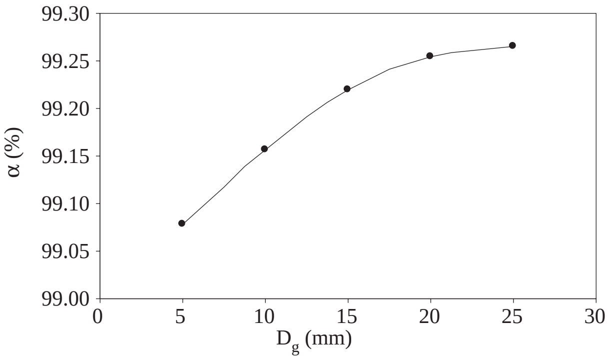

Figure 4. Variation of reliability with particle size of bed material under $d_f = 12$ m.

Conclusions

Local scouring reliability downstream of outlet facilities has been investigated using the Monte Carlo simulation technique. It is based on the generation of random numbers for variables involved in the safety margin by assigning appropriate coefficients of variations and probability density functions. In the calibration of the model, Okyay's (1973) scour prediction equation was used. Rajaratnam's (1982) scour equation was also used to test the validity of the reliability model calibrated using Okyay's (1973) data. With the use of the model proposed in this study, a reliability-based assessment of the safety level of foundation depths of outlet facilities against scouring action can be carried out. The safety factor was defined as the ratio of the foundation depth to the maximum scour depth that can occur under vertical jet action. Use of the model was demonstrated in an application in which both scour equations yielded close results. It was observed that reliability increases as the safety factor rises. Furthermore, reliability increases with increasing mean size of foundation material under a given foundation depth. The final decision on the required depth of foundation for the given bed material or the required size of armoring countermeasure under a given foundation depth should be made by further considering the structural requirements as well as the local geologic and topographic conditions.

Nomenclature

b	jet thickness;
D_g	geometric mean size of bed material;
d_f	depth of foundation;
d_s	maximum depth of scour downstream of an outlet facility;
F_r	Froude number;
H	height of jet above tailwater stage;
P	probability;
PDF	probability density function;
SF	safety factor;
SM	safety margin;
u	jet velocity;
y	depth of tailwater;
W_f	fall velocity of bed material;
α	reliability;
λ_m	model correction factor;
Ω	coefficient of variation and
Ω_r	coefficient of variation of reliability.

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