

The Relative Roles of Diapycnal and Isopycnal Mixing on Subsurface Water Mass Conversion

TREVOR J. McDOUGALL

CSIRO, Division of Oceanography, Hobart, Tasmania, 7001, Australia

(Manuscript received 20 May 1983, in final form 9 July 1984)

ABSTRACT

Fluid motion in the sea is known to occur predominantly along quasi-horizontal neutral surfaces but the very small diapycnal (i.e., across isopycnal) velocities often make a significant contribution to the conservation equations of heat, salt and tracer. By eliminating the diapycnal advection term between the conservation equations for (i) heat and (ii) salt, an equation is derived for the rate of change (Lagrangian derivative) of potential temperature θ on a neutral surface which has terms caused by (a) turbulent mixing along isopycnal surfaces (i.e., isopycnal mixing), (b) diapycnal turbulent mixing and (c) double-diffusive convection. Because of the nature of the isopycnal reference frame, the diapycnal mixing terms do not take their expected forms. For example, the diapycnal turbulent mixing term is proportional to the diapycnal eddy diffusivity D multiplied by the curvature of the θ - S curve, $d^2S/d\theta^2$, rather than the usual form $(D\theta_z)_z$. If the θ - S curve is locally straight, small-scale turbulent mixing can have no effect on the temperature (or salinity) measured on an isopycnal surface. For values of the θ - S curvature appropriate to the Central Waters of the World's Oceans, the rate at which diapycnal turbulent mixing changes potential temperature on isopycnals is a fraction of $D\theta_{zz}$ (say $0.15D\theta_{zz}$). This surprising result is due to the ability of the isopycnal surface to migrate quasi-vertically through the water column (or equivalently, for water to move diapycnally through the isopycnal surface) in response to the divergence ($\nabla \cdot$) of the fluxes of both heat and salt. In the interpretation of oceanographic data sets, it is not yet possible to estimate the diapycnal advection velocity and so it is customarily omitted from the conservation equations. It is the main aim of this paper to show that by so neglecting the diapycnal advective terms, the diapycnal mixing processes enter the conservation equations in greatly altered forms. The conservation equations for scalars (both active and passive) on a neutral surface which we develop are then the appropriate equations to be used in future studies of subsurface water mass conversion.

1. Introduction

The procedure of analyzing hydrographic data by interpolating the temperature, salinity, etc., to a selected number of isopycnal surfaces is becoming more common and the present paper is concerned with the conservation equations which apply to properties on these isopycnal surfaces. Because the diapycnal velocity is very small in relation to those in the isopycnal surface, it is tempting simply to ignore the diapycnal advection term, and indeed, this has been done in the past. However the diapycnal velocity multiplies the largest spatial gradient (the diapycnal gradient) and the diapycnal advection term is the same order of magnitude as the other diapycnal terms in the conservation equations. By eliminating the diapycnal advection term between the conservation equations for heat and salt, conservation equations are derived which do not contain the diapycnal velocity, but as a result, the other diapycnal mixing terms are changed. Some of the consequences of these changes are discussed in this paper for various regions of the ocean.

Velocity and temperature microstructure data from both dropped and towed bodies have been used for

more than a decade to estimate the importance of small-scale mixing processes in the ocean. Individual mixing events are highly intermittent in both space and time and the amount of microstructure data that is needed for statistically significant averages needs careful consideration (Gibson, 1982). However, there seems to be general agreement that values of the diapycnal eddy diffusivity in the upper kilometer of the open ocean are less than $1 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$; perhaps much less (Garrett, 1979). Garrett (1984) has recently reviewed measurements of the diapycnal eddy diffusivity and she suggests that it is proportional to the reciprocal of the buoyancy frequency N . Another recent review of small-scale mixing processes by Caldwell (1983) discusses the increasing evidence for the importance of salt fingers and double-diffusive interleaving in the ocean. While double-diffusive convection has been known for some time to be important in isolated regions of the ocean, Schmitt and Evans (1978) were the first to estimate the flux of salt due to salt fingers in a region of the open ocean, namely the Central Water of the North Atlantic. They used the laboratory-derived fluxes of heat and salt together with finescale profiles of temperature and salinity to

estimate an apparent eddy diffusivity for salt (due to fingers) of up to $0.5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$. Schmitt (1981) has used a linearized analysis of the vertical flux divergence due to salt-fingering to show that deviations of the stability ratio R_ρ in the water column will quickly decay. This is an appealing mechanism to explain the smoothness of the T - S curve of the Central Waters, and this model is discussed in more detail later in this paper.

Computer models of the general circulation of the oceans generally have a strictly horizontal diffusivity and a strictly vertical diffusivity. A sloping isopycnal surface will have a diapycnal flux driven across it both by the horizontal and the vertical fluxes so that diapycnal mixing occurs even if the vertical diffusivity is zero. In this way, the importance of diapycnal mixing processes is understated by merely quoting the best-fit vertical diffusivity. Sarmiento (1983) has also shown that strictly horizontal and strictly vertical mixing results in less vertical penetration of tritium into the thermocline than would be achieved by isopycnal and diapycnal mixing. There are two different approaches to overcoming this problem. In one approach (Bleck and Boudra, 1981), the computer grid is made to coincide with isopycnals and so the diffusivities operate in the isopycnal and diapycnal directions. The other approach (Redi, 1982; Solomon, 1971) retains the original horizontal and vertical spacing of grid points in the computer and expresses the flux divergence caused by given isopycnal and diapycnal diffusivities by using a suitable local rotation of the coordinates. It is hoped that these physically more realistic parameterizations will be used in future numerical models so that diapycnal mixing will be caused by the diapycnal diffusivity alone.

2. The conservation equations in the isopycnal coordinate frame

The definition of neutral surfaces along which fluid parcels can flow isentropically is complicated by the temperature dependence of the compressibility of sea water (Bray and Fofonoff, 1981; Schmitt and Georgi, 1982). Shepherd (1984) has considered the exchange of fluid parcels along neutral surfaces and has shown that a non-steady horizontal flow field leads to diapycnal advection caused by variation of the compressibility of sea water with potential temperature. He has coined the expression "fuzziness of neutral surfaces" to express the fact that variations of potential temperature on a neutral surface in conjunction with isopycnal motion lead to an ambiguity in defining the position of the surface. In this paper the conservation equations are cast in terms of time-mean properties and the turbulent fluxes due to the correlation of velocity with property variations are parameterized by diffusion coefficients. The neutral surfaces can be unambiguously defined for the mean field.

We regard the *in situ* density ρ to be a function of salinity S , potential temperature θ and pressure p . The reference pressure for potential temperature could be any fixed pressure for the purpose of this paper, but it is convenient to define this as atmospheric pressure. Denoting the two dimensional gradient operator in the tangent plane of the local neutral surface by ∇_i we have

$$\rho^{-1} \nabla_i \rho = \beta \nabla_i S - \alpha \nabla_i \theta + \gamma \nabla_i p,$$

where

$$\alpha = \alpha(S, \theta, p) = -\rho^{-1} \left. \frac{\partial \rho}{\partial \theta} \right|_{S,p},$$

$$\beta = \beta(S, \theta, p) = \rho^{-1} \left. \frac{\partial \rho}{\partial S} \right|_{\theta,p},$$

$$\gamma = \gamma(S, \theta, p) = \rho^{-1} \left. \frac{\partial \rho}{\partial p} \right|_{\theta,S},$$

and θ , S , p and ρ denote the values of the mean field. Note that these definitions of α , β and γ are different to the normal ones which are defined in terms of the *in situ* temperature T rather than the potential temperature θ . Gill (1982) derives the following relationships between our α and β and the normal values $\hat{\alpha}$ [$\equiv -\rho^{-1}(\partial \rho / \partial T)_{S,p}$] and $\hat{\beta}$ [$\equiv \rho^{-1}(\partial \rho / \partial S)_{T,p}$]

$$\alpha = \frac{\hat{\alpha}}{(\partial \theta / \partial T)_{S,p}}, \quad \beta = \hat{\beta} + \alpha(\partial \theta / \partial S)_{T,p}.$$

Similarly, it can be shown that our compressibility coefficient γ is related to the normal term $\hat{\gamma}$ [$\equiv \rho^{-1}(\partial \rho / \partial p)_{T,S}$] by

$$\gamma = \hat{\gamma} + \alpha \left(\frac{\partial \theta}{\partial p} \right)_{T,S}.$$

The normal values $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are readily obtained by differentiating the International equation of state of Millero and Poisson (1981). The partial derivatives of θ with respect to T , S and p which appear above can be evaluated from the polynomial for $\theta(T, S, p)$ of Bryden (1973).

