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Development of a Mathematical Model Using Dimensional Analysis for Predicting the Friction Losses in Drip Irrigation Laterals with Cylindrical Type In-Line Emitters

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Abstract: This paper presents a prediction model developed using dimensional analysis for friction losses in drip irrigation laterals. Drip irrigation laterals with cylindrical type in-line emitters placed at different spacings ranging from 0.2 to 1 m were considered. The parameters affecting the variation in friction losses in laterals were defined as dimensionless terms using Buckingham's pi theorem. In order to develop a model, experimental friction loss data from 14 different drip irrigation pipes, and some domestic and some imported ones with the above-mentioned spacings were used. The model developed in this study accounted for 98.7% of the variation in the data. The results showed that the mathematical model may be used to determine friction losses in drip laterals with cylindrical type in-line emitters with spacings ranging from 0.2 to 1 m.

Key Words: Drip irrigation, emitters, friction loss, mathematical model, dimensional analysis

İçine Geçik Silindirik Tip Damlatıcılı Damla Sulama Borularındaki Sürtünme Kayıplarının Kestiriminde Kullanılabilecek Bir Matematiksel Modelin Boyutsal Analiz Yöntemiyle Geliştirilmesi

Özet: Bu çalışmada, damla sulama laterallerinde oluşan sürtünme kayıplarının tahmininde kullanılabilecek bir matematiksel modelin geliştirilmesi amaçlanmıştır. Matematiksel modelin geliştirilmesinde boyutsal analiz yöntemi kullanılmıştır. Çalışmada, silindirik tip içine geçik damlatıcıların 0.2 - 1 m arasında değişen aralıklarda yerleştirildiği damla sulama boruları ele alınmıştır. Borularda sürtünme kayıplarının değişimine neden olan parametreler, Buckingham Pi-teoremi uyarınca boyutsuz terimler olarak tanımlanmıştır. Modeli geliştirmek amacıyla yerli ve yabancı yapım 14 farklı damla sulama borusundan ölçülen deneysel sürtünme kaybı verileri kullanılmıştır. Çalışma sonucunda geliştirilen matematiksel modelin, tüm sürtünme kaybı verilerindeki varyasyonun %98.7'ini açıklayabildiği saptanmıştır. Geliştirilen bu matematiksel model, silindirik tip içine geçik damlatıcıların 0.2 - 1 m sınırları arasındaki her bir damlatıcı aralığı için sürtünme kaybı değerlerinin tahmini amacıyla kullanılabilir.

Anahtar Sözcükler: Damla sulama, damlatıcı, sürtünme kaybı, matematiksel modelleme, boyutsal analiz

Introduction

A drip irrigation system consists of a main line, a sub main, manifolds, lateral lines and emitters. Laterals are designed to distribute irrigation water throughout the field with an acceptable degree of uniformity. Although drip irrigation systems have several advantages over other irrigation systems, it is impossible to obtain ideal water distribution along the lateral line due to variations in emitter discharge caused by operating pressure, water temperature differences, emitter manufacturing variations, emitter clogging and pressure variations caused by slope and friction losses. Hence, for the design of an appropriate drip irrigation system, the properties of the system components, especially the emitter properties and friction losses in laterals for uniformity, must be known (Howell and Barinas, 1980; Wu et al., 1989; Demir and Yürdem, 2002).

Drip irrigation lines made of plastic are usually regarded as a smooth pipe. Generally, the Darcy-Weisbach equation and the Hazen-Williams empirical

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equation can be used to determine friction losses along the lateral line; these equations are given below (Watters and Keller, 1978; Howell et al., 1983; Anyoji, 1986; Wu et al., 1986).

