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The Multipliers on Two Sorts of Function Spaces

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Abstract: This paper deals with the pointwise multipliers from space $F(p,q,s)$ to space β^α on the unit ball B of C^n. The multiplier spaces $M(F(p,q,s),\beta^\alpha)$ are fully characterized.

Key words: pointwise multiplier; characterization; $F(p,q,s)$ space; Bloch type space; unit ball.
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1. Introduction

Let $\mathrm{d}v$ be the normalized Lebesegue measure on the unit ball B of C^n such that $v(B)$=1, and $d\sigma$ be the normalized rotation invariant measure on the boundary ∂B of B such that $\sigma(\partial B)$=1. By $H(B)$ we denote the class of all holomorphic functions on B. H^∞ denotes the class of all bounded holomorphic functions on B.

For $a \in B$, let $g(z,a) = \log|\varphi_a(z)|^{-1}$ be Green's function for B with logarithmic singularity at a, where φ_a is the Möbius transformation of B satisfying $\varphi_a(0) = a$, $\varphi_a(a) = 0$, $\varphi_a = \varphi_a^{-1}$.

Let $0 < p,\ s < \infty$, $-n-1 < q < \infty$. We say $f \in F(p,q,s)$ provided that $f \in H(B)$ and

$$\|f\|_{F(p,q,s)} = |f(0)| + \{\sup_{a\in B}\int_B |Rf(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z)\}^{\frac{1}{p}} < \infty,$$

where

$$Rf(z) = \sum_{j=1}^n z_j \frac{\partial f(z)}{\partial z_j}.$$

$F(p,q,s)$ is a Banach space. If $q+s \le -1$, then $F(p,q,s)$ reduces to the space of constant functions. The space $F(p,q,s)$ was first introduced by Zhao[1]. The space includes many function spaces if we take some specific parameters of p,q and s such as Bloch type space, Q_s space and BMOA space. It may also include Bergman space and Besove space etc.

For $\alpha \in (-\infty,+\infty)$, f is said to be in the Bloch type space β^α provided that $f \in H(B)$ and

$$\|f\|_{\beta^\alpha} = |f(0)| + \sup_{z\in B}(1-|z|^2)^\alpha|Rf(z)| < \infty.$$

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β^p is a Banach space and β^α reduces to the space of constant functions as $\alpha < 0$. The spaces β^1 and β^α $(0 < \alpha < 1)$ are just the Bloch space β and the Lipschitz space $L_{1-\alpha}$, respectively.

Let X, Y be two spaces of holomorphic functions on B. We call φ a pointwise multiplier from X to Y if $\varphi f \in Y$ for all $f \in X$. The collection of all pointwise multipliers from X to Y is denoted by $M(X, Y)$.

The multiplier theory of function spaces has been studied for a long time and a lot of results have been obtained. We know that multiplier theory is one of the important parts in the studies of the Gleason problem, function space properties and general operator theory. Taylor and Stegenga in [2–3] first characterized the pointwise multipliers between Dirichlet type spaces in the unit disc, and the same problem on the unit ball of C^n was discussed in [4]. In [5], by using Bergman metric, Zhu studied the pointwise multipliers on Bloch space, little Bloch space, BMO space and VMO space. As a general rule, for the same type of function spaces, because of the norms being very much the same, we may discuss the pointwise multiplier from a space to another space. Naturally, we want to know if we can discuss the same problems for two different type of function spaces. We have tried in [6] and get some results. It is known that Bloch type space β^p is the generalization of Bloch space and general function space $F(p, q, s)$ includes many function spaces. If we get the pointwise multiplier between $F(p, q, s)$ and β^p, then, in fact, we obtain the results in many function spaces. In this paper, our principal work is to characterize $M(F(p, q, s),\ \beta^\alpha)$ for all kinds of cases about parameters p, q, s.