The buoyancy frequency N of a water column is a well understood quantity which is defined (Gill, 1982, sections 3.6 and 3.7) as $-g/\rho$ times the vertical *in situ* density gradient of the water column minus the vertical gradient of *in situ* density that a fluid parcel would experience if moved isentropically. It follows that

$$g^{-1} N^2 \equiv -\rho^{-1} \rho_z + \rho^{-1} \rho_z|_{\text{isentropic}} = \hat{\alpha}(T_z + \Gamma) - \hat{\beta} S_z,$$

where Γ is the adiabatic lapse rate. The corresponding definition when the *in situ* density ρ is regarded as a function of θ , S and p (rather than T , S and p) is

$$g^{-1} N^2 = \gamma \rho_z - \rho^{-1} \rho_z = \alpha \theta_z - \beta S_z$$

since both θ and S remain unchanged during an isentropic process. Neutral vertical static stability is

defined as $N^2 = 0$, that is, when $\alpha\theta_z = \beta S_z$. Note that this is an exact relationship which includes all the subtle effects of the compressibility of sea water. These stability arguments can be extended to consider the movement of water parcels in directions other than vertical. In particular, if a water parcel is moved isentropically (implying conservation of θ and S) along a neutral surface, the gradient of *in situ* density in this direction will be given by $\rho^{-1}\nabla_i\rho = \gamma\nabla_i p$. The spatial plane on which this identity holds is the local tangent plane of the neutral surface or isopycnal surface. When θ and S vary along the neutral surface, the local tangent plane is still given by $\rho^{-1}\nabla_i\rho = \gamma\nabla_i p$ and this is exactly equivalent to the equation $\beta\nabla_i S = \alpha\nabla_i\theta$. The fact that this relation can be unambiguously used to trace a neutral surface in 3-D space from oceanographic CTD casts, does not mean that such a surface has associated with it a constant value of any particular property. For example, potential density, referred to any particular pressure varies significantly along a neutral surface. Given the mean fields of θ , S and p as single-valued functions of space, the neutral surfaces of the mean field are uniquely defined everywhere by the relation $\beta\nabla_i S = \alpha\nabla_i\theta$. We will use the term "isopycnal surface" synonymously with "neutral surface" but will totally avoid the use of potential density.

We define the diapycnal coordinate d in the direction normal to the local tangent plane of a neutral surface. The unit vector in the d direction is \mathbf{f} and is directed in the "upward" direction (see Fig. 1). The mixing of potential temperature θ and salinity S along a neutral surface is parameterized by an isopycnal eddy-diffusion coefficient K . The mixing of properties across neutral surfaces (i.e., diapycnal mixing) occurs by (i) small-scale turbulent processes which are parameterized by a diapycnal eddy diffusivity D and (ii), by double-diffusive fluxes of salt F^S and of potential temperature F^θ (both defined positive in the diapycnal direction \mathbf{f}). The conservation equations for potential temperature and salinity on an isopycnal surface are

$$\theta_t + \mathbf{V}^i \cdot \nabla_i \theta + e\theta_d = \nabla_i \cdot (K\nabla_i \theta) + (D\theta_d)_d - F_d^\theta, \quad (1)$$

$$S_t + \mathbf{V}^i \cdot \nabla_i S + eS_d = \nabla_i \cdot (K\nabla_i S) + (DS_d)_d - F_d^S, \quad (2)$$

where \mathbf{V}^i is the (two-dimensional) velocity in the local isopycnal plane and e is the diapycnal speed of fluid past (or through) the isopycnal surface in the diapycnal

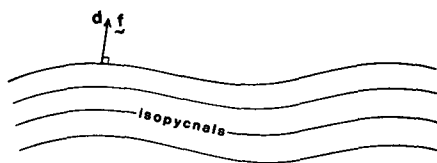


FIG. 1. A sketch of the diapycnal coordinate d which increases in the "upward" direction.

direction \mathbf{f} . The subscripts t and d denote differentiation with respect to time and the diapycnal spatial coordinate respectively. It is perhaps easier to think of the diapycnal velocity as (minus) the sinking velocity of an isopycnal surface through the water column due to the other terms in (1) and (2). Processes in which neutral surfaces move with water particles (such as internal gravity waves) do not contribute to e . The diapycnal speed e corresponds to sdp/ds in Bleck and Boudra's (1981) terminology.

It is important to note that the symbols θ and S in (1) and (2) are the potential temperature and salinity *evaluated on a given neutral surface* rather than at a fixed depth in the water column.

By a similar argument which led to $\beta\nabla_i S = \alpha\nabla_i\theta$ before, we find that $\rho^{-1}\rho_t = \gamma p_t$ on a neutral surface, and so

$$\beta S_t = \alpha\theta_t; \quad \beta\nabla_i S = \alpha\nabla_i\theta \quad (3)$$

on a neutral surface. These constraints can be used with (1) and (2) to produce two other useful equations. Taking β times Eq. (2) minus α times Eq. (1), we obtain

$$(e - D_d)g^{-1}N^2 = K(\alpha\nabla_i^2\theta - \beta\nabla_i^2S) + D(\alpha\theta_{dd} - \beta S_{dd}) + \beta F_d^S(1 - r), \quad (4)$$

where $r \equiv \alpha F_d^\theta / (\beta F_d^S)$ and $g^{-1}N^2 = \alpha\theta_d - \beta S_d$.

Since the diapycnal velocity through isopycnals, $e\mathbf{f}$, is too small to be directly measured, we can obtain another useful relation from (1) and (2) by eliminating e between them and using (3) to obtain a more practically useful equation for the rate of change of potential temperature on a neutral surface,

$$\theta_t + (\mathbf{V}^i - \nabla_i K) \cdot \nabla_i \theta = K\nabla_i^2\theta \frac{[(\theta_d/S_d)(\nabla_i^2 S/\nabla_i^2\theta) - 1]}{[R_\rho - 1]} + \frac{D\theta_{dd}}{(R_\rho - 1)} \left[\frac{\theta_d S_{dd}}{S_d \theta_{dd}} - 1 \right] - \frac{\beta}{\alpha} F_d^S \frac{[R_\rho - r]}{[R_\rho - 1]}. \quad (5)$$

Here R_ρ is defined as $\alpha\theta_d/\beta S_d$ and is equal to 1 when the water column is neutrally stable ($N^2 = 0$).

We now consider the relative magnitudes of the terms that appear in (4) and (5) and then we apply (5) (or, in fact, its simplified form (5') below) to various regions in the ocean. The reader who wishes to avoid the mathematical details could proceed straight to Section 2c.

a. *The terms in the diapycnal velocity equation (4)*

Equation (4) can be regarded as giving the speed $-e$ at which neutral surfaces migrate through the water column on account of both diapycnal mixing processes and a contribution $K(\alpha\nabla_i^2\theta - \beta\nabla_i^2S)$ from isopycnal mixing which would be zero if the equation of state were linear. The left-hand side of (4) contains the diapycnal advection terms due to e and D_d (which behaves like a diapycnal velocity).

1) THE CABBELING AND COMPRESSIBILITY TERMS
IN (4)

To understand better the first term on the right-hand side of (4), we take the isopycnal divergence of the identity $\beta \nabla_i S = \alpha \nabla_i \theta$ to obtain

$$K(\alpha \nabla_i^2 \theta - \beta \nabla_i^2 S) = -K|\nabla_i \theta|^2 \left[\frac{\partial \alpha}{\partial \theta} + 2 \frac{\alpha}{\beta} \frac{\partial \alpha}{\partial S} - \frac{\alpha^2}{\beta^2} \frac{\partial \beta}{\partial S} \right] - K \nabla_i \theta \cdot \nabla_i p \left[\frac{\partial \alpha}{\partial p} - \frac{\alpha}{\beta} \frac{\partial \beta}{\partial p} \right]. \quad (4A)$$

The term $\partial \alpha / \partial \theta$ is approximately $10^{-5} \text{ } ^\circ\text{C}^{-2}$ and the other terms in the first square bracket add up to about 10% of $\partial \alpha / \partial \theta$ (Garrett and Horne, 1978). The term $\partial \alpha / \partial p$ is approximately $2 \times 10^{-8} \text{ } (^\circ\text{C})^{-1} \text{ (db)}^{-1}$ and $-(\alpha/\beta)(\partial \beta / \partial p)$ is approximately $2 \times 10^{-9} \text{ } ^\circ\text{C}^{-1} \text{ db}^{-1}$ (from Gill 1982). In what follows we adopt the shorthand practice of denoting the two square brackets by their leading terms, $\partial \alpha / \partial \theta$ and $\partial \alpha / \partial p$. This is not an approximation procedure but merely an effort to economize on the amount of new nomenclature in the paper.

The contribution of $-K|\nabla_i \theta|^2(\partial \alpha / \partial \theta)$ to (4) is due to the cabbeling process which can be understood by considering the mixing along a neutral surface between two water columns as shown in the θ - S diagram of Fig. 2(a). When a water parcel b mixes in equal proportions with water parcel e, the mixture has properties given by point h which is on the straight line joining points b and e on Fig. 2(a) (since both θ and S are conservative quantities). In this way isopycnal mixing leads to diapycnal displacement of the mixed water parcel. The kink in the CTD casts abc and def migrates downwards across the neutral surface (from b and e to h) on account of the cabbeling process. From (4) and (4A) it can be seen that the diapycnal velocity associated with the cabbeling process is $e_c = -gN^{-2}K|\nabla_i \theta|^2(\partial \alpha / \partial \theta)$. This diapycnal velocity affects the conservation equations for θ and S in the ratio $\theta_d : S_d$ (see (1) and (2)). The fluxes of heat and salt across neutral surfaces by this mechanism are $e_c \theta$ and $e_c S$.

