严格反馈非线性系统的自适应模糊控制

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Abstract: A new robust adaptive indirect fuzzy controller is designed based on backstepping method for a class of uncertain nonlinear systems.The adaptive compensation term of the optimal approximation error and a new robust term are adopted to minify the influence of modeling error and parameter estimation error.The approach does not require the optimal approximation error to be square-integrable or the supremum of the optimal approximation error to be known.It's proved that the final closed-loop system must be globally stable in the sense that all signals involved must be uniformly bounded and simulation results show that the closed-loop fuzzy control system is proved to be globally stable,with tracking error converging to the arbitrarily small neighborhood of the origin.

Key words: nonlinear systems;backstepping;fuzzy control;the optimal approximation error

摘 要:针对一类不确定非线性系统,基于 backstepping 方法提出了一种新的鲁棒自适应模糊控制器设计方案。该方案通过引入最优逼近误差的自适应补偿项和新的鲁棒项,削减建模误差和参数估计误差的影响,从而在稳定性分析中取消了要求逼近误差平方可积或逼近误差的上确界已知的条件。理论分析证明了闭环系统状态有界,跟踪误差收敛到零的较小邻域内。仿真结果表明了该方法的有效性。

关键词:非线性系统;后推;模糊控制;最优逼近误差

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1 引言

针对被控对象的不确定性和时变性,人们提出了自适应控制理论。当前,线性系统的自适应控制问题已基本解决,但非线性系统的自适应控制还存在很多难点。反步法一经提出,便得到广泛地关注,并被推广到自适应控制、鲁棒控制、滑模变结构控制等领域。在设计不确定系统的鲁棒或自适应控制器方面,特别是当存在干扰或不确定不匹配条件时,反步法具有明显的优越性,并取得了许多研究成果[1-5]。文献[6]利用模糊系统的逼近能力和监督控制方案解决了一类典型反馈系统的鲁棒跟踪和稳定性问题,为将模糊逻辑系统用于非线性系统做了开创性的工作,但该系统未考虑外界干扰且要求逼近误差平方可积,这在实际系统中很难满足。文献[3]通过引入逼近误差的自适应补偿项消除了逼近误差平方可积这一限制,设计的控制器不仅使得全局状态稳定,且跟踪误差收敛到零。文献[4-5]讨论了一类严格反馈非线性系统,控制器设计的优点是无过参数设计,但设计过程中要求控制增益部分已知且虚拟控制增益函数是常量。

受上述文献的启发,该文运用二型模糊逻辑系统的逼近能力,基于 backstepping 方法,提出了一种自适应模糊控制器的设计方案,通过引入逼近误差的自适应补偿项,证明了闭环系统状态有界,跟踪误差收敛到零的较小邻域内。仿真结果表明,提出的自适应模糊控制算法具有较强的鲁棒性和良好的跟踪性能。

2 系统的描述及基本假设

考虑如下不确定非线性系统:

$$\begin{cases}\dot{x}_1=x_2+\boldsymbol{\varphi}_1^{\mathrm{T}}(x_1)\boldsymbol{\theta}+\chi_1(x,t)\\ \dot{x}_2=x_3+\boldsymbol{\varphi}_2^{\mathrm{T}}(x_1,x_2)\boldsymbol{\theta}+\chi_2(x,t)\\ \vdots\\ \dot{x}_n=f(x)+g(x)u+\boldsymbol{\varphi}_n^{\mathrm{T}}(x,t)\boldsymbol{\theta}+\chi_n(x,t)\\ y=x_1\end{cases} \tag{1}$$

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其中 $\boldsymbol{x}=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}\in R^n$ 为系统的状态且假设可以通过量测得到，式中 f,g 是未知连续函数，$u\in R,y\in R$ 分别为系统的输入和输出，$\chi_i(x,t),i=1,2,\cdots,n$ 为外界干扰。

参照文献[6]中的讨论，对系统（1）作如下假设：

（1）$|f(x)|\leqslant F(x),\boldsymbol{x}\in R^n$；

（2）$0<K_1(x)\leqslant g(x)\leqslant K_2(x),\boldsymbol{x}\in R^n$；

（3）$\|(y_r,\dot{y}_r,\cdots,y_r^{(n)})^{\mathrm{T}}\|\leqslant M_r$；

（4）$|\chi_i(x,t)|\leqslant\rho_i(x_1,\cdots,x_i),\forall t\geqslant 0$；

（5）$\|\boldsymbol{\theta}\|\leqslant\|\boldsymbol{\theta}\|_{\max}\leqslant p$。

其中 $F(x),K_1(x),K_2(x),\rho_i(x_1,\cdots,x_i)$ 是已知正的连续函数，M_r，p 是已知的正常数，定义 $\Omega_x=\{\boldsymbol{x}\mid\|\boldsymbol{x}\|\leqslant M_x\}$，$\Omega_x$ 为一有界闭区域。