2. Some lemmas

In the following, $z = (z_1, \ldots, z_n)$, $a = (a_1, \ldots, a_n)$, $\langle z, a\rangle = \sum_{j=1}^n z_j \overline{a}_j$. We will use the symbol c to denote a finite positive number which does not depend on variables z, a, w but may depend on some norms and parameters p, q, n, s, α, x etc., not necessarily the same at each occurrence.

In order to prove the main result, we first give some lemmas.

Lemma 2.1 *For $0 < p,\ s < \infty$, $-n-1 < q < \infty$ and $q + s > -1$, if $f \in F(p, q, s)$, then $f \in \beta^{\frac{n+1+q}{p}}$.*

Proof Suppose $f \in F(p, q, s)$. For fixed $0 < r_0 < 1$, since $(Rf)\circ\varphi_a \in H(B)$, $|(Rf)\circ\varphi_a|^p$ is subharmonic in $r_0 B$, that is,

$$\begin{aligned} |Rf(a)|^p = |(Rf)\circ\varphi_a(0)|^p &\le \frac{1}{r_0^{2n}} \int_{r_0 B} |(Rf)\circ\varphi_a(w)|^p \mathrm{d}v(w) \\ &= \frac{1}{r_0^{2n}} \int_{\varphi_a(r_0 B)} |Rf(z)|^p \frac{(1-|a|^2)^{n+1}}{|1-\langle z, a\rangle|^{2n+2}} \mathrm{d}v(z). \end{aligned} \tag{2.1}$$

For $z \in \varphi_a(r_0 B)$, from (5) in [7], we have

$$g(z, a) = \log \frac{1}{|\varphi_a(z)|} \ge \log \frac{1}{r_0}$$

and

$$\frac{1-r_0}{1+r_0}(1-|a|^2)\leq 1-|z|^2\leq \frac{1+r_0}{1-r_0}(1-|a|^2).$$

Thus

$$\frac{(1-|a|^2)^{n+1}}{|1-\langle z,a\rangle|^{2n+2}(1-|z|^2)^q g^s(z,a)}\leq \frac{4^{n+1}}{(1-|a|^2)^{n+1+q}}(\frac{1+r_0}{1-r_0})^{|q|}\log^{-s}\frac{1}{r_0}. \tag{2.2}$$

From (2.1) and (2.2), we get

$$\begin{aligned}|Rf(a)|^p &\leq \frac{1}{r_0^{2n}}\int_{\varphi_a(r_0B)}|Rf(z)|^p\frac{(1-|a|^2)^{n+1}}{|1-\langle z,a\rangle|^{2n+2}}\mathrm{d}v(z)\\ &= \frac{1}{r_0^{2n}}\int_{\varphi_a(r_0B)}|Rf(z)|^p(1-|z|^2)^q g^s(z,a)\frac{(1-|a|^2)^{n+1}}{|1-\langle z,a\rangle|^{2n+2}(1-|z|^2)^q g^s(z,a)}\mathrm{d}v(z)\\ &\leq \frac{4^{n+1}r_0^{-2n}}{(1-|a|^2)^{n+1+q}}(\frac{1+r_0}{1-r_0})^{|q|}\log^{-s}\frac{1}{r_0}\|f\|^p_{F(p,\ q,\ s)}.\end{aligned}$$

This shows that $f\in\beta^{\frac{n+1+q}{p}}$.

Lemma 2.2 *(1) Let $f\in\beta^p$. Then* $|f(z)|=\begin{cases}O(1), & p<1,\\ O(\log 2(1-|z|^2)^{-1}), & p=1,\\ O((1-|z|^2)^{1-p}), & p>1.\end{cases}$

(2) For $p\geq 0$, if $f\in H(B)$ and

$$|f(z)|=O(\frac{1}{(1-|z|^2)^p}),\quad then\quad |Rf(z)|=O(\frac{1}{(1-|z|^2)^{p+1}}).$$

Proof This is Lemma 2.2 in [6].