The contribution of the term $-K \nabla_i \theta \cdot \nabla_i p (\partial \alpha / \partial p)$ to (4) is due to the variation of the compressibility of sea water with potential temperature. (Note that $(\partial \alpha / \partial p) - (\alpha/\beta)(\partial \beta / \partial p) = -(\partial \gamma / \partial \theta) - (\alpha/\beta)(\partial \gamma / \partial S)$.) This effect can be illustrated by considering the quasi-horizontal movement of two parcels of fluid with different temperatures. Figure 2(b) illustrates the fate of two parcels of fluid that begin on the same neutral isopycnal surface a distance L apart, move quasi-horizontally, meet and subsequently mix at a distance half-way between their original locations. The neutral surface is defined in terms of the mean S , θ and p single-valued functions of space and the vertical axis of figure 2(b) is the difference between the *in situ* density and that of the neutral surface at the same

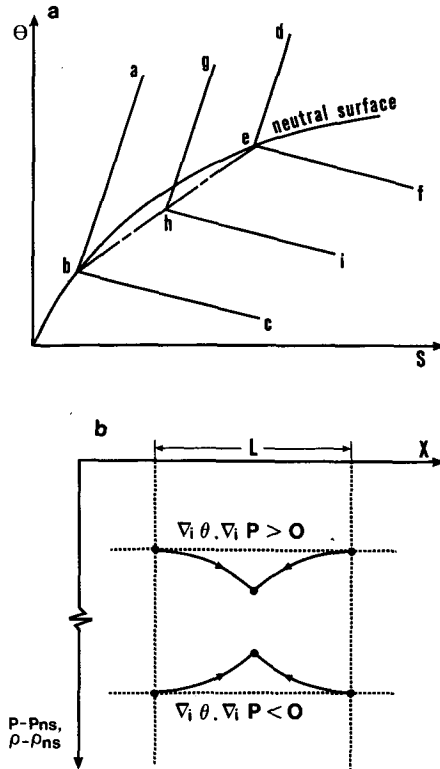


FIG. 2. (a) Temperature (θ) and salinity (S) diagram showing how the cabbeling process causes the production of more dense water by isopycnal mixing between water masses b and e; abc and def are adjacent CTD casts and by mixing in equal proportions along straight lines (on the θ - S diagram) the resultant water column is given by ghi. (b) Diagram showing the effect of the temperature dependence of compressibility ($\partial \gamma / \partial \theta = -\partial \alpha / \partial p$) acting in conjunction with quasi-horizontal motion of temperature anomalies. Water parcels a distance L apart are moved isentropically towards each other and their paths are shown by the arrowed loci as the difference between the *in situ* density $\rho(x)$ and that of the neutral surface $\rho_{ns}(x)$ of the mean properties.

location. If $\nabla_i \theta \cdot \nabla_i p > 0$, the parcels move below the neutral surface and if $\nabla_i \theta \cdot \nabla_i p < 0$ they rise above it. The *in situ* density at the mid-point $\rho(L/2)$, is readily shown to be

$$\rho\left(\frac{L}{2}\right) - \rho_{ns}\left(\frac{L}{2}\right) = \nabla_i \theta \cdot \nabla_i p \frac{\partial \alpha}{\partial p} \frac{1}{2} \left(\frac{L}{2}\right)^2 [\rho + \gamma \rho^2 g^2 N^{-2}]$$

[to first order in $(L/2)^2$]. The definition of N^2 [$= -g(\rho^{-1} \rho_d - \gamma p_d)$] and the hydrostatic relation $p_d = -g\rho$ leads to the following expression for the pressure difference

$$p - p_{ns} = \rho g^2 N^{-2} \frac{\partial \alpha}{\partial p} \nabla_i \theta \cdot \nabla_i p \frac{1}{2} \left(\frac{L}{2}\right)^2.$$

The final intimate mixing between the water parcels will not occur by the mesoscale eddies that cause the isopycnal diffusivity K , but rather by shear dispersion by inertial and internal waves. The small-scale iso-

pycnal diffusivity due to this process is about $(N^2/f^2)D$ where f is the Coriolis frequency (Young *et al.*, 1982; Garrett, 1983).

From (4) and (4A) it is apparent that the compressibility (or "fuzziness of neutral surfaces") process contributes to the diapycnal velocity by the amount $e_f = -gN^{-2}K\nabla_\theta \cdot \nabla_i p (\partial\alpha/\partial p)$. This process can be represented on the θ - S diagram in the same way as the cabbelling process [Fig. 2(a)]. The curvature of the neutral surfaces on the θ - S diagram is now due to the variation of α with pressure rather than to the variation of α with θ which is the cause of cabbelling. The curvature of the neutral surface on the θ - S diagram is now proportional to $\nabla_\theta \cdot \nabla_i p (\partial\alpha/\partial p)$ and so can be positive or negative. The fluxes of heat and salt across a neutral surface due to this "fuzziness" process are given by $e_f\theta$ and e_fS .

2) THE DIAPYCNAL MIXING TERMS IN (4)

The second term on the right-hand side of (4) is due to diapycnal turbulent mixing. Taking the derivative with respect to the diapycnal coordinate d of $g^{-1}N^2 = \alpha\theta_d - \beta S_d$, we find

$$D(\alpha\theta_{dd} - \beta S_{dd}) = Dg^{-1}(N^2)_d - D\theta_d^2 \left[\frac{\partial\alpha}{\partial\theta} + \frac{2}{R_p} \frac{\alpha}{\beta} \frac{\partial\alpha}{\partial S} - \frac{\alpha^2}{\beta^2 R_p^2} \frac{\partial\beta}{\partial S} \right] - D\theta_d p_d \left[\frac{\partial\alpha}{\partial p} - \frac{\alpha}{\beta R_p} \frac{\partial\beta}{\partial p} \right]. \quad (4B)$$

If p has units of decibars and d is in meters, then $p_d \approx -1 \text{ db m}^{-1}$. The square brackets in this equation are again dominated by their leading terms and we use this fact below simply to reduce the number of terms in Eq. (4').

The last term in Eq. (4) is due to double-diffusive convection. Restricting attention to parts of a water column where potential temperature and salinity change monotonically with depth we may assume that the buoyancy flux ratio $R_f = \alpha F^\theta / \beta F^S$ is constant, and for salt fingers, equal to about 0.6 (Schmitt, 1979; McDougall and Taylor, 1984). By differentiating this expression for R_f with respect to the diapycnal coordinate d , it is found that

$$\beta F_d^S (1 - r) = \beta F_d^S (1 - R_f) + \left(\alpha_d - \frac{\alpha}{\beta} \beta_d \right) F^\theta. \quad (4C)$$

b. The terms in the potential-temperature conservation equation

The first term on the right-hand side of (5) can be easily manipulated to the form

$$K\nabla_i^2\theta \frac{[(\theta_d/S_d)(\nabla_i^2 S/\nabla_i^2\theta) - 1]}{[R_p - 1]} = K\nabla_i^2\theta + \frac{R_p}{\alpha(R_p - 1)} K[\beta\nabla_i^2 S - \alpha\nabla_i^2\theta]. \quad (5A)$$

The second part of this expression can be recognized from (4A) as due to the cabbelling and compressibility terms.

The diapycnal mixing term in (5) is found to be proportional to the curvature of the temperature-salinity relation. We obtain the following expression for the curvature $d^2S/d\theta^2$ in terms of diapycnal derivatives by differentiating the identity $dS/d\theta = S_d/\theta_d$ with respect to θ .

$$\frac{d^2S}{d\theta^2} = \frac{\theta_{dd}S_d}{\theta_d^3} \left(\frac{\theta_d S_{dd}}{S_d \theta_{dd}} - 1 \right). \quad (6)$$

Strictly, this is the curvature of the θ - S locus as one measures θ and S in the diapycnal direction, but, for all practical purposes, it can be regarded as the curvature obtained from a normal CTD cast even though such a cast is not exactly normal to each isopycnal surface. The second term on the right-hand side of (5) is then

$$\frac{D\theta_{dd}}{(R_p - 1)} \left[\frac{\theta_d S_{dd}}{S_d \theta_{dd}} - 1 \right] = DgN^{-2}\theta_d^3\beta \frac{d^2S}{d\theta^2}. \quad (5B)$$

The double-diffusive term in (5) does not need any further explanation.

c. Equations 4 and 5 rewritten

The results of Sections 2a and 2b are here used to rewrite equations (4) and (5), obtaining

$$(e - D_d)g^{-1}N^2 = Dg^{-1}(N^2)_d - \frac{\partial\alpha}{\partial\theta} (K|\nabla_\theta|^2 + D\theta_d^2) - \frac{\partial\alpha}{\partial p} (K\nabla_\theta \cdot \nabla_i p + D\theta_d p_d) + \beta F_d^S (1 - r) \quad (4')$$

and

$$\theta_i + (\mathbf{V}^i - \nabla_i K) \cdot \nabla_i \theta = K\nabla_i^2\theta + DgN^{-2}\theta_d^3\beta \frac{d^2S}{d\theta^2} - \frac{\beta}{\alpha} F_d^S \frac{(R_p - r)}{(R_p - 1)} + KgN^{-2}\theta_d \left\{ \frac{\partial\alpha}{\partial\theta} |\nabla_\theta|^2 + \frac{\partial\alpha}{\partial p} \nabla_\theta \cdot \nabla_i p \right\}. \quad (5')$$

By way of reminder, θ_i here is the rate of change of potential temperature on a neutral surface. The cabbelling and compressibility terms in (5') come from (4A) and so are $\partial\alpha/\partial\theta \approx 1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-2}$ and $\partial\alpha/\partial p \approx 2.2 \times 10^{-8} \text{ }^\circ\text{C}^{-1} \text{ db}^{-1}$ in this equation. Taking 1°C as a maximum variation in potential temperature on a neutral surface over a horizontal scale of 100 km, it can be shown that the cabbelling term in (5') is approximately one fifth of $K\nabla_i^2\theta$. As explained in the next section, the "fuzziness" or compressibility term is likely to be smaller than the cabbelling term.

d. The physical reasoning involved in equation (5')

Both Eqs. (1) and (5') are conservation equations for potential temperature on a neutral surface and

both equations are correct. Equation (5') is the more convenient equation for diagnostic studies of mixing processes on isopycnal surfaces because it contains one less physical process than Eq. (1). The diapycnal advection mixing process has been eliminated by using the salinity conservation Eq. (2) and the identities (3). Since it is probably not possible to estimate the diapycnal velocity $e\mathbf{f}$ with existing data sets (see Section 2e), it is advantageous to use the form of the conservation equation which does not contain this term, namely (5'). Previous diagnostic studies of mixing across and along isopycnal surfaces have neglected the diapycnal advection term, but have not realized that by so doing, the diapycnal flux divergence terms are altered to those appearing in (5') rather than those in (1).