设 $f(x,\theta_f),g(x,\theta_g)$ 是两个Ⅱ型模糊逻辑系统在区域 Ω_x 上分别对 $f(x),g(x)$ 的一个逼近，即

$$f(x,\theta_f)=\frac{\sum_{l=1}^{M}y_f^l\left[\prod_{i=1}^{n}\exp\left(-\frac{(x_i-a_{if}^l)^2}{(b_{if}^l)^2+b_{0f}}\right)\right]}{\sum_{l=1}^{M}\prod_{i=1}^{n}\exp\left(-\frac{(x_i-a_{if}^l)^2}{(b_{if}^l)^2+b_{0f}}\right)} \tag{2}$$

$$g(x,\theta_g)=\frac{\sum_{l=1}^{N}y_g^l\left[\prod_{i=1}^{n}\exp\left(-\frac{(x_i-a_{ig}^l)^2}{(b_{ig}^l)^2+b_{0g}}\right)\right]}{\sum_{l=1}^{N}\prod_{i=1}^{n}\exp\left(-\frac{(x_i-a_{ig}^l)^2}{(b_{ig}^l)^2+b_{0g}}\right)} \tag{3}$$

M,N 是两个模糊逻辑系统中的规则数目。

$$\boldsymbol{\theta}_f=(y_f^1,\cdots,y_f^M,b_{1f}^1,\cdots,b_{nf}^1,\cdots,b_{1f}^M,\cdots,b_{nf}^M,a_{1f}^1,\cdots,a_{nf}^1,\cdots,a_{1f}^M,\cdots,a_{nf}^M)^{\mathrm{T}}$$

$$\boldsymbol{\theta}_g=(y_g^1,\cdots,y_g^N,b_{1g}^1,\cdots,b_{ng}^1,\cdots,b_{1g}^N,\cdots,b_{ng}^N,a_{1g}^1,\cdots,a_{ng}^1,\cdots,a_{1g}^N,\cdots,a_{ng}^N)^{\mathrm{T}}$$

是可调参数。正数 b_{0f},b_{0g} 是设计参数。令

$$\Omega_f=\{\boldsymbol{\theta}_f:\|\boldsymbol{\theta}_f\|\leqslant M_f\}$$

$$\Omega_g=\{\boldsymbol{\theta}_g:\|\boldsymbol{\theta}_g\|\leqslant M_g,y_g^l\geqslant\varepsilon,l=1,\cdots,N\}$$

$$\boldsymbol{\theta}_f^*=\arg\min_{\theta_f\in\Omega_f}[\sup_{x\in\Omega_x}|f(x,\theta_f)-f(x)|]$$

$$\boldsymbol{\theta}_g^*=\arg\min_{\theta_g\in\Omega_g}[\sup_{x\in\Omega_x}|g(x,\theta_g)-g(x)|]$$

其中正常数 M_f,M_g,ε 是设计参数。设 $\hat{\theta}_f(t)\in\Omega_f,\hat{\theta}_g(t)\in\Omega_g$ 分别是 $\boldsymbol{\theta}_f^*,\boldsymbol{\theta}_g^*$ 在 t 时刻的估计值，将 $f(x,\theta_f^*),g(x,\theta_g^*)$ 在 $\hat{\theta}_f(t),\hat{\theta}_g(t)$ 的邻域内展开成泰勒展式得

$$f(x,\theta_f^*)-f(x,\hat{\theta}_f(t))=-\boldsymbol{\phi}_f^{\mathrm{T}}(t)\frac{\partial f(x,\hat{\theta}_f)}{\partial\hat{\theta}_f}+O(\|\boldsymbol{\phi}_f(t)\|^2) \tag{4}$$

$$g(x,\theta_g^*)-g(x,\hat{\theta}_g(t))=-\boldsymbol{\phi}_g^{\mathrm{T}}(t)\frac{\partial g(x,\hat{\theta}_g)}{\partial\hat{\theta}_g}+O(\|\boldsymbol{\phi}_g(t)\|^2) \tag{5}$$