3. The main results

Theorem 3.1 *Let $0<p,\ s<\infty$, $-n-1<q<\infty$, $q+s>-1$. If $n+1+q>p\alpha$, then $\varphi\in M(F(p,q,s),\ \beta^\alpha)$ if and only if $\varphi\equiv 0$.*

Proof Suppose $\varphi\in M(F(p,q,s),\ \beta^\alpha)$ and $a\in B$. We set

$$x=\frac{(1-|a|^2)(1-|z|^2)}{|1-\langle z,a\rangle|^2}=1-|\varphi_a(z)|^2.$$

Then

$$g(z,a)=-\frac{1}{2}\log(1-x)\leq\frac{x}{2}[1+\frac{3}{4}+(\frac{3}{4})^2+\cdots]=2x\quad\text{as}\quad \frac{1}{2}\leq|\varphi_a(z)|<1. \tag{3.1}$$

For any $w\in B$, let

$$f_w(z)=\frac{1-|w|^2}{(1-\langle z,w\rangle)^{\frac{n+1+q}{p}}}.$$

First, we consider the case $s>n$. By (3.1) and Proposition 1.4.10 in [8], we have

$$\begin{aligned}&\int_{1/2\leq|\varphi_a(z)|<1}|Rf_w(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z)\\ &\leq\int_{1/2\leq|\varphi_a(z)|<1}\frac{2^s(n+1+q)^p(1-|w|^2)^p(1-|a|^2)^s(1-|z|^2)^{q+s}}{p^p|1-\langle z,w\rangle|^{n+1+q+p}\ |1-\langle z,a\rangle|^{2s}}\mathrm{d}v(z)\\ &\leq\frac{2^{s+n+1+q+p}(n+1+q)^p}{p^p}\int_B\frac{(1-|a|^2)^s(1-|z|^2)^{s-n-1}}{|1-\langle z,a\rangle|^{2s}}\mathrm{d}v(z)\leq c.\end{aligned} \tag{3.2}$$

Next, we consider the case $0 < s \le n$. We choose constants x, x' and λ satisfying

$$\max\{1, \frac{n}{q+n+1}\} < x < \frac{n}{n-s}, \lambda = q+n+1-\frac{n+1}{x}, \frac{1}{x}+\frac{1}{x'} = 1$$

such that $\lambda x > -1$, $(q+s-\lambda)x' > -1$. Applying (3.1) and Proposition 1.4.10 in [8], we have

$$\begin{aligned}
&\int_{1/2\le|\varphi_a(z)|<1} |Rf_w(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z)\\
&\le \int_{1/2\le|\varphi_a(z)|<1} \frac{2^s\ (n+1+q)^p(1-|w|^2)^p\ (1-|a|^2)^s(1-|z|^2)^{q+s}}{p^p|1-\langle z,w\rangle|^{n+1+q+p}|1-\langle z,a\rangle|^{2s}}\mathrm{d}v(z)\\
&\le \frac{2^s(n+1+q)^p}{p^p}\ (1-|w|^2)^p\{\int_B \frac{(1-|z|^2)^{\lambda x}}{|1-\langle z,w\rangle|^{(n+1+q+p)x}}\mathrm{d}v(z)\}^{\frac{1}{x}}\times\\
&\quad (1-|a|^2)^s\{\int_B \frac{(1-|z|^2)^{(q+s-\lambda)x'}}{|1-\langle z,a\rangle|^{2sx'}}\mathrm{d}v(z)\}^{\frac{1}{x'}} \le c. \qquad (3.3)
\end{aligned}$$

