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Electromagnetic perturbations of small Schwarzschild anti-De sitter black holes: quasinormal modes

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Abstract: In this paper, the evolution of a Maxwell field propagating on the background of small Schwarzschild anti-de Sitter black holes is studied by numerical simulation. The pictures show that the quasinormal frequencies of a Maxwell field around a small anti-de Sitter black hole are different from that of a scalar field for a small black hole or an electromagnetic field for a large black hole.

Key words: electromagnetic perturbations; Quasinormal Modes; small black holes

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1 Introduction

In the past few years, quasinormal modes (QNMs) of anti-de Sitter black holes in the scalar field have been investigated. Chan and Mann first studied the quasinormal ringing for a conformally coupled scalar field in anti-de Sitter (AdS) space [1]. And the evolution of d- dimensional small Schwarzschild anti-de Sitter black holes has also been studied [2], etc. Today, quasinormal modes of AdS black holes in an electromagnetic field are being noticed. It is well known that the qusinormal ringing in an electromagnetic field is very important to further study black holes, owing to the AdS/CFR conjecture. So Cardoso and Lemos [3] began to discuss an exact solution for the QNMs of scalar, electromagnetic and Weyl perturbations of a Banados-Teitelboim-Zanelli (BTZ) black hole. Then, quasinormal modes of electromagnetic and gravitational perturbations of a Schwarzschild black hole in ansymptotically AdS space-time [4] were studied . Recently, E. Beti and K. D. Kokkotas [5] begin to study scalar ,electromagnetic and gravitational perturbations of a Reissner-Nordstrom-anti de Sitter (RN-AdS) space-time, and compute their quasinormal modes. They say that electromagnetic perturbations of large black holes are characterized by the existence of purely damped modes. Nowadays, the relation between the cosmological constant and different kinds of fields' propagations in the Schwarzschild AdS and the RN-AdS black holes is researched [6]. However, for the evolution of the Maxwell

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field around the small AdS black hole background, there is no survey yet.

In this paper, we analyze in detail here the wave propagation of the Maxwell field around small four-dimensional Schwarzschild AdS black holes. By numerical simulation, we get two pictures. One shows the relationship between the evolution of the Maxwell field and the event horizon. The other tells us the information of quasinormal ringing which changes with l.

2 Maxwell perturbations

We consider the evolution of a Maxwell field in a Schwarzschild-anti-de Sitter spacetime with metric given by

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{1}$$

In the equation (1), $f(r)$ is defined as

$$f(r) = \frac{r^2}{R^2} + 1 - \frac{2M}{r^2}, \tag{2}$$

where R is the AdS radius and M is the black hole mass. The black hole horizon is at $r = r_+$, the largest root of $f(r) = 0$. In this paper, we discuss the small black hole with $r_+ < R$.

Let us consider a Maxwell field in the Schwarzschild AdS space-time, obeying the wave equation:

$$F^{\mu\nu};\nu = 0, \text{ with } F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}, \tag{3}$$

where four-dimensional vector A of electromagnetic potential as following:

$$A_t = 0,\ A_r = 0,\ A_\theta = 0,\ A_\phi = \psi(r,t)\sin\theta\frac{dP_l(\cos\theta)}{d\theta}. \tag{4}$$

Putting Equation (4) into Maxwell's Equation (3), we get a second order differential equation for perturbation:

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial r^{*2}} = V(r)\psi, \tag{5}$$

where the effective potential

$$V(r) = f(r)\left[\frac{l(l+1)}{r^2}\right], \tag{6}$$

and the tortoise coordinate r^* is defined as

$$dr^* = \frac{dr}{f(r)}. \tag{7}$$

Obviously, the effective potential of a Maxwell field $V(r)$ is not the same as that of a scalar field or that of large holes in an electromagnetic field. For perturbations with $l > 0$, we can show explicitly that the effective potential is positive definite. It vanishes at the horizon, which corresponds to $r^* \to -\infty$, and is a finite value at $r \to \infty$ corresponding to a finite value of r^* which requires that ψ vanished at infinity. The boundary conditions are satisfied with the wave equation.

3 Numerical Simulation

We introduced light-cone variables $u = t - r^*$ and $v = t + r^*$, in terms of which the equation (5) can be written as

$$-4\frac{\partial^2\psi}{\partial u\partial v}=V(r)\psi\ . \tag{8}$$

Considering that the decay of the test field is independent of the initial conditions (This fact is confirmed in Ref. [2]). We begin at a point (u_0 , v_0) with a Gaussian pulse of width σ , centered on v_c (v_c is quite far away from v_0) and on $u = u_0$, and set the field to zero on $v = v_0$. In addition, we can freely set the value of v_c , because it has an insignificant effect on the evolution of the test field.

$$\psi\ (u=u_0,v)=\exp\left[-\frac{(v-v_c)^2}{2\sigma^2}\right], \tag{9}$$

$$\psi\ (u,v=v_0)=0. \tag{10}$$

We can discretize the equation (8) and then implement a finite differencing scheme to solve it numerically. Using Taylor's theorem, it is discretized as

$$\psi_N=\psi_E+\psi_W-\psi_S-\delta u\delta vV\left(\frac{v_N+v_W-u_N-u_E}{4}\right)\frac{\psi_W+\psi_E}{8}+O(\Delta^4), \tag{11}$$

where we define the points as: N: $(u+\Delta, v+\Delta)$, W: $(u+\Delta, v)$, E: $(u, v+\Delta)$ and S: (u,v). Inifically, we calculate the values of $\psi\ (u_0,v)$ for various values of v in term of equation (9). Secondly, in light of equation (10), the point in the field can be calculated by using the former three points in the $u-v$ plane. After the intergration is completed, the values of $\psi\ (u_{max}, v)$ and $\psi\ (u, v_{max})$ are obtained, where the point of the u_{max} and v_{max} is the summit on the numerical grid. Taking sufficiently large u_{max} and v_{max}, we have a good approximation for the wave function at the quasinormal modes for AdS space. We fix $R = 1$ in the following.