其中 $\boldsymbol{\phi}_f(t)=\hat{\theta}_f(t)-\boldsymbol{\theta}_f^*,\boldsymbol{\phi}_g(t)=\hat{\theta}_g(t)-\boldsymbol{\theta}_g^*$ 定义最优逼近误差：

$$\omega=[f(x,\theta_f^*)-f(x)]+[g(x,\theta_g^*)-g(x)]u_c-O(\|\boldsymbol{\phi}_f(t)\|^2)-O(\|\boldsymbol{\phi}_g(t)\|^2)u_c \tag{6}$$

u_c 待定。

$$\varepsilon_\omega=\max_{x\in\Omega_x,\hat{\theta}_f\in\Omega_f,\hat{\theta}_g\in\Omega_g}|f(x,\theta_f^*)-f(x)+(g(x,\theta_g^*)-g(x))u_c-O(\|\boldsymbol{\phi}_f(t)\|^2)-O(\|\boldsymbol{\phi}_g(t)\|^2)u_c| \tag{7}$$

则 ε_ω 是未知有界常数。

控制目标：使系统输出跟踪给定的参考信号 $y_r(t)$，且使跟踪误差收敛到期望的有界范围内，并保证闭环系统所有信号全局有界。

3　自适应模糊控制器的设计

引入如下坐标变换：

$$\begin{cases}z_1=x_1-y_r\\z_2=x_2-\alpha_1\\\vdots\\z_n=x_n-\alpha_{n-1}\end{cases} \tag{8}$$

设计控制器过程如下。

第1步：$i=1$ 时，

$$\dot{z}_1=x_2+\boldsymbol{\varphi}_1^{\mathrm{T}}(x_1)\boldsymbol{\theta}+\chi_1(x,t)-\dot{y}_r=z_2+\alpha_1+\boldsymbol{\omega}_1^{\mathrm{T}}\boldsymbol{\theta}+\chi_1(x,t)-\dot{y}_r$$

其中 $\boldsymbol{\omega}_1=\boldsymbol{\varphi}_1(x_1)$，$\alpha_1$ 待定。

取　$$V_1=\frac{1}{2}z_1^2+\frac{1}{2}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^{\mathrm{T}}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}) \tag{9}$$

$\hat{\boldsymbol{\theta}}$ 是对 $\boldsymbol{\theta}$ 的估计，下文同理。则

$$\dot{V}_1=z_1\dot{z}_1-(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}=z_1[z_2+\alpha_1+\boldsymbol{\omega}_1^{\mathrm{T}}\hat{\boldsymbol{\theta}}+\chi_1(x,t)-\dot{y}_r]+\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(z_1\boldsymbol{\omega}_1-\dot{\hat{\boldsymbol{\theta}}})$$

其中 $\tilde{\boldsymbol{\theta}}=\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}$。取

$$\tau_1=z_1\boldsymbol{\omega}_1-l\hat{\boldsymbol{\theta}}\ (l>0\text{ 为设计常数}) \tag{10}$$

$$\alpha_1=-c_1z_1-\boldsymbol{\omega}_1^{\mathrm{T}}\hat{\boldsymbol{\theta}}+\dot{y}_r-\frac{z_1\rho_1^2}{2k},\ (k>0\text{ 为设计常数}) \tag{11}$$

则　$$\dot{V}_1\leqslant-c_1z_1^2+z_1z_2+\frac{k}{2}+\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(z_1\boldsymbol{\omega}_1-\dot{\hat{\boldsymbol{\theta}}})=-c_1z_1^2+z_1z_2+\frac{k}{2}+l\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\hat{\boldsymbol{\theta}}+\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\tau_1-\dot{\hat{\boldsymbol{\theta}}})$$

又因为

$$2\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\hat{\boldsymbol{\theta}}=2(\boldsymbol{\theta}-\tilde{\boldsymbol{\theta}})^{\mathrm{T}}\tilde{\boldsymbol{\theta}}=2\boldsymbol{\theta}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}-2\|\tilde{\boldsymbol{\theta}}\|^2=2\boldsymbol{\theta}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}-\|\tilde{\boldsymbol{\theta}}\|^2-\|\boldsymbol{\theta}\|^2+\|\boldsymbol{\theta}\|^2-\|\tilde{\boldsymbol{\theta}}\|^2\leqslant\|\boldsymbol{\theta}\|^2-\|\tilde{\boldsymbol{\theta}}\|^2$$

所以　$$\dot{V}_1\leqslant-c_1z_1^2+z_1z_2+\frac{k}{2}+\frac{l}{2}(\|\boldsymbol{\theta}\|^2-\|\tilde{\boldsymbol{\theta}}\|^2)+\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\tau_1-\dot{\hat{\boldsymbol{\theta}}})$$