The physical reason for the different forms of the diapycnal mixing terms in (5') as compared to those in (1) is that diapycnal mixing (both due to small scale turbulence as parameterized by D and due to double-diffusive convection, as given by F_d^S) causes a diapycnal velocity $e\mathbf{f}$ [see (4')] which in turn also contributes to the conservation equation (1). Since we are unable to measure e , the most sensible thing to do is to lump these "extra" contributions to (1) due to e , into the diapycnal mixing terms which cause e to exist in the first place. This is exactly what equation (5') does. The diapycnal diffusivity D contributes to the conservation equation for potential temperature on a neutral surface by an amount proportional to the curvature of the θ - S relation ($d^2S/d\theta^2$) rather than through the expected term ($D\theta_d$). Hence if the θ - S curve obtained from a vertical CTD cast is locally straight, diapycnal eddy diffusion does not alter θ at all on this isopycnal surface. Note also that there is no D_d (or D_z) term in (5') and so only the value of D on the particular isopycnal surface is relevant.

Note also the appearance in the water-mass conversion equation (5') of the cabbeling and compressibility (or "fuzziness") terms. While these terms are proportional to the isopycnal diffusivity K , they are in fact diapycnal advective terms. Some fractions of the diapycnal advective terms in (1) and (2) ($e\theta_d$ and eS_d) are due to the cabbeling and the fuzziness processes. When the $(e - D_d)\theta_d$ and $(e - D_d)S_d$ terms are eliminated between (1) and (2), the cabbeling and compressibility terms reappear explicitly in (5'). Note that those terms cannot be represented by a diapycnal diffusivity acting on θ_{ad} and S_{ad} since they are inherently diapycnal advective terms which are generated by isopycnal turbulence. We show below that the six terms appearing on the right-hand side of (4') are often of comparable magnitude. This conclusion also applies to the five terms on the right-hand side of (5'). In particular, this implies that the diapycnal terms in (5') due to cabbeling and compressibility may well be as important for subsurface water-mass

conversion as the diapycnal turbulent term. This rather provocative result cannot be adequately explored in the present paper, but rather we examine the use of (5') in two different types of locations; first, places where $\nabla\theta = 0$ and, second, neutral surfaces that are ventilated only at one or two isolated regions.

e. Is equation (4') useful?

Some discussion of the oceanographic use of the diapycnal velocity equation (4') is warranted. It is tempting to equate the velocity of fluid through isopycnals $e\mathbf{f}$ with the Ekman downwelling velocity w which is about $-1 \times 10^{-6} \text{ m s}^{-1}$ in mid-gyre (Keffer and Niiler, 1982). This temptation must be resisted because such a small downwelling velocity is consistent with an isopycnal slope (to the horizontal) of only 10^{-4} and a "downslope" horizontal velocity of 10^{-2} m s^{-1} , without requiring any velocity across isopycnals. Upwelling and diapycnal velocities are identical only if the neutral surface is locally tangent to the geopotential surface.

The dominant terms on the right-hand side of (4') are likely to be $Dg^{-1}(N^2)_d$ and $-D(\partial\alpha/\partial\theta)\theta_d^2$. These terms have opposite signs and magnitudes of approximately $D \times (10^{-9} \text{ m}^{-2})$ in the main thermocline. Since these two terms tend to cancel each other, the $-D(\partial\alpha/\partial p)\theta_d p_d$ term which is about $D \times (0.2 \times 10^{-9} \text{ m}^{-2})$ (taking $\theta_d = 10^{-2} \text{ }^\circ\text{C m}^{-1}$) becomes relatively more important. Where double-diffusive "staircases" exist, they are likely to be the largest term in (4'). Taking $\nabla\theta$ to be 1°C in 1000 km and a slope $\nabla_p p$ of 10^{-4} (dimensionless as p is expressed in db, not Pa) we find that the cabbeling term $-K(\partial\alpha/\partial\theta)|\nabla\theta|^2$ is five times as important as the "fuzziness" or compressibility term $-K(\partial\alpha/\partial p)\nabla\theta \cdot \nabla_p p$ and the total of these two terms is of order $-K \times (10^{-17} \text{ m}^{-2})$. Taking $K \approx 10^3 \text{ m}^2 \text{ s}^{-1}$, we see that the terms due to isopycnal mixing (the K terms) make a contribution to the right-hand side of (4') as large as or larger than the diapycnal D terms when $D \leq 0.2 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$. Since this diapycnal diffusivity is now considered to be a medium to large value, it is apparent that the cabbeling and compressibility terms in (4') will often be important. This discussion has used arbitrarily chosen values of $\nabla\theta$, etc., but it has shown that no term on the right-hand side can be ignored. The diapycnal velocity e is then due to a delicate balance between all these six separate physical processes and e is likely to be in the range $\pm 10^{-8} \text{ m s}^{-1}$ but may occasionally be as large as $\pm 10^{-7} \text{ m s}^{-1}$ if D is as large as $10^{-4} \text{ m}^2 \text{ s}^{-1}$. These values of the diapycnal velocity are only a few percent of the downwelling or upwelling velocity w past a given depth (or geopotential surface) and are obviously impossible to measure directly.

We conclude that it is not possible to use (4') to distinguish between the relative roles of mixing pro-

cesses in the ocean (i.e., to estimate D and K) because e is so small as to be unmeasurable. Rather, (4') must be regarded as an equation for e , once the coefficients D and K have been estimated.

3. Some examples of the use of equation (5')

a. Reinterpretation of interannual data from the North Pacific

White and Bernstein (1981) have used the results of two large-scale hydrographic surveys in the central North Pacific to estimate the vertical eddy diffusivity in the main pycnocline. The two surveys were conducted one year apart and the rates of change of potential temperature θ on three different isopycnal surfaces, together with the advective terms $\mathbf{V}^i \cdot \nabla \theta$ were regressed with $(D\theta_z)_z$ to estimate the vertical eddy diffusivity D . In other words their work was designed to explain $\theta_t + \mathbf{V}^i \cdot \nabla \theta$ by a diapycnal mixing eddy-diffusion coefficient alone. This procedure lumps the contributions from neglected processes (e.g., isopycnal mixing) into the diapycnal diffusion coefficient D . In this way, the value of D that one obtains cannot be unequivocally associated with the diapycnal mixing process itself, but is the value of this coefficient that best explains the data, having ignored the other physical processes. We also ignore isopycnal mixing (i.e., assume $K = 0$) in this section of the paper and we seek to explain White and Bernstein's data for $\theta_t + \mathbf{V}^i \cdot \nabla \theta$ by the (now correct) turbulent diapycnal mixing term and the double-diffusive flux term in (5'). That is, the truncated equation that we use is

$$\theta_t + \mathbf{V}^i \cdot \nabla \theta = DgN^{-2}\theta_d^3\beta \frac{d^2S}{d\theta^2} - \frac{\beta}{\alpha} F_z^S \frac{(R_\rho - r)}{(R_\rho - 1)}, \tag{5''}$$

and we consider the application of this equation to White and Bernstein's isopycnal surfaces one at a time beginning with the lowest (most dense) surface, $\sigma_\theta = 27.2$.

1) THE $\sigma_\theta = 27.2$ SURFACE

In order to evaluate the curvature $d^2S/d\theta^2$, the tabulated values of the θ - S curve of Emery and Dewar (1982) were used. Although θ - S curves are shown for the whole North Pacific in $5^\circ \times 5^\circ$ squares, the tabulations are for groups of stations. We have taken the θ - S group 20 as typical for the latitude band 35 - 40°N between 170 and 150°W . Figure 3 of White and Bernstein (1981) shows that the potential temperature is 3.75°C on the $\sigma_\theta = 27.2$ surface in this geographical region and from Emery and Dewar's Table 2, Area 20, we find $d^2S/d\theta^2 = 0.048\text{‰ }^\circ\text{C}^{-2}$. This θ - S curve is shown in Fig. 3. The water column at this low temperature of 3.75°C is stably stratified in both heat and salt and so double-diffusive convection is not possible on the mean vertical gradients of

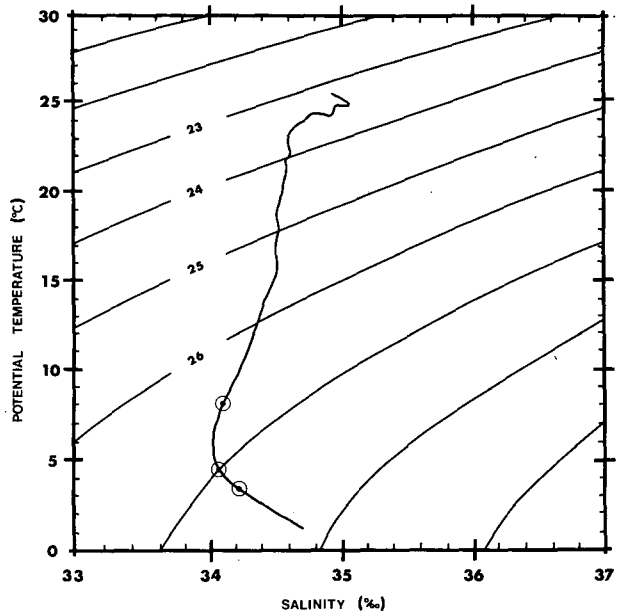


FIG. 3. Potential temperature (θ)-salinity (S) diagram from Emery and Dewar (1982), area 20, North Pacific. The three circled points correspond to the three potential density surfaces of White and Bernstein (1981).