At the same time,

$$\begin{aligned}
&\int_{|\varphi_a(z)|<1/2} |Rf_w(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z)\\
&\le c\int_{|\varphi_a(z)|<1/2} \frac{(1-|w|^2)^p(1-|z|^2)^q}{|1-\langle z,w\rangle|^{n+1+q+p}}\log^s\frac{1}{|\varphi_a(z)|}\mathrm{d}v(z)\\
&= c\int_{|u|<1/2} \frac{(1-|w|^2)^p(1-|\varphi_a(u)|^2)^q}{|1-\langle\varphi_a(u),w\rangle|^{n+1+q+p}}\frac{(1-|a|^2)^{n+1}}{|1-\langle u,a\rangle|^{2n+2}}\log^s\frac{1}{|u|}\mathrm{d}v(u)\\
&= c\int_{|u|<1/2} \frac{(1-|w|^2)^p(1-|a|^2)^{n+1+q}}{|1-\langle\varphi_a(u),w\rangle|^{n+1+q+p}}\frac{(1-|u|^2)^q}{|1-\langle u,a\rangle|^{2n+2+2q}}\log^s\frac{1}{|u|}\mathrm{d}v(u)\\
&\le c\int_{|u|<1/2} \frac{(1-|a|^2)^{n+1+q}}{(1-|\varphi_a(u)|^2)^{n+1+q}}\frac{(1-|u|^2)^q}{|1-\langle u,a\rangle|^{2n+2+2q}}\log^s\frac{1}{|u|}\mathrm{d}v(u)\\
&= c\int_{|u|<1/2} \frac{1}{(1-|u|^2)^{n+1}}\log^s\frac{1}{|u|}\mathrm{d}v(u)\\
&\le c\int_B \log^s\frac{1}{|u|}\mathrm{d}v(u) = c\int_0^1 2nr^{2n-1}\log^s\frac{1}{r}\mathrm{d}r\int_{\partial B}\mathrm{d}(\xi) \le c. \qquad (3.4)
\end{aligned}$$

Thus, from (3.2), (3.3) and (3.4), we get

$$\begin{aligned}
&\int_B |Rf_w(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z)\\
&= (\int_{1/2\le|\varphi_a(z)|<1} + \int_{|\varphi_a(z)|<1/2})|Rf_w(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z) \le c.
\end{aligned}$$

This shows that $\|f_w\|_{F(p,q,s)} \le c$.

By the Closed Graph Theorem, $M_\varphi: f \to \varphi f$ is a bounded linear operator from $F(p,q,s)$ to β^α. Therefore,

$$(1-|z|^2)^\alpha|R[\varphi(z)f_w(z)]| \le ||\varphi f_w||_{\beta^\alpha} \le ||M_\varphi||.||f_w||_{F(p,q,s)} \le c. \qquad (3.5)$$

From (3.5) and Lemma 2.2, we obtain

$$|\varphi(z)f_w(z)| \le \begin{cases} c(1-|z|^2)^{1-\alpha}, & \alpha > 1,\\ c\log 2(1-|z|^2)^{-1}, & \alpha = 1,\\ c, & \alpha < 1. \end{cases}$$

Let $z=w$. Then

$$|\varphi(w)| \leq \begin{cases} c(1-|w|^2)^{\frac{n+1+q}{p}-\alpha}, & \alpha>1, \\ c(1-|w|^2)^{\frac{n+1+q}{p}-1}\log 2(1-|w|^2)^{-1}, & \alpha=1, \\ c(1-|w|^2)^{\frac{n+1+q}{p}-1}, & \alpha<1. \end{cases} \tag{3.6}$$

If $n+1+q>p$, from (3.6), we can get $|\varphi(w)|\to 0(|w|\to 1)$. Furthermore, we have $\varphi\equiv 0$ by Maximum Modulus Principle. If $n+1+q\leq p$, then $\alpha<1$ and from (3.5)

$$\Big|\frac{(n+1+q)(1-|w|^2)\langle z,w\rangle\varphi(z)}{p(1-\langle z,w\rangle)^{\frac{n+1+q}{p}+1}}+\frac{(1-|w|^2)R\varphi(z)}{(1-\langle z,w\rangle)^{\frac{n+1+q}{p}}}\Big|\leq \frac{c}{(1-|z|^2)^{\alpha}}.$$

Let $z=w$. We have

$$|\varphi(w)|\leq \frac{p}{(n+1+q)|w|^2}\{c(1-|w|^2)^{\frac{n+1+q}{p}-\alpha}+(1-|w|^2)|R\varphi(w)|\}.$$

Since $M(F(p,q,s),\ \beta^{\alpha})\subseteq\beta^{\alpha}$, $|R\varphi(w)|\leq c(1-|w|^2)^{-\alpha}$. We get

$$|\varphi(w)|\leq \frac{p}{(n+1+q)|w|^2}\{c(1-|w|^2)^{\frac{n+1+q}{p}-\alpha}+c(1-|w|^2)^{1-\alpha}\}\to 0\ \ (|w|\to 1).$$

This implies $\varphi\equiv 0$.