第2步：$i=2$ 时，$\dot{z}_2=\dot{x}_2-\dot{\alpha}_1=z_3+\alpha_2+\boldsymbol{\theta}^{\mathrm{T}}\varphi_2+\chi_2-\dot{\alpha}_1$，$\alpha_2$ 待定

$$\dot{\alpha}_1(x_1,\hat{\theta}_1,y_r,\dot{y}_r)=\frac{\partial\alpha_1}{\partial x_1}\dot{x}_1+\frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}}\dot{\hat{\boldsymbol{\theta}}}+\frac{\partial\alpha_1}{\partial y_r}\dot{y}_r+\frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r=\bar{\dot{\alpha}}_1+\frac{\partial\alpha_1}{\partial x_1}\boldsymbol{\varphi}_1^{\mathrm{T}}\boldsymbol{\theta}+\frac{\partial\alpha_1}{\partial x_1}\chi_1+\frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}}\dot{\hat{\boldsymbol{\theta}}}$$

其中 $\bar{\dot{\alpha}}_1=\frac{\partial\alpha_1}{\partial x_1}x_2+\frac{\partial\alpha_1}{\partial y_r}\dot{y}_r+\frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r$，即 $\dot{\alpha}_1$ 为中的已知部分

令　$$V_2=V_1+\frac{1}{2}z_2^2$$

$$\boldsymbol{\omega}_2=\varphi_2-\frac{\partial\alpha_1}{\partial x_1}\varphi_1$$

$$\tau_2=\tau_1+z_2\boldsymbol{\omega}_2 \tag{12}$$

所以
$$\dot V_2\leqslant -c_1z_1^2+z_1z_2+\frac{k}{2}+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_1-\dot{\hat{\boldsymbol\theta}})+z_2\Big[z_3+\alpha_2+\boldsymbol\omega_2^{\mathrm T}\boldsymbol\theta+(\chi_2-\frac{\partial\alpha_1}{\partial x_1}\chi_1)-\bar{\dot\alpha}_1-\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}\dot{\hat{\boldsymbol\theta}}\Big]$$

取
$$\alpha_2=-c_2z_2-z_1-\boldsymbol\omega_2^{\mathrm T}\hat{\boldsymbol\theta}+\bar{\dot\alpha}_1+\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}\tau_2-\frac{z_2\rho_2^2}{2k}-\frac{z_2(\frac{\partial\alpha_1}{\partial x_1}\rho_1)^2}{2k} \tag{13}$$

所以
$$\dot V_2\leqslant -c_1z_1^2-c_2z_2^2+z_2z_3+\frac{3}{2}k+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_2-\dot{\hat{\boldsymbol\theta}})+z_2\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}(\tau_2-\dot{\hat{\boldsymbol\theta}})$$

第3步：$\dot z_3=\dot x_3-\dot\alpha_2=z_4+\alpha_3+\boldsymbol\theta^{\mathrm T}\varphi_3+\chi_3-\dot\alpha_2$，$\alpha_3$待定。

$$\dot\alpha_2=\frac{\partial\alpha_2}{\partial x_1}\dot x_1+\frac{\partial\alpha_2}{\partial x_2}\dot x_2+\frac{\partial\alpha_2}{\partial y_r}\dot y_r+\frac{\partial\alpha_2}{\partial\dot y_r}\ddot y_r=$$
$$\bar{\dot\alpha}_2+(\frac{\partial\alpha_2}{\partial x_1}\varphi_1+\frac{\partial\alpha_2}{\partial x_2}\varphi_2)^{\mathrm T}\boldsymbol\theta+\frac{\partial\alpha_2}{\partial x_1}\chi_1+\frac{\partial\alpha_2}{\partial x_2}\chi_2+\frac{\partial\alpha_2}{\partial\hat{\boldsymbol\theta}}\dot{\hat{\boldsymbol\theta}})$$

$\bar{\dot\alpha}_2=\frac{\partial\alpha_2}{\partial x_1}x_2+\frac{\partial\alpha_2}{\partial x_2}x_3+\frac{\partial\alpha_2}{\partial y_r}\dot y_r+\frac{\partial\alpha_2}{\partial\dot y_r}\ddot y_r$，即为$\dot\alpha_2$中的已知部分

令 $V_3=V_2+\frac{1}{2}z_3^2$

$$\boldsymbol\omega_3=\boldsymbol\varphi_3-\frac{\partial\alpha_2}{\partial x_2}\boldsymbol\varphi_2-\frac{\partial\alpha_2}{\partial x_1}\boldsymbol\varphi_1$$
$$\tau_3=\tau_2+z_3\boldsymbol\omega_3 \tag{14}$$