The Darcy-Weisbach equation is

$$H_f = f \frac{L}{D} \frac{V^2}{2g} \tag{1}$$

where H_f is the friction loss of the pipe (m), L is the pipe length (m), D is the pipe's internal diameter (m), V is the velocity (m s^{-1}), g is the acceleration of gravity (9.81 m s^{-2}), and f is the friction coefficient. For smooth pipes the friction coefficient is characterized by the Blasius equation for turbulent flow as

$$f = 0.3164\ R_e^{-0.25} \qquad (4\,000 \le R_e \le 100\,000) \tag{2}$$

where $R_e = VD/\nu$ is the Reynolds number and ν is the kinematic viscosity of water ($\nu = 1.01 \times 10^{-6}$ for 20 °C water temperature) (Howell et al., 1983; Anyoji, 1986; Wu et al., 1986).

The Hazen-Williams empirical equation is

$$H_f = 6.840 \frac{L}{C^{1.852}} \frac{V^{1.852}}{D^{1.167}} \tag{3}$$

where H_f, L, D are expressed in m and V is expressed in m s^{-1}, and C is the roughness coefficient. The roughness coefficient at a maximum and recommended velocity value of 1.5 m s^{-1} depends upon pipe diameter, with C = 130 for 14-15 mm, C = 140 for 18-19 mm, and C = 150 for 25-27 mm for smooth plastic pipes (Howell et al., 1983; Anyoji, 1986). On the other hand, some researchers correlated the change in the roughness coefficient to the type of emitter and spacing in the lateral line by comparing the Hazen-Williams and Blasius equations. They reported that the roughness coefficient (C) can be taken as 100-130 for drip irrigation pipes in 14-16 mm diameter depending upon the emitter type and spacing (Watters and Keller, 1978; Korukçu, 1980; Braud and Soom, 1981; Tüzel, 1990; Demir and Uz, 1995).

Generally, in the design of drip irrigation systems, friction losses are found based on the assumption that the flow rates of emitters are the same along the laterals and the flow is turbulent (Bralts and Wu, 1979). Due to friction losses, some variations occur in flow rates and, as a result of this, some important variations occur even if the conditions are the same when Darcy-Weisbach and Hazen-Williams equations are used (Watters and Keller, 1978; Allen, 1996; Demir and Uz, 1999).

Drip irrigation laterals are not only smooth pipes; these pipes also have multiple outlets depending upon the emitter type and spacing. Since emitters discharge water along the lateral line, the flow rate in the lateral line decreases with respect to length. The friction loss between the emitters of the lateral line can be calculated by the equation derived by Christiansen (1942):

$$\Delta H_f = K \Delta L \frac{\Delta Q^m}{D^{2m+n}} \tag{4}$$

where ΔH_f is the friction loss in ΔL lateral section (m), K is the constant based on dimensions, ΔL is the emitter spacing (m), ΔQ is the flow rate in ΔL lateral section (m^3 s^{-1}), D is the lateral internal diameter (m), n is the pipe diameter exponent and m is the friction coefficient in which m = 1 for laminar flow and m = 1.75 for turbulent flow in a smooth pipe, and m = 1.852 for turbulent flow using the Hazen-Williams equation and m = 2 for a fully turbulent flow where the friction coefficient is constant (Wu and Gitlin, 1975; Anyoji, 1986; Wu et al., 1986).

For multiple outlet pipes, the total friction loss is equal to the sum of the losses between the outlets. Therefore, the friction loss along the lateral line can be calculated by multiplying equation (4) by the Christiansen adjustment value, F (Christiansen, 1942).

Wu and Gitlin (1975) derived the hydraulic energy gradient line for determining the emitter flow variation and uniformity along a lateral line. Since the flow rate in the lateral line decreases with respect to length due to emitter discharges from the laterals, an energy gradient line along the line is not straight but curved with an exponential shape. The shape of the energy gradient curve along the lateral line was expressed by the dimensionless energy gradient line as

$$R_i = 1 - (1 - i)^{m+1} \tag{5}$$

where $R_i = \Delta H_i / \Delta H$ is the energy drop ratio at a given length ratio i, ΔH is the total friction loss at the end of the lateral line, ΔH_i is the total friction loss at a given length ratio i, i = l/L is the length ratio, L is the total length of the line, and l is the given length from the inlet.