Theorem 3.2 *Let $0<p,\ s<\infty$, $-n-1<q<\infty$, $q+s>-1$, $n+1+q\leq p\alpha$.*

(1) If $n+1+q>p$, then $\varphi\in M(F(p,q,s),\ \beta^{\alpha})$ if and only if $\varphi\in H(B)$ and

$$|\varphi(z)|=O\Big(\frac{1}{(1-|z|^2)^{\alpha-\frac{n+1+q}{p}}}\Big).$$

(2) If $n+1+q<p$, then $\varphi\in M(F(p,q,s),\ \beta^{\alpha})$ if and only if $\varphi\in\beta^{\alpha}$.

(3) If $n+1+q=p$, $s>n$, then $\varphi\in M(F(p,q,s),\ \beta^{\alpha})$ if and only if $\varphi\in I_{\alpha}$, where

$$I_{\alpha}=\{\varphi:\ \varphi\in H^{\infty}\ \ \textit{and}\ \ \sup_{z\in B}(1-|z|^2)|R\varphi(z)|\log\frac{2}{1-|z|^2}<\infty\},\quad \textit{for}\ \ \alpha=1;\quad \textit{or}$$

$$I_{\alpha}=\{\varphi:\ \varphi\in H(B)\ \ \textit{and}\ \ \sup_{z\in B}(1-|z|^2)^{\alpha}|R\varphi(z)|\log\frac{2}{1-|z|^2}<\infty\},\quad \textit{for}\ \ \alpha>1.$$

Proof Suppose $\varphi\in M(F(p,q,s),\ \beta^{\alpha})$ and $a\in B$.

(1) For $n+1+q>p$, we have $\alpha>1$. From (3.6), we can obtain

$$|\varphi(w)|\leq \frac{c}{(1-|w|^2)^{\alpha-\frac{n+1+q}{p}}}.$$

(2) For $n+1+q<p$, we know $M(F(p,q,s),\ \beta^{\alpha})\subseteq\beta^{\alpha}$. Hence $\varphi\in\beta^{\alpha}$.

(3) For $n+1+q=p$, $s>n$, we have $\alpha\geq 1$.

Let

$$f_w(z)=\log\frac{2}{1-\langle z,w\rangle}.$$

Then

$$\begin{aligned}&\int_{1/2<|\varphi_a(z)|<1}|Rf_w(z)|^p(1-|z|^2)^q g^s(z,a)\mathrm{d}v(z)\\ &\le c\int_{1/2<|\varphi_a(z)|<1}\frac{(1-|z|^2)^{q+s}(1-|a|^2)^s}{|1-\langle z,w\rangle|^{q+n+1}\ |1-\langle z,a\rangle|^{2s}}\mathrm{d}v(z)\\ &\le c\int_B\frac{(1-|z|^2)^{s-n-1}(1-|a|^2)^s}{|1-\langle z,a\rangle|^{2s}}\mathrm{d}v(z)\le c.\end{aligned}$$

This implies $||f_w||_{F(p,\ q,\ s)}\le c$. Applying Lemma 2.2 to (3.5), we get

$$|\varphi(z)f_w(z)|\le\begin{cases}c(1-|z|^2)^{1-\alpha}, & \alpha>1,\\ c\log 2(1-|z|^2)^{-1}, & \alpha=1.\end{cases}$$

If we set $z=w$, then

$$|\varphi(w)|\le\begin{cases}c(1-|w|^2)^{1-\alpha}\log^{-1}[2(1-|w|^2)^{-1}], & \alpha>1,\\ c, & \alpha=1.\end{cases}\tag{3.7}$$