则
$$\dot V_3\leqslant-\sum_{j=1}^{2}c_jz_j^2+z_2z_3+\frac{3}{2}k+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_2-\dot{\hat{\boldsymbol\theta}})+z_2\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}(\tau_2-\dot{\hat{\boldsymbol\theta}})+z_3\Big[z_4+\alpha_3+\boldsymbol\omega_3^{\mathrm T}\boldsymbol\theta+(\chi_3-\frac{\partial\alpha_2}{\partial x_1}\chi_1-\frac{\partial\alpha_2}{\partial x_2}\chi_2)-\bar{\dot\alpha}_2-\frac{\partial\alpha_2}{\partial\hat{\boldsymbol\theta}}\dot{\hat{\boldsymbol\theta}}\Big]$$

取
$$\alpha_3=-c_3z_3-z_2-\boldsymbol\omega_3^{\mathrm T}\hat{\boldsymbol\theta}+\bar{\dot\alpha}_2+\frac{\partial\alpha_2}{\partial\hat{\boldsymbol\theta}}\tau_3+z_2\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_3-\frac{z_3\rho_3^2}{2k}-\frac{z_3(\frac{\partial\alpha_2}{\partial x_1}\rho_1)^2}{2k}-\frac{z_3(\frac{\partial\alpha_2}{\partial x_2}\rho_2)^2}{2k} \tag{15}$$

所以
$$\dot V_3\leqslant-\sum_{j=1}^{3}c_jz_j^2+z_3z_4+3k+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_3-\dot{\hat{\boldsymbol\theta}})+z_2\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}(\tau_2-\dot{\hat{\boldsymbol\theta}})+z_3\frac{\partial\alpha_2}{\partial\hat{\boldsymbol\theta}}(\tau_3-\dot{\hat{\boldsymbol\theta}})+z_3z_2\frac{\partial\alpha_1}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_3$$

第i步：$\dot z_i=\dot x_i-\dot\alpha_{i-1}=z_{i+1}+\alpha_i+\boldsymbol\theta^{\mathrm T}\boldsymbol\varphi_i+\chi_i-\dot\alpha_{i-1}$，$3\leqslant i\leqslant n$，$\alpha_{i-1}$待定。

$$\dot\alpha_{i-1}=\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\dot x_j+\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)}=$$
$$\bar{\dot\alpha}_{i-1}+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\boldsymbol\varphi_j^{\mathrm T}\boldsymbol\theta+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\chi_j+\frac{\partial\alpha_{i-1}}{\partial\hat{\boldsymbol\theta}}\dot{\hat{\boldsymbol\theta}}$$

其中$\bar{\dot\alpha}_{i-1}=\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1}+\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)}$，即为$\dot\alpha_{i-1}$中的已知部分。

令
$$V_i=V_{i-1}+\frac{1}{2}z_i^2$$
$$\boldsymbol\omega_i=\boldsymbol\varphi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j},\ 2\leqslant i\leqslant n$$
$$\tau_i=\tau_{i-1}+z_i\boldsymbol\omega_i \tag{16}$$
$$\alpha_i=-c_iz_i-z_{i-1}-\boldsymbol\omega_i^{\mathrm T}\hat{\boldsymbol\theta}+\bar{\dot\alpha}_{i-1}+\frac{\partial\alpha_i}{\partial\hat{\boldsymbol\theta}}\tau_i+\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_i-\frac{z_i}{2k}(\rho_i^2+\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j}\rho_j)^2) \tag{17}$$

则
$$\dot V_i\leqslant-\sum_{j=1}^{i}c_jz_j^2+z_iz_{i+1}+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_i-\dot{\hat{\boldsymbol\theta}})+\sum_{j=1}^{i-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\boldsymbol\theta}}(\tau_{j+1}-\dot{\hat{\boldsymbol\theta}})+\sum_{k=3}^{i}(z_k\sum_{j=2}^{k-1}z_j\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_k)+\frac{i(i+1)}{4}k$$

第$n-1$步：
$$\dot V_{n-1}\leqslant-\sum_{j=1}^{n-1}c_jz_j^2+z_nz_{n-1}+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_{n-1}-\dot{\hat{\boldsymbol\theta}})+\sum_{j=1}^{n-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\boldsymbol\theta}}(\tau_{j+1}-\dot{\hat{\boldsymbol\theta}})+\sum_{k=3}^{n-1}(z_k\sum_{j=2}^{k-1}z_j\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_k)+\frac{n(n-1)}{4}k$$