Kang and Nishiyama (1996) presented a method for the design of micro irrigation laterals based on average emitter discharge and the required uniformity of water application. They used finite element method to analyze the hydraulics of a lateral.

Drip irrigation pipes having cylindrical type in-line emitters at different spacings ranging from 0.2 to 1 m are the most common type manufactured at present. In the design of drip irrigation systems it is well known that emitter spacing is determined according to soil infiltration value. However, emitter spacing determined based on soil infiltration could be different than those of products readily available. In this case, the irrigation system is designed by selecting an upper side emitter spacing even though manufacturers are willing to produce pipes with requested emitter spacing. Selecting an upper side emitter spacing could also adversely affect water distribution in the field.

Another important aspect of system design is that the lateral lengths are supposed to conform with optimum lateral lengths that would supply uniform water distribution. The extent of friction loss is a prerequisite in the determination of optimum lateral lengths. For this purpose, laboratory data are required; however, this requires much effort, resources and time since a great deal of experiments are needed for different emitter spacings. Additionally, friction losses, optimum pipe lengths, and emitter flow rates are determined according to the soil infiltration value. This requires comprehensive laboratory experiments, which are not desired by manufacturers. A practical way of calculating the friction losses, therefore, is needed. Hence, a study using the most common domestic emitters and some imported ones was conducted.

The objective of this study was to develop a prediction model for friction losses in drip irrigation laterals with cylindrical type in-line emitters at different spacings.

Materials and Methods

Different drip irrigation pipes with different emitters spacings were used (Table 1). The general specifications of the pipes and emitters are shown in Figure 1.

The study was carried out using a special test apparatus built in the Pump Testing Laboratory of the Department of Agricultural Machinery, Faculty of Agriculture, Ege University.

Table 1. Properties of the emitters and drip irrigation pipes used in the study.

Emitter no.	Emitter flow rate* q ($l\ h^{-1}$)	Emitter's exterior diameter d_o (m)	Emitter's interior diameter d_i (m)	Emitter length L_e (m)	Pipe's exterior diameter D_o (m)	Pipe's interior diameter D_i (m)	Emitter spacing ΔL (m)
1	4.20	0.0159	0.0120	0.0688	0.0160	0.0136	0.2, 0.25, 0.33, 0.4, 0.5
2	2.77	0.0165	0.0118	0.0600	0.0160	0.0136	0.75
3	2.14	0.0160	0.0120	0.0395	0.0158	0.0138	0.2, 0.25, 0.33
4	4.21	0.0160	0.0120	0.0395	0.0158	0.0138	0.2, 0.25, 0.33, 0.4, 0.75
5	3.00	0.0155	0.0116	0.0315	0.0148	0.0130	0.2, 0.25, 0.33, 0.4, 0.5, 0.6, 0.75
6	3.04	0.0155	0.0116	0.0315	0.0158	0.0140	0.2, 0.25, 0.33, 0.4, 0.5, 0.6, 0.75, 1.0
7	3.08	0.0159	0.0120	0.0320	0.0150	0.0134	0.2, 0.25, 0.33, 0.4, 0.5, 0.6, 0.75
8	1.72	0.0158	0.0118	0.0680	0.0162	0.0136	0.33
9	3.46	0.0164	0.0114	0.0650	0.0162	0.0136	0.33
10	3.54	0.0163	0.0115	0.0675	0.0160	0.0136	0.33
11	3.46	0.0165	0.0115	0.0650	0.0160	0.0136	0.33
12	4.39	0.0158	0.0117	0.0680	0.0160	0.0136	0.33
13	2.56	0.0157	0.0117	0.0675	0.0158	0.0136	0.33
14	2.98	0.0150	0.0115	0.0682	0.0158	0.0130	0.33, 0.4, 0.5, 0.6, 0.75, 1.0