θ and S . Of course it is possible to have double-diffusive convection in such a region if strong horizontal gradients of θ and S occur along isopycnals, but in the middle of an ocean we do not expect much intrusive frontal behavior. Equation (5'') then becomes

$$\theta_t + \mathbf{V}^i \cdot \nabla \theta = D\theta_{zz} \left[\frac{(\beta/\alpha)(d^2S/d\theta^2)(\theta_z^2 R_\rho)}{(R_\rho - 1)\theta_{zz}} \right]. \tag{5'''}$$

The square bracket on the right hand side of this equation can now be evaluated from (i) the value of $d^2S/d\theta^2$ obtained above from Emery and Dewar's data, (ii) the ratio $R_\rho = \alpha\theta_z/(\beta S_z)$ of approximately -0.6 taken from the slope of the θ - S curve and by using the appropriate values of α and β , and (iii) values of θ_z and θ_{zz} of $0.325 \times 10^{-2} \text{ }^\circ\text{C m}^{-1}$ and $1.25 \times 10^{-5} \text{ }^\circ\text{C m}^{-2}$ from White and Bernstein's Fig. 4. The square bracket in this case is equal to 0.12 and this means that the value of the eddy diffusivity D must be a factor of $(0.12)^{-1}$ larger than that found by White and Bernstein, namely $D \approx 1.7 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$.

2) THE $\sigma_\theta = 27.0$ SURFACE

From Fig. 3 of White and Bernstein, the potential temperature on this surface between 35 and 40°N is 4.55°C and the tabulated θ - S curve 20 of Emery and Dewar gives $d^2S/d\theta^2 = 0.136\text{‰ }^\circ\text{C}^{-2}$. The R_ρ is -1.2 and using $\theta_z = 0.7 \times 10^{-2} \text{ }^\circ\text{C m}^{-1}$ and $\theta_{zz} = 2.5 \times 10^{-5} \text{ }^\circ\text{C m}^{-2}$ we obtain a value of 0.98 for the multiplying square bracket in (5'''). As $R_\rho < 0$,

double-diffusive convection is not possible on the mean gradients and since the square bracket in (5'') is close to 1.0, the appropriate value of the eddy diffusivity D is the value obtained by White and Bernstein, namely $0.2 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$.

3) THE $\sigma_\theta = 26.6$ SURFACE

The appropriate potential temperature for this potential density surface is 7.8°C and at this temperature $d^2S/d\theta^2$ is only $0.004\text{‰ }^\circ\text{C}^{-2}$, which is consistent with the θ - S curve being virtually a straight line at this temperature (see Fig. 3). Here R_ρ is 3.5, and since it is greater than 1.0, "finger" double-diffusive convection is possible on the mean property gradients. However we will investigate for the moment the possibility of a balance involving diapycnal eddy diffusion but not double diffusion. White and Bernstein observed a negative value of $\theta_i + \mathbf{V}^i \cdot \nabla_i \theta$, positive θ_z ($1.6 \times 10^{-2} \text{ }^\circ\text{C m}^{-1}$) and negative θ_{zz} ($-3 \times 10^{-5} \text{ }^\circ\text{C m}^{-2}$). Now, from (5''') we see that the sign of θ_{zz} is immaterial, once $d^2S/d\theta^2$ is known and that if (5''') is applicable at this potential density level, D must be negative. The square bracket in (5''') takes the value -0.25 which means that D would need to be $-0.8 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ to explain the data of White and Bernstein on this potential density surface. Such a negative value of D in midocean is implausible and so we consider double-diffusive convection. Adopting the form

$$F^S = -A^S(R_\rho)S_z \quad (10)$$

for the salt flux it is readily shown that F_z^S also has the wrong sign to achieve a balance in (5'). The isopycnal term $K\nabla_i^2\theta$ does have the correct sign and so we conclude that at least on the $\sigma_\theta = 26.6$ surface, we are not justified in neglecting isopycnal mixing.

In summary, by using the correct conservation equation for potential temperature on a neutral surface (5'), this data set from the North Pacific no longer gives a constant value of the vertical diffusivity of $0.2 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$, but rather it gives a larger value on the deepest surface and an inconsistency for the shallowest surface. We interpret this as indicating that the isopycnal mixing terms in (5') are not insignificant and that their neglect is unjustified.

b. The salinity maximum in the Tasman Sea

Boland (1971) has plotted the salinities observed in the Tasman Sea on the surface $\sigma_t = 26.40$ ($\theta \approx 13.5^\circ\text{C}$) and this is reproduced in Fig. 4. This figure shows the existence of a salinity maximum in the interior of the Tasman Sea well away from any boundaries or seamounts. At such a salinity maximum ($\nabla_i S = 0$) the advective term $(\mathbf{V}^i - \nabla_i K) \cdot \nabla_i \theta$ and the cabbeling and compressibility terms in (5') are zero and we can analyze the possible balance of terms in

this equation. Assuming a steady state ($\theta_t = 0$), it is apparent that at the salinity maximum, (5') contains only the first three terms on the right-hand side. The isopycnal Laplacian of θ , $(\nabla_i^2\theta)$ is estimated from Fig. 4 to be $-6 \times 10^{-12} \text{ }^\circ\text{C m}^{-2}$. The curvature of the θ - S relation is evaluated by using the average of Pearce's (1981) θ - S curves for his areas B and C, giving $d^2S/d\theta^2 = -0.025\text{‰ }^\circ\text{C}^{-2}$. Note that this is negative, in contrast to the Central Water of most of the World's Oceans. The stability ratio R_ρ is 2.5 at this depth, $\beta/\alpha = 3.66^\circ\text{C } \text{‰}^{-1}$ and $\theta_d \approx 0.03^\circ\text{C m}^{-1}$, and so the terms in (5') become

$$0 = -K [\text{m}^2 \text{ s}^{-1}] \times 6 \times 10^{-12} [^\circ\text{C m}^{-2}] - D [\text{m}^2 \text{ s}^{-1}] \\ \times 1.1 \times 10^{-4} [^\circ\text{C m}^{-2}] - \frac{\beta}{\alpha} F_d^S \frac{(R_\rho - r)}{(R_\rho - 1)}$$

from which it is apparent that no balance can exist between isopycnal mixing and diapycnal turbulent diffusion (assuming $K > 0$, $D > 0$). The double-diffusive term must be invoked to balance the equation. Note that if we had erroneously taken the turbulent diapycnal term to be $(D\theta_z)_z$ (without including the resulting diapycnal advection term) a different conclusion would have been reached since $\theta_{zz} > 0$ and a (false) balance could have been achieved without invoking double-diffusive convection.

c. Comments on Reid's (1965) isopycnal maps of salinity for the Pacific Ocean

Reid (1965) has presented beautiful color maps of salinity contours on two different isopycnal surfaces for the whole Pacific Ocean. The deeper one of these surfaces is reproduced in Fig. 5. At the equator, this surface corresponds to the depth of the salinity minimum at about 800 m whereas both north and south of the equator the surface lies below the depth of the salinity minimum. This isopycnal surface is ventilated farther south than 60°S and does not come into contact with the sea surface or the mixed layer elsewhere in the whole ocean. In particular, the shallowest depth of this surface in the Northern Hemisphere is 400 m. For a steady state ocean in which no diapycnal mixing occurred, the salinity would be constant on such an isopycnal surface which is ventilated at only one location (assuming the properties along the ventilation line are uniform). The variation of salinity on Fig. 5 is then either due to unsteadiness (which we dismiss) or to diapycnal mixing caused by the diapycnal turbulent diffusivity D or by double-diffusive convection. Without either of these two diapycnal mixing processes, the isopycnal diffusivity K , will ensure that θ is made uniform on the neutral surface. In this uniform state, the diapycnal mixing terms due to cabbeling and compressibility will also be zero. Some of this diapycnal mixing must occur on the boundaries (Armi, 1978) as is evident