第n步：令
$$V_n=V_{n-1}+\frac{1}{2}z_n^2+\frac{1}{2\gamma_1}\boldsymbol\phi_f^{\mathrm T}\boldsymbol\phi_f+\frac{1}{2\gamma_2}\boldsymbol\phi_g^{\mathrm T}\boldsymbol\phi_g+\frac{1}{2\gamma_3}(\hat\varepsilon_\omega-\varepsilon_\omega)^2 \tag{18}$$
$$\dot V_n=\dot V_{n-1}+z_n(f(x)+g(x)u+\boldsymbol\theta^{\mathrm T}\varphi_n+\chi_n(x,t)-\dot\alpha_{n-1})+\frac{1}{\gamma_1}\boldsymbol\phi_f^{\mathrm T}\dot{\hat{\boldsymbol\theta}}_f+\frac{1}{\gamma_2}\boldsymbol\phi_g^{\mathrm T}\dot{\hat{\boldsymbol\theta}}_g+\frac{1}{\gamma_3}(\hat\varepsilon_\omega-\varepsilon_\omega)\dot{\hat\varepsilon}_\omega \tag{19}$$
$$\dot\alpha_{n-1}=\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\dot x_j+\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(j)}}y_r^{(j+1)}=$$
$$\bar{\dot\alpha}_{n-1}+(\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\boldsymbol\varphi_j)^{\mathrm T}\boldsymbol\theta+\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\chi_j+\frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol\theta}}\dot{\hat{\boldsymbol\theta}}$$

其中$\bar{\dot\alpha}_{n-1}=\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1}+\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(j)}}y_r^{(j+1)}$，即为$\dot\alpha_{n-1}$中的已知部分。

采用如下控制律：
$$u_c=\frac{1}{\hat g(x)}\Big[-\hat f(x)-c_nz_n-z_{n-1}-\boldsymbol\omega_n^{\mathrm T}\hat{\boldsymbol\theta}+\bar{\dot\alpha}_{n-1}+\frac{\partial\alpha_{n-1}}{\partial\hat{\boldsymbol\theta}}\tau_n+\sum_{j=1}^{n-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_n-\frac{z_n}{2k}(\rho_n^2+\sum_{j=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_j}\rho_j)^2)\Big] \tag{20}$$
$$u=u_c-\frac{\hat\varepsilon_\omega}{k_1(x)}\mathrm{sgn}(z_n) \tag{21}$$

所以
$$\dot V_n\leqslant-\sum_{j=1}^{n}c_jz_j^2+\frac{l}{2}(\|\boldsymbol\theta\|^2-\|\tilde{\boldsymbol\theta}\|^2)+\tilde{\boldsymbol\theta}^{\mathrm T}(\tau_n-\dot{\hat{\boldsymbol\theta}})+\sum_{j=1}^{n-1}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\boldsymbol\theta}}(\tau_{j+1}-\dot{\hat{\boldsymbol\theta}})+\frac{n(n+1)}{4}k+\sum_{k=3}^{n}(z_k\sum_{j=2}^{k-1}z_j\frac{\partial\alpha_{j-1}}{\partial\hat{\boldsymbol\theta}}\boldsymbol\omega_k)+$$

$$z_n[-\boldsymbol{\omega}-\boldsymbol{\phi}_f^{\mathrm{T}}(t)\frac{\partial\hat{f}(x,\hat{\boldsymbol{\theta}}_f)}{\partial\hat{\boldsymbol{\theta}}_f}-\boldsymbol{\phi}_g^{\mathrm{T}}(t)\frac{\partial\hat{g}(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_g}u_c]-$$

$$\frac{g(x)}{K_1(x)}\hat{\varepsilon}_\omega|z_n|+\frac{1}{\gamma_1}\boldsymbol{\phi}_f^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_f+\frac{1}{\gamma_2}\boldsymbol{\phi}_g^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_g+\frac{1}{\gamma_3}(\hat{\varepsilon}_\omega-\varepsilon_\omega)\dot{\hat{\varepsilon}}_\omega \tag{22}$$