* operating pressure: 1.0 bar

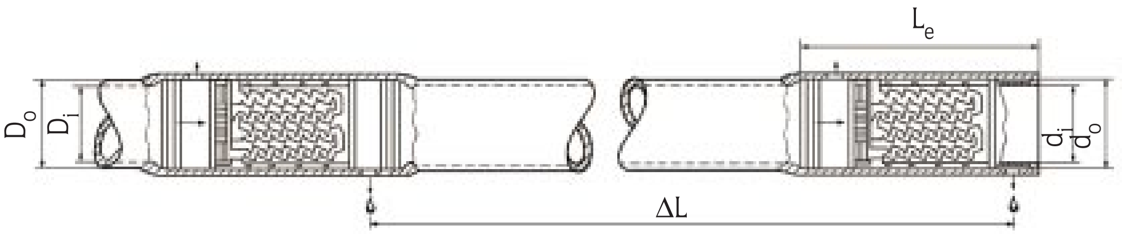

Figure 1. General specifications of the pipes and emitters (see Table 2 for explanation of the symbols).

The friction losses in laterals with different emitter spacings were measured using piezometric tubes at various flow rates and 20 °C (±2 °C) water temperature. The laterals used for the measurement of friction losses were 10 m long (Howell and Hiller, 1974; Korukçu, 1980; Braud and Soom, 1981).

Dimensional Analysis of Friction Losses

Dimensional analysis is a useful tool for developing prediction equations for various physical systems. It reduces the physical quantities pertinent to a system to dimensionless groups called pi terms (Langhaar, 1987). An application of the dimensional analysis known as Buckingham's pi theorem is used in finding the friction losses in laterals with in-line type emitters, and this theorem requires the pertinent, non-redundant quantities affecting the physical system. These quantities, shown in Table 2, are considered to be operating and lateral/emitter geometry related.

The resulting dimensionless groups are given in Table 3.

Table 2. Variables affecting friction losses in drip irrigation laterals.

Type of variable	Symbol	Variable	Dimension	Unit
Dependent	ΔH_f	Friction loss in emitter spacing	L	m
Independent	D_i	Lateral interior diameter	L	m
	d_i	Emitter interior diameter	L	m
	L_e	Emitter length	L	m
	ΔL	Emitter spacing	L	m
	g	Acceleration of gravity	$L\ T^{-2}$	$m\ s^{-2}$
	V	Velocity	$L\ T^{-1}$	$m\ s^{-1}$
	ν	Kinematic viscosity of water	$L^2\ T^{-1}$	$m^2\ s^{-1}$

Table 3. Dimensionless groups and their significance.

Dimensionless terms (π terms)	Significance
$\Delta H_f / \Delta L$ (π_1)	Friction loss in emitter spacing
$V D_i / \nu$ (π_2)	The ratio of inertia forces to viscous forces, known as the Reynolds number
$V^2/g D_i$ (π_3)	The ratio of inertia force to gravity force, known as the Froude number
$L_e / \Delta L$ (π_4)	The ratio of emitter length and lateral length
$\Delta L / D_i$ (π_5)	Dimensionless term that includes the operating condition and manufacturing properties of the lateral
d_i / D_i (π_6)	Ratio of emitter's internal diameter to lateral internal diameter (called the shrink ratio)
L_e / d_i (π_7)	Dimensionless term and ratio of emitter length and emitter's internal diameter

The dependent pi term, friction loss in emitter spacing, can be expressed as a function of dimensionless groups as shown below:

$$\Delta H_f/\Delta L = f\,(VD/\nu,\ V^2/gD_i,\ L_e/\Delta L,\ \Delta L/D_i, d_i/D_i,\ L_e/d_i) \quad (6)$$

The friction loss data obtained in the laboratory from 14 different emitters placed at different spacings were tabulated in a spreadsheet and the dimensionless groups shown in Table 3 were calculated. The dimensionless groups were then transformed by taking the log of each dimensionless term and transferring it to Minitab, a statistical program for fitting a multiple linear regression model stepwise.