On the other hand, from (3.5), we have

$$(1-|z|^2)^\alpha|R\varphi(z)|\ |f_w(z)|\le c+(1-|z|^2)^\alpha|\varphi(z)|\ |Rf_w(z)|.$$

Thus, using (3.7), we get

$$\begin{aligned}&(1-|w|^2)^\alpha|R\varphi(w)|\ log\frac{2}{1-|w|^2}\le c+(1-|w|^2)^\alpha|\varphi(w)|\frac{|w|^2}{1-|w|^2}.\\ &\le\begin{cases}c+c|w|^2\log^{-1}[2(1-|w|^2)^{-1}], & \alpha>1,\\ c, & \alpha=1\end{cases}\le c.\end{aligned}\tag{3.8}$$

Combing (3.7) with (3.8), the proof of the necessary conditions is completed.

Conversely, for any $f\in F(p,q,s)$, we have $f\in\beta^{\frac{n+1+q}{p}}$ from Lemma 2.1.

(1) For $n+1+q>p$, suppose

$$\varphi\in H(B)\ \text{ and }\ |\varphi(z)|\le\frac{c}{(1-|z|^2)^{\alpha-\frac{n+1+q}{p}}}.$$

Applying Lemma 2.2, we get

$$\begin{aligned}&(1-|z|^2)^\alpha|R[\varphi(z)f(z)]|\le(1-|z|^2)^\alpha(|f(z)R\varphi(z)|+|\varphi(z)Rf(z)|)\\ &=(1-|z|^2)^{\frac{n+1+q}{p}-1}|f(z)|(1-|z|^2)^{\alpha-\frac{n+1+q}{p}+1}|R\varphi(z)|+\\ &\quad(1-|z|^2)^{\frac{n+1+q}{p}}|Rf(z)|(1-|z|^2)^{\alpha-\frac{n+1+q}{p}}|\varphi(z)|\\ &\le c\Rightarrow\varphi f\in\beta^\alpha\Rightarrow\varphi\in M(F(p,q,s),\beta^\alpha).\end{aligned}$$

Applying Lemma 2.2 and conditions, we can prove (2) and (3). And the details are omitted.

Since $F(2,1-n,n)=\mathrm{BMO}A$, from Theorem 3.1, we can get

Corollary *For $\alpha<1$, $\varphi\in M(\mathrm{BMO}A,\ \beta^\alpha)$ if and only if $\varphi\equiv 0$.*

Theorem 3.3 *Let $0 < p,\ s < \infty$, $0 < s \le n$, $-s-1 < q < \infty$, $n+1+q = p$, $\alpha \ge 1$. If $\varphi \in I_\alpha$, then $\varphi \in M(F(p,q,s),\ \beta^\alpha)$. Conversely, if $\varphi \in M(F(p,q,s),\ \beta^\alpha)$, then, for any $0 < \varepsilon < \min\{1/n-(n-s)/pn, s/pn\}$, $\varphi \in J_{\alpha,\varepsilon}$, where*

$$J_{\alpha,\ \varepsilon} = \{\varphi : \varphi \in H(B) \ \ and \ \ \sup_{z\in B}(1-|z|^2)^\alpha\ |R\varphi(z)|[\log\frac{2}{1-|z|^2}]^{1-\frac{n-s}{pn}-\varepsilon} < \infty\};$$

$$I_\alpha = \{\varphi : \varphi \in H^\infty \ \ and \ \ \sup_{z\in B}(1-|z|^2)|R\varphi(z)|\log\frac{2}{1-|z|^2} < \infty\} \ \ as \ \ \alpha = 1 \ \ or$$

$$I_\alpha = \{\varphi : \varphi \in H(B) \ \ and \ \ \sup_{z\in B}(1-|z|^2)^\alpha|R\varphi(z)|\log\frac{2}{1-|z|^2} < \infty\} \ \ as \ \ \alpha > 1.$$

Proof If $\varphi \in I_\alpha$, then $\varphi \in M(F(p,q,s),\ \beta^\alpha)$ from the proof of Theorem 3.2.