采用如下自适应律：

$$\dot{\hat{\boldsymbol{\theta}}}=\tau_n \tag{23}$$

其中

$$\tau_n=\tau_{n-1}+\boldsymbol{\omega}_n z_n \tag{24}$$

$$\dot{\hat{\boldsymbol{\theta}}}_f=\begin{cases}\gamma_1 z_n\dfrac{\partial f(x,\hat{\boldsymbol{\theta}}_f)}{\partial\hat{\boldsymbol{\theta}}_f} \\ \quad 当\|\hat{\boldsymbol{\theta}}_f\|<M_f或(\|\hat{\boldsymbol{\theta}}_f\|=M_f且z_n\hat{\boldsymbol{\theta}}_f^{\mathrm{T}}\dfrac{\partial f(x,\hat{\boldsymbol{\theta}}_f)}{\partial\hat{\boldsymbol{\theta}}_f}\leqslant 0) \\ \gamma_1 z_n\dfrac{\partial f(x,\hat{\boldsymbol{\theta}}_f)}{\partial\hat{\boldsymbol{\theta}}_f}-\gamma_1 z_n\dfrac{\hat{\boldsymbol{\theta}}_f\boldsymbol{\theta}_f^{\mathrm{T}}}{\|\hat{\boldsymbol{\theta}}_f\|^2}\dfrac{\partial f(x,\hat{\boldsymbol{\theta}}_f)}{\partial\hat{\boldsymbol{\theta}}_f} \\ \quad 当\|\hat{\boldsymbol{\theta}}_f\|=M_f且z_n\hat{\boldsymbol{\theta}}_f^{\mathrm{T}}\dfrac{\partial f(x,\hat{\boldsymbol{\theta}}_f)}{\partial\hat{\boldsymbol{\theta}}_f}>0\end{cases} \tag{25}$$

当$\hat{y}_g^l(t)=\varepsilon$时

$$\dot{\hat{y}}_g^l=\begin{cases}\gamma_2 z_n u_c\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{y}_g^l}, & 当z_n u_c\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{y}_g^l}\geqslant 0 \\ 0, & 当z_n u_c\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{y}_g^l}<0\end{cases} \tag{26}$$

否则

$$\dot{\hat{\boldsymbol{\theta}}}_{g+}=\begin{cases}\gamma_2 z_n u_c\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g+}} \\ \quad 当\|\hat{\boldsymbol{\theta}}_g\|<M_g或(\|\hat{\boldsymbol{\theta}}_g\|=M_g且 \\ z_n u_c[\hat{\boldsymbol{\theta}}_{g+}^{\mathrm{T}}\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g+}}+\hat{\boldsymbol{\theta}}_{g\varepsilon 1}^{\mathrm{T}}\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g\varepsilon 1}}]\leqslant 0) \\ \gamma_2 z_n u_c\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g+}}-\gamma_2 z_n u_c\hat{\boldsymbol{\theta}}_{g+}\dfrac{[\hat{\boldsymbol{\theta}}_{g+}^{\mathrm{T}}\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g+}}+\hat{\boldsymbol{\theta}}_{g\varepsilon 1}^{\mathrm{T}}\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g\varepsilon 1}}]}{\|\hat{\boldsymbol{\theta}}_{g+}\|^2} \\ \quad 当\|\hat{\boldsymbol{\theta}}_g\|=M_g且z_n u_c[\hat{\boldsymbol{\theta}}_{g+}^{\mathrm{T}}\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g+}}+\hat{\boldsymbol{\theta}}_{g\varepsilon 1}^{\mathrm{T}}\dfrac{\partial g(x,\hat{\boldsymbol{\theta}}_g)}{\partial\hat{\boldsymbol{\theta}}_{g\varepsilon 1}}]>0\end{cases} \tag{27}$$

$$\dot{\hat{\varepsilon}}_\omega=\gamma_3(|z_n|-\sigma\hat{\varepsilon}_\omega) \tag{28}$$

其中$\varepsilon>0$是一常数，$\gamma_1>0$，$\gamma_2>0$，$\gamma_3>0$均为自适应律，$\hat{\boldsymbol{\theta}}_{g+}(t)$是$\hat{\boldsymbol{\theta}}_g(t)$将中删除满足式(26)的所有分量后所得的参数估计向量，$\hat{\boldsymbol{\theta}}_{g\varepsilon 1}(t)$是$\hat{\boldsymbol{\theta}}_g(t)$中满足式(26)第一行的所有分量所构成的列向量。

由式(10)，(12)，(14)，(16)及式(23)~(28)得：

$$\dot{V}_n\leqslant-\sum_{j=1}^{n}c_j z_j^2+\frac{l}{2}(\|\boldsymbol{\theta}\|^2-\|\tilde{\boldsymbol{\theta}}\|^2)+$$

$$\frac{n(n+1)}{4}k+\frac{\sigma}{2}[\varepsilon_\omega^2-(\hat{\varepsilon}_\omega-\varepsilon_\omega)^2] \tag{29}$$