Results

The following prediction model was developed from 849 laboratory measurements:

$$\Delta H_f/\Delta L = 0.05046\,(V^2/gD_i)^{0.864}\,(\Delta L/D_i)^{-0.28}(d_i/D_i)^{-2.816}(L_e/d_i)^{0.027} \quad (7)$$

where ΔH_f is the friction loss in emitter spacing (m), V is velocity (m s^{-1}), g is the acceleration of gravity (m s^{-2}), D_i is the pipe's internal diameter (m), ΔL is emitter spacing (m), d_i is the emitter's interior diameter (m), and L_e is emitter length (m).

The results from the regression analysis are given in Table 4. The developed model is significant, indicating a high correlation, and each term in the regression equation is significant at 95% probability. During the stepwise regression, a Froude number was selected and entered into the model as the first and the most significant dimensionless term. The Reynolds number was not included since both terms had the same varying quantities during the laboratory experiments. What was obtained from the regression analysis is meaningful since the water temperature was kept constant. Therefore the kinematic viscosity did not vary and therefore only velocity and the pipe's interior diameter changed, hence making the Reynolds and Froude numbers the related dimensionless terms. In addition to the Reynolds number, the $L_e/\Delta L$ dimensionless term was insignificant at 95% probability and therefore it was not included in the model.

Table 4. Results of multiple regressions analysis.

	Friction loss in a unit length of lateral (ΔH_f / ΔL)		
	Exponential term	Standard error	R^2 (%)
Log (constant)	-1.297	0.0109	-
V^2 / gD_i	0.864	0.0034	93.22
$\Delta L / D_i$	-0.280	0.0055	97.29
d_i / D_i	-2.816	0.0930	98.68
L_e / d_i	0.027	0.0067	98.70

The friction loss model developed can be written in a simplified form by taking the acceleration due to gravity as 9.81 m s^{-2}

$$\Delta H_f = 0.007017\ V^{1.728}\ D_i^{-2.232}\ \Delta L^{0.72}\ d_i^{-2.843}\ L_e^{0.027} \quad (8)$$

where ΔH_f , D_i , ΔL, d_i and L_e are expressed in m, and V is in m s^{-1}.

If one wants to calculate friction losses using the flow rate, equation 8 can be written as

$$\Delta H_f = 0.007017\ Q^{1.728}\ D_i^{-1.224}\ \Delta L^{0.72}\ d_i^{-2.843}\ L_e^{0.027} \quad (9)$$

where Q is in m^3 s^{-1}.

As seen from the model, friction losses are affected by the lateral/emitter properties and emitter spacing.

The model gives satisfactory results within the range of operational and lateral/ emitter related variables and it is valid for the following conditions:

$0.2 \le \Delta L \le 1$ m

$13 \le D_i \le 14$ mm

$11.4 \le d_i \le 12.0$ mm

$31.5 \le L_e \le 68.8$ mm

Model verification and discussion follow.

The prediction model was verified against the data obtained in the laboratory and the comparison is illustrated in Figure 2. A good agreement between the model and the laboratory measurements exists. The results from the comparison of the measured data against the Darcy-Weisbach (Equation 1) and the Hazen-Williams models (Equation 3) along with the model developed (Equation 8) in this study can be seen in Figures 3 and 4. The developed model predicts friction losses better than the others. Similar comparisons made using other emitters used in this study resulted in the same trend. Figures 3 and 4 are vivid examples of why a friction loss model including emitter properties and emitter spacings as developed in this study was needed.

One of the main findings from this study is the exponent of the flow rate as shown in Equation 9. The

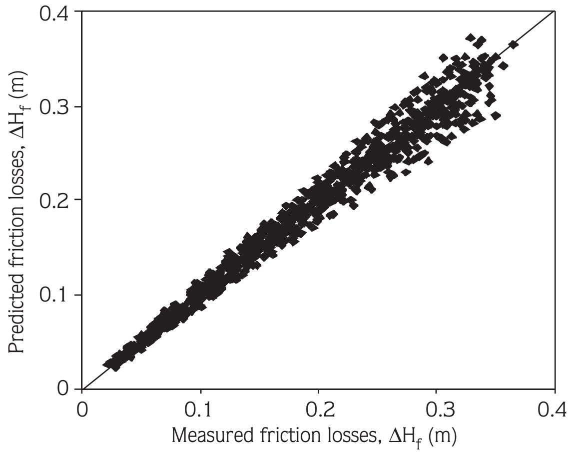

Figure 2. Comparison of the measured and predicted friction loss (ΔH_f) values.