Now we start to report the results of our numerical simulations of evolving electromagnetic field on small Schwarzschild AdS black hole background.

For the four-dimensional small black hole ($r_+ < R$), quasinormal ringings are displayed in Fig. 1 for selected values of r_+ and multipule index $l = 1$.

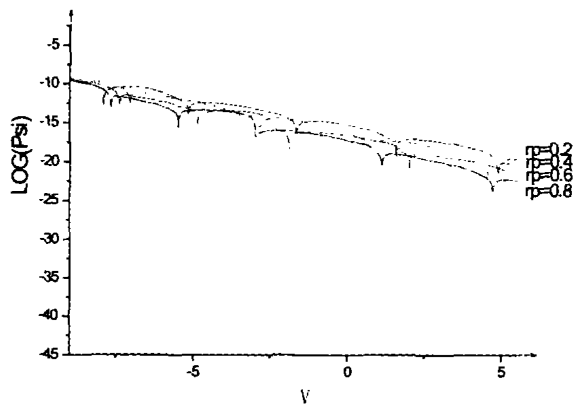

Figure 1 The wave functions for small AdS black holes for $l = 1$, with $r_+ = 0.2, 0.4, .06, 0.8$

As is shown in Fig. 1, the oscillation time scale increases slightly with the event horizon r_+ increasing, which means that the smaller the black holes is, the bigger the real part of frequency (ω_R) is. This phenomenon is distinct from that in Ref. [2] where it is illustrated that the oscillation time scale in scalar field almost keeps as a constant for various of r_+ . And this behavior also differs from that of large black holes in electromagnetic field [4] where it is said that some quasinormal modes of large black holes in electromagnetic field do not oscillate, which only decay since they have pure imaginary frequencies. Here in Fig. 1, we present the evolution pictures for $r_+ = 0.2, 0.4, 0.6, 0.8$ respectively. We have also calculated the evolution for several other event horizons. However, we haven't found anything like that described in Ref. [4].

From the picture, we can also see that the rates of the damping for different event horizons are almost identical. And this result is not similar to that in Ref. [2] or that in Ref. [4]. It is shown in Ref. [2] that the rates of damping vary with r_+. While in Fig. 1 we see only little difference of damping rate for different r_+. It tells us that the dependency of decay on r_+ in electromagnetic field is not as sensitive as that in scalar field. According to Ref. [4], the imaginary part of the frequency scales linearly with r_+. So the evolution of an electromagnetic field around small Schwarzschild AdS black holes with r_+ has its own features.

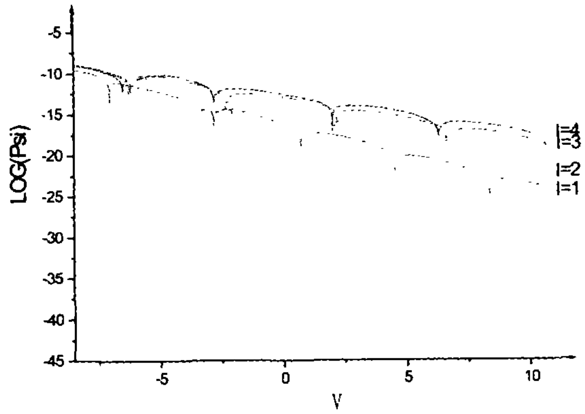

Figure 2　The wave functions for small AdS black holes for defferent l with $r_+ = 0.4$

The small black holes of the lowest multipole index $l = 1$ has been discussed. Then we show how wave dynamics behaves for a Maxwell field on the background of the AdS small black holes with different multipole index in Fig. 2. We can see in the picture that the oscillation time scale and the damping time scale for the quasinormal modes are varying with l.

On the one hand, we learn that the period of oscillation increases with l, which means that the real part of the qusinormal frequency (ω_R) decreases. This result is opposite to the conclusion in Ref. [2]. On the other hand, the damping time scale also increases(ω_I decreases). It is similar to that in Ref. [2]. And also, the connection of ω_I and l in Fig. 2 is not alike to that of large black holes in the electromagnetic field which shows that the imaginary part of the frequency is nearly independent of the multipole index l. If the multipole index is large enough, the rates of the damping are almost the same. It is natural and reasonable that the results we obtain are quite different from that in Refs. [2] and [4], since they have different effective potentials.

4　Conclusion

In a word, we have studied the evolution around small Schwarzschild AdS black holes and got different results from that of large AdS black holes in a Maxwell field and that of small black holes in a scalar field. We show the result of the increase of the oscillation time scales and the nearly same of the damping time scales with r_+. At the same time, we find that increasing l, the real part of frequency decreases with $r_+ = 0.4$. Certainly, they have many properties that are identical to small black holes in the scalar field and large black holes in the electromagnetic field. All these features are constructive to the study of black holes.

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Schwarzschild anti - de sitter 小黑洞的电磁扰动

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摘要: 通过数值模拟研究了在 Schwarzschild anti-de Sitter 小黑洞周围电磁场的演化情况. 得到的两幅图显示出小黑洞周围电磁场的拟正则模和小黑洞的标量场及大黑洞的电磁场的拟正则模的不同之处.

关键词: 电磁场扰动; 拟正则模; 小黑洞