由式(25)~(27)得$\boldsymbol{\phi}_f^{\mathrm{T}}\boldsymbol{\phi}_f\leqslant 2M_f$，$\boldsymbol{\phi}_g^{\mathrm{T}}\boldsymbol{\phi}_g\leqslant 2M_g$，取$c_{\min}=\min(c_1,c_2,\cdots,c_n)$，$\min(l,\gamma_3\sigma)>2c_{\min}$并取$k$足够小，则$\exists M>0$，$\ni\frac{l}{2}\|\boldsymbol{\theta}\|^2+\frac{n(n+1)}{4}k+\frac{\sigma_\omega}{2}\varepsilon_w^2+\frac{2c_{\min}}{\gamma_1}M_f+\frac{2c_{\min}}{\gamma_2}M_g<M$，并令$\lambda=2c_{\min}$，则

$$\dot{V}_n\leqslant-\lambda V_n+M \tag{30}$$

因为 $\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{e}^{\lambda t}V_n)\leqslant\lambda\mathrm{e}^{\lambda t}V_n+\mathrm{e}^{\lambda t}(-\lambda V_n+M)=M\mathrm{e}^{\lambda t}$

所以 $\mathrm{e}^{\lambda t}V_n\leqslant V_n(0)+\int_0^t M\mathrm{e}^{\lambda t}\mathrm{d}t\leqslant V_n(0)+\frac{M}{\lambda}(\mathrm{e}^{\lambda t}-1)$

所以 $V_n(t)\leqslant\frac{V_n(0)}{\mathrm{e}^{\lambda t}}+\frac{M}{\lambda}-\frac{M}{\lambda\mathrm{e}^{\lambda t}}$

所以，当$t\to\infty$，$V_n(t)\to\frac{M}{\lambda}$为一有界量。

所以，V_n有界，$z_1,\cdots,z_n$，$\alpha_1,\cdots,\alpha_{n-1}$，$\hat{\boldsymbol{\theta}}$有界。故$x$，$u_c$有界。

4　稳定性分析

定理　在假设(1)~(5)下，考虑系统(1)，用控制器(20)~(21)，参数自适应律(23)~(28)，则闭环系统所有信号有界，且$|y_1-y_r|\leqslant\sqrt{2V(0)\mathrm{e}^{-\lambda t}}+\sqrt{\frac{M}{\lambda}}$。

证明　由上述分析过程易得，不再赘述。

5　仿真结果

考虑不确定非线性系统

$$\begin{cases}\dot{x}_1=x_2+\boldsymbol{\theta}x_1^2+x_1\sin(t)\sin(x_2) \\ \dot{x}_2=x_1^2\sin x_2+(2-0.5\sin^2(x_1))u+\boldsymbol{\theta}x_2^2+0.5x_2\sin(t) \\ y=x_1\end{cases}$$

参考信号为$y_r=0.5(\sin(t)+\sin(0.5t))$，分析可知$F(x)=x_1^2$，$K_1(x)=1.5$，$K_2(x)=2$，$\rho_1(x)=|x_1|$，$\rho_2(x)=|x_2|$，跟踪误差$e_1=y-y_r$，$\boldsymbol{x}(0)=[0,0.5]^{\mathrm{T}}$，模糊系统的固定参数选取$M_1=M_2=2$，$\boldsymbol{a}_1=\boldsymbol{a}_2=[-2,-1,0,1,2]^{\mathrm{T}}$，$b_1=b_2=0.6$，$b_{0f}=b_{0g}=0.001$　，其他参数选取$c_1=1.5$；$c_2=1.5$，$l=4$，$k=0.5$，$\hat{\boldsymbol{\theta}}_f(0)=2*\mathrm{rand}(5,1)-1$；$\hat{\boldsymbol{\theta}}_g(0)=2*\mathrm{rand}(5,1)$；$\hat{\boldsymbol{\theta}}(0)=0.5$，$\gamma_1=2.5$，$\gamma_2=2.5$，$\gamma_3=1.5$，$\hat{\varepsilon}(0)=0.1$，$\sigma=3$。仿真结果如图1，图2所示。

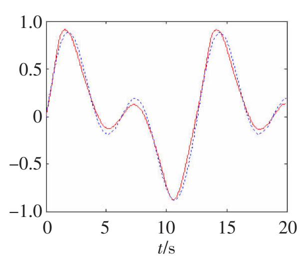

图1　系统输出y、跟踪曲线y_d

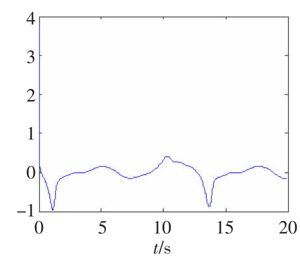

图2　控制律u

6　结论

针对不确定非线性系统，利用模糊逻辑系统逼近非线性不确定项，同时基于backstepping方法设计了控制律，并通过引入逼近误差补偿项，证明了跟踪误差收敛到期望的性能指标。

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