Conversely, let $\varphi \in M(F(p,q,s),\ \beta^\alpha)$. For any $w, a \in B$, we set

$$g_w(z) = \log\frac{2}{1-\langle z,w\rangle}.$$

Since $q+s+1 = p-n+s > 0$, for any $0 < \varepsilon < \min\{1/n-(n-s)/pn, s/pn\}$, we can choose constants x, x', λ satisfying

$$\max\{1, \frac{n}{p}\} < x = \frac{n}{n-s+\varepsilon pn} < \frac{n}{n-s}, \quad \lambda = p - \frac{n+1}{x}, \quad \frac{1}{x}+\frac{1}{x'} = 1$$

such that $\lambda x > -1$, $(q+s-\lambda)x' > -1$. We have

$$\begin{aligned}
&\int_{1/2<|\varphi_a(z)|<1} |Rg_w(z)|^p(1-|z|^2)^q\ g^s(z,a)\mathrm{d}v(z)\\
&\le c\int_B \frac{(1-|a|^2)^s(1-|z|^2)^{q+s}}{|1-\langle z,w\rangle|^p|1-\langle z,a\rangle|^{2s}}\mathrm{d}v(z)\\
&\le c(1-|a|^2)^s\{\int_B \frac{(1-|z|^2)^{\lambda x}}{|1-\langle z,w\rangle|^{px}}\mathrm{d}v(z)\}^{\frac{1}{x}}\times\\
&\quad \{\int_B \frac{(1-|z|^2)^{(q+s-\lambda)x'}}{|1-\langle z,a\rangle|^{2sx'}}\mathrm{d}v(z)\}^{\frac{1}{x'}} \le c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{x}}.
\end{aligned}$$

Thus

$$(1-|z|^2)^\alpha|R[\varphi(z)g_w(z)]| \le ||M_\varphi||.||g_w||_{F(p,q,s)} \le c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}}. \tag{3.9}$$

From Lemma 2.2, we get

$$|\varphi(z)g_w(z)| \le \{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}}\begin{cases} c(1-|z|^2)^{1-\alpha}, & \alpha > 1,\\ c\log 2(1-|z|^2)^{-1}, & \alpha = 1.\end{cases}$$

If we set $z = w$, then

$$|\varphi(w)| \le \{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}}\begin{cases} c(1-|w|^2)^{1-\alpha}\log^{-1}[2(1-|w|^2)^{-1}], & \alpha > 1,\\ c, & \alpha = 1.\end{cases} \tag{3.10}$$

From (3.9) and (3.10), we have

$$(1-|w|^2)^\alpha|R\varphi(w)|\log\frac{2}{1-|w|^2} \le c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}} + (1-|w|^2)^\alpha|\varphi(w)|\frac{|w|^2}{1-|w|^2}$$

$$\leq c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}}+\begin{cases} c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}-1}, & \alpha>1 \\ c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}}, & \alpha=1 \end{cases} \leq c\{\log\frac{2}{1-|w|^2}\}^{\frac{1}{px}},$$

that is,

$$(1-|w|^2)^\alpha|R\varphi(w)|\{\log\frac{2}{1-|w|^2}\}^{1-\frac{n-s}{pn}-\varepsilon}=(1-|w|^2)^\alpha|R\varphi(w)|\{\log\frac{2}{1-|w|^2}\}^{1-\frac{1}{px}}\leq c.$$

This completes the proof. □

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两类函数空间上的乘子

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摘要: 本文在 C^n 中单位球上讨论了空间 $F(p,q,s)$ 到 Bloch 型空间 β^α 上的点乘子. 对乘子空间 $M(F(p,q,s),\beta^\alpha)$ 进行了完整刻画.

关键词: 点乘子; 刻画; $F(p,q,s)$ 空间; Bloch 型空间; 单位球.