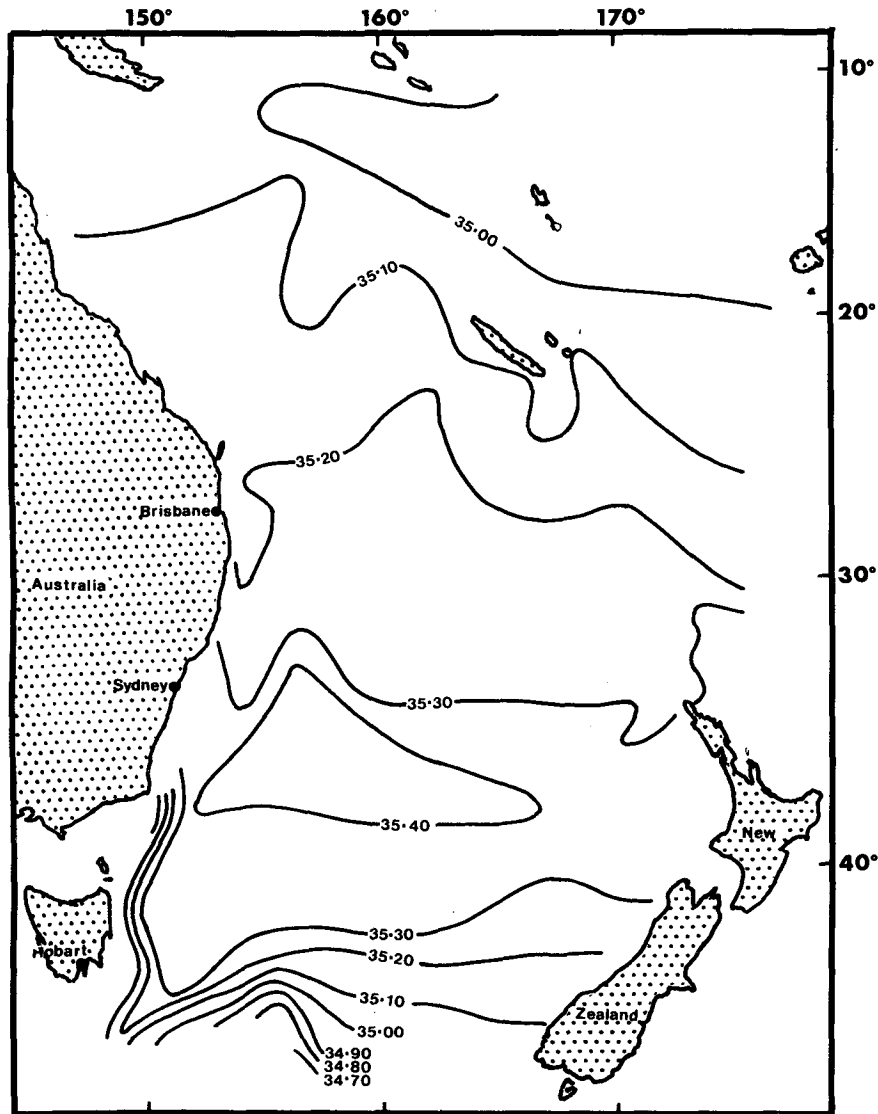


FIG. 4. The salinity on the $\sigma_t = 26.40$ surface in the Tasman and Coral Seas (redrawn from Boland, 1971).

in Fig. 5 near New Caledonia, but it would appear that the bulk of the diapycnal mixing occurs in the ocean interior although this is by no means certain. As freshly ventilated, low-salinity water moves northward towards the equator, its salinity increases, reaching a maximum in the equatorial region. This diapycnal mixing cannot be due to double-diffusive convection because the water column is stably stratified in both heat and salt on this isopycnal; rather it must be due to the small-scale turbulent processes which we have parameterized with the eddy diffusivity D . For the many θ - S curves of Emery and Dewar (1982) in the equatorial Pacific, the curvature $d^2S/d\theta^2$ is positive and this is consistent (from 5') with the observed increase in salinity as the equator is approached. The lower salinities throughout the

Northern Hemisphere (lower than near the equator) must be generated where $d^2S/d\theta^2$ is negative and the θ - S curves of Emery and Dewar indicate that this occurs in the northwestern corner of the North Pacific, approximately in the region to the northwest of the 34.25‰ contour on Fig. 5. Isopycnal mixing acts to smooth out the property distributions between the three most important "source" regions we have identified, namely (i) the region of ventilation south of 60°S, (ii) the equatorial region where D is conventionally thought to be large, consistent with significant upwelling (although beware, $e \neq w$), and (iii) the northwestern corner of the North Pacific Ocean.

The shallower isopycnal surface considered by Reid (1965) ($\delta_T = 125$ cl/ton) is ventilated at high latitudes in both the Northern and the Southern Hemispheres.

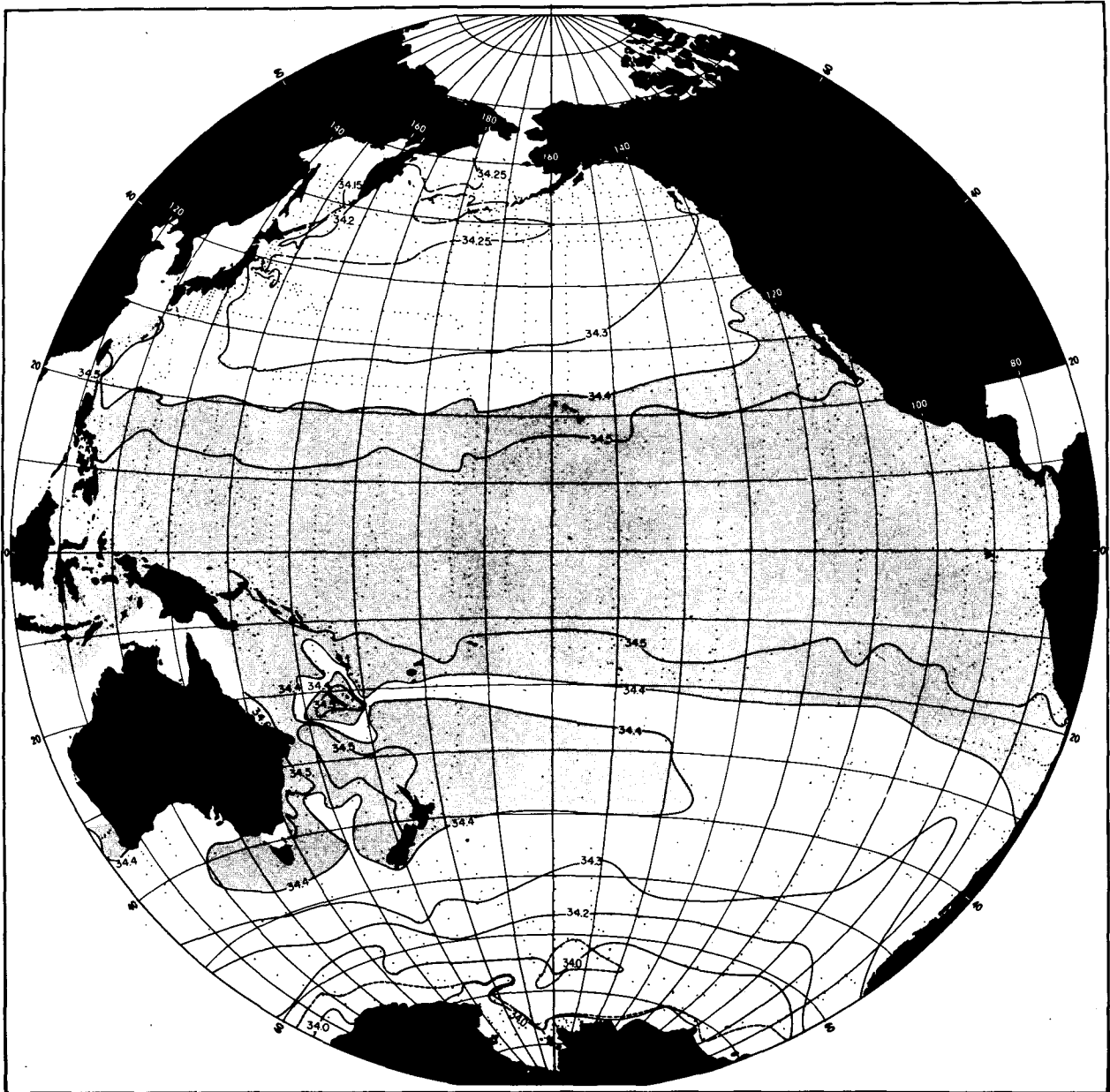


FIG. 5. Salinity (‰) on the surface where $\delta\sigma_t = 80$ cl/ton. Reproduced from Reid (1965).

There is a salinity maximum ridge along the equator which cannot be due to isopycnal mixing and so significant diapycnal mixing must again be invoked here.

Another question that comes to mind when looking at Reid's salinity distribution in the Pacific (our Fig. 5) is why the distribution is apparently unaffected by the Hawaiian Island group whereas intense isopycnal salinity gradients occur near the New Caledonian islands. Can the diapycnal diffusivity D be so different at the two locations? The answer undoubtedly lies mainly in the different values of $d^2S/d\theta^2$ at the two locations rather than being due to variations of

D or of θ_{dd} and S_{dd} separately. Near New Caledonia, $d^2S/d\theta^2$ is approximately $0.032\text{‰ }^\circ\text{C}^{-2}$ on this isopycnal surface (Tomczak, personal communication, 1984) whereas, near Hawaii, $d^2S/d\theta^2$ is one quarter of this value (Emery and Dewar, 1982).