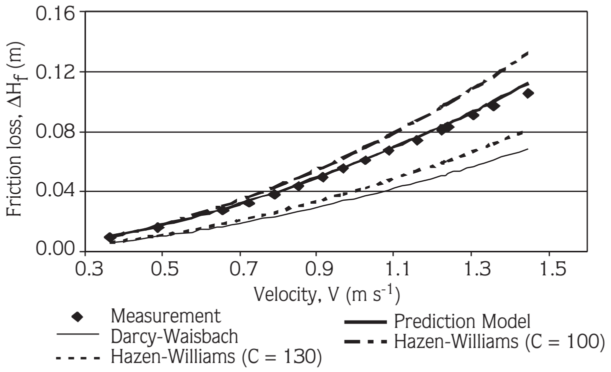

Figure 3. Comparison of the measured data against the Darcy-Weisbach, Hazen-Williams and prediction models at different velocities (Emitter no: 3; Emitter spacing (ΔL): 0.33 m).

exponent of flow rate was 1.728. This value is close to the m = 1.852 defined in the Hazen-Williams equation and m = 1.75 found by Blasius. The flow rate exponent in the model indicates turbulent flow as seen in other widely used models.

The pipe diameters exponent (n) in the Hazen-Williams (Wu et al., 1986) and Blasius (Watters and Keller, 1978) models are 4.871 and 4.75, respectively. In order to define friction losses as a function of pipe and emitter properties and operating conditions, the developed model includes both the pipe and emitter diameters separately. Once the pipe is considered smooth, this assumption results in 4.067, which is the sum of the

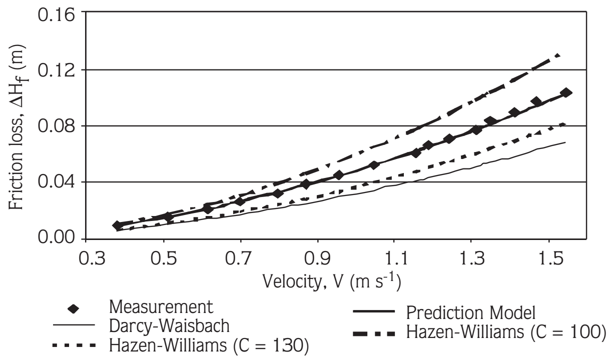

Figure 4. Comparison of the measured data against the Darcy-Weisbach, Hazen-Williams and prediction models at different velocities (Emitter no: 6; Emitter spacing (ΔL): 0.75 m).

exponent value of the interior diameters of the pipe and emitter. Even though the exponent value of the pipe diameter from this study is in agreement with other models, the differences between this model and other, well known models may cause some significant differences in friction losses (Watters and Keller, 1978; Allen, 1996; Demir and Uz, 1999). Additionally, assuming a constant C value for each flow condition and using the Hazen-Williams equation may cause some different findings as implied by some researchers (Watters and Keller, 1978; Braud and Soom, 1981; Demir and Uz, 1995). These are shown in Figures 3 and 4.

Conclusion

1. The developed model in this study will estimate the friction loss for different in-line, cylindrical type emitters possible with an acceptable accuracy.

2. It is also thought that the model can be used to calculate friction losses for the design of similar types of new emitters at different spacings by considering the soil infiltration value.

3. More sophisticated models that include the effect of water temperature and some new models for other types of emitters are needed. Hence, the authors plan to work on these models since drip irrigation is expected to expand and be used in larger areas where projects should be undertaken with accurate calculations and approaches.

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