It is instructive also to consider the balance of terms in (5') in the equatorial Pacific (Fig. 5) where $\nabla\theta = \nabla_i S = 0$. Assuming a steady state, there are only the isopycnal and diapycnal turbulent diffusion terms in (5') since the diapycnal property gradients are not favorable to double-diffusion. From Fig. (5), $\nabla^2\theta \approx -7.5 \times 10^{-14} \text{ }^\circ\text{C m}^{-2}$ and from Emery and Dewar we find $\theta_a \approx 4 \times 10^{-3} \text{ }^\circ\text{C m}^{-1}$, $d^2S/d\theta^2$

$\approx 0.022\text{‰ } ^\circ\text{C}^{-2}$ and $R_\rho/(R_\rho - 1) \approx 1$. These values imply that

$$D \approx 0.36 \times 10^{-7} K.$$

By arbitrarily applying Armi and Stommel's (1983) value of $K \approx 10^3 \text{ m}^2 \text{ s}^{-1}$ to the equator, this relation gives $D \approx 0.36 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$, close to the canonical value of $10^{-4} \text{ m}^2 \text{ s}^{-1}$ due to Munk (1966). These data can also be used to find the diapycnal velocity by (4'). The cabbeling and fuzziness terms are zero at the equator where $\nabla\theta = 0$ and the data of Emery and Dewar lead us to

$$(e - D_d) \approx 1.9 \times 10^{-3} \text{ m}^{-1} D,$$

and using the value of D we have found above, this gives $(e - D_d) \approx 7 \times 10^{-8} \text{ m s}^{-1}$. This is very close to the canonical upwelling velocity value of 10^{-7} m s^{-1} of Munk (1966). Notice that the above estimates of D and $(e - D_d)$ have assumed only (i) that a steady-state exists and (ii) a value of K . The other oceanographic data we have used are rather well known and it is the method of manipulating Eqs. (1) and (2) into (4') and (5') which has allowed the above balance of physical processes to be inferred. For example, by using simply equation (1) at this location, three processes remain $[(e - D_d)\theta_d, K\nabla_i^2\theta$ and $D\theta_{dd}]$ and assuming a value of K only gives a linear relation between $(e - D_d)$ and D . Our approach uses extra information from the local θ - S relation (namely, the curvature $d^2S/d\theta^2$) to find the value of both $(e - D_d)$ and D .

d. Isopycnal advection versus diapycnal salt fingering in the Central Water

The water in the main thermocline is thought to form at the surface in late winter and to move equatorward (and downwards) as shown in Fig. 6. Iselin (1939) proposed this explanation on the basis of the similarity between the θ - S curve of vertical casts in the Central Water thermocline and the θ - S characteristics of the ocean mixed layer in winter at various latitudes. Stommel (1979) examined the connection between the ocean mixed layer and the geostrophic flow below, and he has explained why

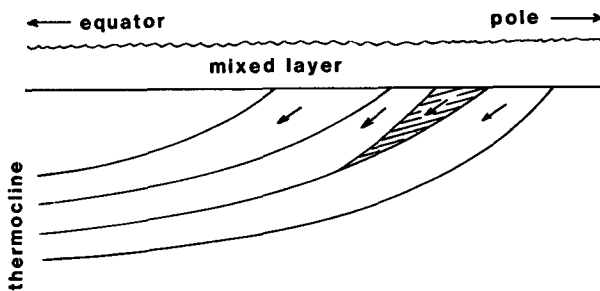


FIG. 6. Sketch showing the spreading of water along isopycnals from the mixed layer to greater depths in the Central Water thermocline. The shaded region is an anomalous patch of water.

the water which forms the thermocline has the properties of the late winter mixed-layer water. Schmitt (1981) advanced an alternative explanation for the form of the θ - S curve in the Central Water. He has shown that the strong dependence of double-diffusion on R_ρ implies that deviations from a constant R_ρ quickly decay. Inevitable annual and spatial variations in the water properties at the formation region (at the base of the mixed layer in late winter, see Fig. 6) are not seen in the θ - S curve of the Central Water at depth and Schmitt's mechanism can eliminate these variations of R_ρ .

Schmitt (1981) has shown that the θ - S curve of the Central Water is much better fitted by a constant R_ρ curve than by a straight line. With this in mind, we briefly consider the sizes of the two diapycnal mixing terms in (5') in relation to their normal expressions, $(D\theta_z)_z$ and $-F_z^\theta$ for an ocean with $R_\rho = \text{constant}$. Differentiating the definition $R_\rho = \alpha\theta_z/\beta S_z$ with respect to z , the equation

$$\frac{dR_\rho}{dz} = \frac{\alpha\theta_{zz}}{\beta S_z} - \frac{\alpha\theta_z\beta S_{zz}}{(\beta S_z)^2} + \frac{\alpha_z\theta_z}{\beta S_z} - \frac{\beta_z S_z\alpha\theta_z}{(\beta S_z)^2} \quad (11)$$

is obtained, and using (6), this relation can be expressed as

$$\frac{(\theta_z)^2}{S_z} \frac{d^2S}{d\theta^2} = -\frac{1}{R_\rho} \frac{dR_\rho}{dz} + \left[\frac{\alpha_z}{\alpha} - \frac{\beta_z}{\beta} \right]. \quad (12)$$

For $dR_\rho/dz = 0$, the ratio of the term in (5') due to diapycnal turbulent mixing to $D\theta_{zz}$ is

$$\begin{aligned} & \frac{1}{D\theta_{zz}} \cdot DgN^{-2}\theta_d^3 \frac{\beta d^2S}{d\theta^2} \\ &= \frac{1}{(R_\rho - 1)} \cdot \frac{\theta_z}{\theta_{zz}} \cdot \left[\frac{\alpha_z}{\alpha} - \frac{\beta_z}{\beta} \right] \approx 0.15, \quad (13) \end{aligned}$$

where we have used $R_\rho = 1.7$, $\theta_z = 10^{-2} \text{ } ^\circ\text{C m}^{-1}$, $\theta_z/\theta_{zz} = 200 \text{ m}$ and both cabbeling and compressibility terms which appear in the expansion of the square bracket in (13) have been included. On the other hand, the ratio of the double-diffusive term in (5') to $-F_z^\theta$ is

$$\frac{1}{F_z^\theta} \times \frac{\beta F_z^S}{\alpha} \times \frac{(R_\rho - r)}{(R_\rho - 1)} = \frac{1}{r} \times \frac{(R_\rho - r)}{(R_\rho - 1)} \approx 2.6, \quad (14)$$

where $r = 0.6$ has been used. Equations (13) and (14) are interpreted as follows. The vertical divergence of the double-diffusive flux of potential temperature F_z^θ affects the conservation equation (5') for potential temperature on a neutral surface at the rate $-2.6 F_z^\theta$, while, for the case $dR_\rho/dz = 0$, diapycnal eddy diffusion enters this same equation as $0.15 D\theta_{zz}$. These factors of 2.6 and 0.15 occur because the diapycnal advection, which occurs as a natural consequence of both diapycnal mixing processes, has been incorporated into (5') in the diapycnal mixing terms themselves rather than by including a separate

diapycnal advection term as appears in (1). Note also that (5') does not contain a term due to the diapycnal divergence of D , D_d . The numerical factor for fingering, 2.6, is not known accurately because of uncertainties in r which depends not only on the buoyancy flux ratio R_f , but also on the length scale F^S/F_d^S —see (4C). The buoyancy flux ratio R_f has been measured in one-dimensional laboratory experiments across sharp finger interfaces (Schmitt, 1979; McDougall and Taylor, 1984) but the buoyancy-flux ratio expected from double-diffusive interleaving (Stern, 1967; Toole and Georgi, 1981) remains rather uncertain although Garrett (1982) has made a start on this problem. In particular, he has shown that in some circumstances, R_f can be negative because of double-diffusive intrusions.

4. Conclusions

The physical processes which mix properties in the ocean act either along isopycnals (large-scale stirring as evidenced by baroclinic eddies) or across isopycnals (breaking internal waves, boundary mixing, the final stages of baroclinic instability and double-diffusive convection) and so the isopycnal coordinate frame is a very sensible choice for the analysis of oceanographic data. The isopycnal gradient of potential temperature, ∇_θ is often much less than $\theta_x \mathbf{i} + \theta_y \mathbf{j}$ and the analysis of scalar oceanographic data along neutral surfaces will often lead to much better information on the size of various physical processes than a corresponding analysis in the strictly horizontal and vertical directions. For example, Armi and Stommel (1983), by analyzing their data on potential density surfaces, were able to deduce isopycnal Laplacians and isopycnal diffusivities, whereas McWilliams (1983), who used x , y , z coordinates in an area of similar eddy activity, was only able to deduce that the balance of processes was "primarily between mean advection, mostly vertical, and mostly horizontal flux divergences associated with mesoscale eddies". These large terms automatically cancel in an isopycnal analysis, leading to a higher signal-to-noise ratio for the determination of the physically relevant mixing processes. At an earlier stage in the analysis, the density data will of course need to be referenced to geopotential surfaces (i.e., the x , y , z coordinate frame) to determine (by geostrophy) the quasi-horizontal velocity field \mathbf{V}^i . Also, it must always be borne in mind how the variation of K along isopycnals acts like an isopycnal advective velocity (Armi, 1979; Armi and Haidvogel, 1982). The diapycnal advection term is commonly ignored when writing down the conservation equations in the isopycnal coordinate frame and in this paper it is shown that this is unacceptable. Although the diapycnal velocity is tiny compared to the isopycnal velocity, the diapycnal property gradients are much larger than the isopycnal gradients. It is possible to derive conservation equations that do not contain the diapycnal advective terms and this is the form of the

conservation equations that is used in this paper. In this form, the diapycnal mixing terms are significantly changed. The salient points of this paper are

- The diapycnal turbulent diffusivity D appears in the conservation equation for potential temperature θ on a neutral surface in a term which is proportional to the curvature of the potential temperature-salinity relation, $d^2S/d\theta^2$, rather than as the normal term $(D\theta_z)_z$.
- The new water-mass conversion Eq. (5') has been used in a preliminary fashion to determine the relative importance of isopycnal mixing, diapycnal turbulence and double-diffusive convection in three regions of the ocean. Previous data from the North Pacific which had been interpreted in terms of diapycnal turbulent mixing have been shown to be significantly affected by isopycnal mixing. Without the elimination of the unknown diapycnal advection term, such a conclusion is not possible.
- In the Tasman Sea, the diapycnal turbulence mixing terms in the conservation equations for heat and salt on an isopycnal surface have the opposite sign to the terms $(D\theta_z)_z$ and $(DS_z)_z$. At a position in the interior of the Tasman Sea where there is a salinity maximum on a particular isopycnal surface, the new conservation equation has been used to show that double-diffusive convection must be large enough to balance both isopycnal mixing and the small-scale turbulence flux divergence.
- Contours of salinity on a particular isopycnal surface for the whole Pacific Ocean from Reid (1965) are used to pinpoint areas of the ocean where diapycnal mixing must be occurring. This is determined by looking for regions which have the appropriate θ - S curvature $d^2S/d\theta^2$. This procedure is possible because the effects of the previously unknown diapycnal velocity e are included in the relevant conservation equation without actually having to evaluate e . The predictive power of the new method is also demonstrated in the equatorial Pacific where an assumed value of the isopycnal diffusivity leads to the diapycnal velocity and the diapycnal diffusion coefficient.
- By carefully defining the quasi-horizontal neutral surfaces in the ocean (which are not potential density surfaces) it has been shown that both the cabbeling and compressibility (or "fuzziness of neutral surfaces") processes appear in the water-mass conversion equation (5') as diapycnal advective terms which are proportional to the isopycnal diffusivity. These processes cause diapycnal fluxes past neutral surfaces and diapycnal flux divergences of both heat and salt on neutral surfaces. While present indications are that they cannot be ignored it is important that their size be explored more thoroughly in the future.

Acknowledgments. Helpful correspondence with Dr. J. Shepherd is gratefully acknowledged. Thanks are due to Jackie Firth for typing the manuscript and Graham Wells for drafting the figures.

APPENDIX

The Conservation Equation for a Tracer on a Neutral Surface

The conservation equation for a passive tracer of concentration C on a neutral surface is

$$C_t + (\mathbf{V}^i - \nabla_i K) \cdot \nabla_i C + (e - D_d)C_d = K \nabla_i^2 C + DC_{dd} - F_d^C.$$

This is the form of the equation that includes the diapycnal advection term $(e - D_d)C_d$. We assume that salt fingering is the cause of the double-diffusive convection and that the molecular diffusivity of C is much closer to that of salt than of heat. Under these circumstances we can write $F^C = (C_d/S_d)F^S$ and this allows considerable simplification in the form of the conservation equation that does not contain the diapycnal advective term, namely

$$C_t + (\mathbf{V}^i - \nabla_i K) \cdot \nabla_i C = K \nabla_i^2 C + \frac{C_d}{S_d} DgN^{-2}(\theta_d)^3 \alpha \frac{d^2 S}{d\theta^2} - \frac{C_d}{S_d} F_d^S \frac{(R_p - r)}{(R_p - 1)} - \frac{(C_d)^3}{S_d} \frac{d^2 S}{dC^2} \left[D - \frac{F^S}{S_d} \right] + KgN^{-2} C_d \left\{ \frac{\partial \alpha}{\partial \theta} |\nabla_i \theta|^2 + \frac{\partial \alpha}{\partial p} \nabla_i \theta \cdot \nabla_i p \right\}.$$

Note that the salt flux by fingers is often taken to be given by $F^S = -A^S S_d$ and so the square bracket above is $(D + A^S)$ and so is positive.

For completeness, the salinity conservation equation for water-mass conversion that corresponds to (5') is

$$S_t + (\mathbf{V}^i - \nabla_i K) \cdot \nabla_i S = K \nabla_i^2 S + DgN^{-2}(\theta_d)^3 \alpha \frac{d^2 S}{d\theta^2} - F_d^S \frac{(R_p - r)}{(R_p - 1)} + KgN^{-2} S_d \left\{ \frac{\partial \alpha}{\partial \theta} |\nabla_i \theta|^2 + \frac{\partial \alpha}{\partial p} \nabla_i \theta \cdot \nabla_i p \right\}.$$

REFERENCES

- Armi, L., 1978: Some evidence for boundary mixing in the deep Ocean. *J. Geophys. Res.*, **83**, 1971-1979.
- , 1979: Effects of variations in the eddy diffusivity on property distributions in the ocean. *J. Mar. Res.*, **37**, 515-529.
- , and D. B. Haidvogel, 1982: Effects of variable and anisotropic diffusivity in a steady-state diffusion model. *J. Phys. Oceanogr.*, **12**, 785-794.
- , and H. Stommel, 1983: Four views of a portion of the North Atlantic Subtropical Gyre. *J. Phys. Oceanogr.*, **13**, 828-857.
- Bleck, R., and D. B. Boudra, 1981: Initial testing of a numerical ocean circulation model using a hybrid (quasi-isopycnic) vertical coordinate. *J. Phys. Oceanogr.*, **11**, 755-770.
- Boland, F. M., 1971: Temperature-salinity anomalies at depths between 200 m and 800 m in the Tasman Sea. *Aust. J. Mar. Freshwater Res.*, **22**, 55-62.
- Bray, N. A., and N. P. Fofonoff, 1981: Available potential energy for MODE eddies. *J. Phys. Oceanogr.*, **11**, 30-47.
- Bryden, H. L., 1973: New polynomials for thermal expansion, adiabatic temperature gradient and potential temperature of sea water. *Deep-Sea Res.*, **20**, 401-408.
- Caldwell, D. R., 1983: Small-scale physics of the ocean. *Rev. Geophys. Space Phys.*, **21**, 1192-1205.
- Emery, W. J., and J. S. Devar, 1982: Mean temperature-salinity, salinity-depth and temperature-depth curves for the North Atlantic and North Pacific. *Progress in Oceanography*, Vol. 11, Pergamon, 219-305.
- Gargett, A. E., 1984: Vertical eddy diffusivity in the ocean interior. *J. Mar. Res.*, **42**, 359-393.
- Garrett, C., 1979: Mixing in the ocean interior. *Dyn. Atmos. Oceans*, **3**, 239-265.
- , 1982: On the parameterization of diapycnal fluxes due to double-diffusive intrusions. *J. Phys. Oceanogr.*, **12**, 952-959.
- , 1983: On the initial streakiness of a dispersing tracer in two- and three-dimensional turbulence. *Dyn. Atmos. Oceans*, **7**, 265-277.
- , and E. Horne, 1978: Frontal circulation due to cabbeling and double-diffusion. *J. Geophys. Res.*, **83**, 4651-4656.
- Gibson, C. H., 1982: Alternative interpretations for microstructure patches in the thermocline. *J. Phys. Oceanogr.*, **12**, 374-383.
- Gill, A. E., 1982: *Atmosphere-Ocean Dynamics*, Academic Press, 662 pp.
- Iselin, C. O'D., 1939: The influence of vertical and lateral turbulence on the characteristics of the waters at mid-depths. *Eos Trans. Amer. Geophys. Union*, **20**, 414-417.
- Keffer, T., and P. P. Niiler, 1982: Eddy convergence of heat, salt, density and vorticity in the subtropical North Atlantic. *Deep-Sea Res.*, **29**, 201-216.
- McDougall, T. J., and J. R. Taylor, 1984: Flux measurements across a finger interface at low values of the stability ratio. *J. Mar. Res.*, **42**, 1-14.
- McWilliams, J. C., 1983: On the mean dynamical balances of the Gulf Stream recirculation zone. *J. Mar. Res.*, **41**, 427-460.
- Millero, F. J., and A. Poisson, 1981: International one-atmosphere equation of state for sea water. *Deep-Sea Res.*, **28**, 625-629.
- Munk, W. H., 1966: Abyssal recipes. *Deep-Sea Res.*, **13**, 707-730.
- Pearce, A., 1981: Temperature-salinity relationships in the Tasman Sea. CSIRO Div. Fisher. Oceanogr. Rep. No. 135, 41 pp.
- Redi, M. H., 1982: Oceanic isopycnal mixing by coordinate rotation. *J. Phys. Oceanogr.*, **12**, 1154-1158.
- Reid, J. L., 1965: *Intermediate Waters of the Pacific Ocean*, Johns Hopkins Oceanic Studio, No. 2, The Johns Hopkins University Press, 85 pp.
- Sarmiento, J. L., 1983: A simulation of bomb tritium entry into the Atlantic Ocean. *J. Phys. Oceanogr.*, **13**, 1924-1939.
- Schmitt, R. W., 1979: Flux measurements on salt fingers at an interface. *J. Mar. Res.*, **37**, 419-436.
- , 1981: Form of the temperature-salinity relationship in the Central Water: Evidence of double-diffusive mixing. *J. Phys. Oceanogr.*, **11**, 1015-1026.
- , and D. L. Evans, 1978: An estimate of the vertical mixing due to salt fingers based on observations in the North Atlantic Central Water. *J. Geophys. Res.*, **83**, 2913-2919.
- , and D. T. Georgi, 1982: Finestructure and microstructure in the North Atlantic Current. *J. Mar. Res.*, **40**(Suppl), 659-705.
- Shepherd, J. G., 1984: On isopycnal movements, neutral surfaces, and vertical transport in the ocean (unpublished manuscript).
- Solomon, H., 1971: On the representation of isentropic mixing in ocean circulation models. *J. Phys. Oceanogr.*, **1**, 233-234.
- Stern, M. E., 1967: Lateral mixing of water masses. *Deep-Sea Res.*, **14**, 747-753.
- Stommel, H., 1979: Determination of water mass properties of water pumped down from the Ekman layer to the geostrophic flow below. *Proc. Natl. Acad. Sci. USA*, **76**, 3051-3055.
- Toole, J. M., and D. T. Georgi, 1981: On the dynamics and effects of double-diffusively driven intrusions. *Progress in Oceanography*, Vol. 10, Pergamon, 123-145.
- White, W., and R. Bernstein, 1981: Large-scale vertical eddy diffusion in the main pycnocline of the Central North Pacific. *J. Phys. Oceanogr.*, **11**, 434-441.
- Young, W. R., P. B. Rhines and C. J. R. Garrett, 1982: Shear-flow dispersion, internal waves and horizontal mixing in the ocean. *J. Phys. Oceanogr.*, **12**, 515-527.