Development of Mathematical Communication in Problem Solving Groups By Language Minority Students

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Abstract

This article is an examination of mathematical communication in two algebra classes with ESL students. Using videotaped data, the author found that students in one classroom engaged in very little mathematical communication, most of which was oriented toward simple answers and fragmented procedural descriptions. This is attributed to instructional decisions that minimized the authority and use of small groups, and students' difficulties with the mathematical register in both English and Spanish. This contrasted with the other classroom in which students engaged in extensive mathematical communication in small and large group formats. The paper discusses how small groups facilitate the development of communicative competence and the potential value of computers for stimulating discussion.

With the adoption of a constructivist philosophy, mathematics educators now advocate more active learning on the part of students and a more facilitative role for teachers. A key component of most new instructional programs is that students are expected to discuss mathematics with their peers and their teachers. This new emphasis upon mathematical communication is a challenge for teachers and students in classrooms everywhere. For the 1.2 million Limited English Proficiency (LEP) students in California and their counterparts in other states, the challenge is even greater. As recently reported by the California Department of Education, most of these students receive no content instruction in their native language (Macias, 1995). These students are given the double challenge of learning new ways to talk about mathematics while learning a second language.

There is an urgent need for researchers and teachers to work together to find effective ways of including LEP students in the new mathematics programs, using the resources that are currently available, so that linguistic minority students are not left behind as mathematics education is transformed. This study addresses that need by examining how two teachers who were implementing an innovative algebra program called *College Preparatory Mathematics: A Change from Within* (CPM) structured classroom discussions in classes with substantial numbers of second language learners. One teacher, Miss G, taught a class composed totally of native speakers of Spanish; Miss G was not a native Spanish speaker, had a very low level of proficiency in

Spanish, and used only English as a medium of instruction. The teacher consciously used the principles of sheltered instruction (Snow, 1990) and made other adaptations to her teaching style. The school also assigned Spanish-speaking aides to help in this class. The second class, taught by Miss Y, was at the same school and used the same curriculum. A majority of students in the class were native speakers of English, but with a substantial minority having similar language skills to Miss G's students. Miss G's class is of particular interest because it represents one school's attempt to meet the needs of students with limited English proficiency while carrying out a major change in the mathematics curriculum. All names mentioned in this article are pseudonyms.

The goal of this article is to describe what kinds of mathematical communication took place under what instructional conditions in each classroom. In particular, I want to analyze why the students in Miss G's class engaged in almost no mathematical communication in either large group discussions or in their cooperative groups, despite the school's efforts to provide extra resources in support of the class. I will argue that the changes made to accommodate the language differences between the teacher and students in fact undermined the CPM program's intended cooperative group structure. In addition, despite the extra use of Spanish-speaking aides in the class, the students did not receive enough comprehensible language input to develop their communication skills in either English or Spanish. In contrast, the students in Miss Y's class had more opportunities to practice their mathematical communication because of Miss Y's more extensive reliance upon small group instructional formats.

Educational Reform in Mathematics and Linguistic Minority Students

Improving students' ability to communicate mathematics is one of the major goals of the mathematics reform movement. The *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics (1989) states:

> In grades 9-12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can: reflect upon and clarify their thinking about mathematical ideas and relationships; formulate mathematical definitions and express generalizations discovered through investigations; express mathematical ideas orally and in writing; read written presentations of mathematics with understanding; ask clarifying and extending questions related to mathematics they have read or heard about; (and) appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas. (p. 140)

In California, where this research took place, this same theme is incorporated into the *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve* (California State Department of Education, 1992b). Mathematical communication is one of the four large themes of the mathematics curriculum. In addition, "logic and language" is designated as one of the seven content areas for every grade level.

Improved equity is another major goal of the mathematics reform movement. As stated in the *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve:*

> . . . this document reasserts the goal of mathematical power for all students and emphasizes the phrase "for all students." Many of the recommendations here are motivated by a concern for equity-giving every student in California fair access to mathematics education. Included are females and males; rich, poor, and middle class; descendants from all parts of the world; speakers of Mandarin, English, Arabic, Spanish and the more than 200 other first languages of U.S. citizens. (California State Department of Education, 1992b, pp. 2-3)

It has been amply documented that traditional mathematics instruction has not promoted the achievement of minority students, including linguistic minority students. On standardized tests, Latino students and linguistic minority students on average score below the mean in both California (California State Department of Education, 1992a) and the nation (Mullis, Dossey, Owen, & Phillips, 1991). A recent report reveals that fewer than 4% of the Latino students graduating from California's high schools have met the entrance requirements in mathematics for admission to the University of California (University of California Latino Eligibility Task Force, 1993). Many secondary schools make no provision for linguistic minority students in the content areas by offering courses either with sheltered instruction or in students' primary language (Minicucci & Olsen, 1992).

Many of the new mathematics programs have been written with a clear intent of meeting the challenge of mathematics reform while incorporating instructional practices that will promote achievement for a broader range of students. The CPM math program was explicitly designed to meet the *California State Mathematics Framework* (California State Department of Education, 1992b) goal of establishing a high school sequence in algebra and geometry that would make these courses accessible to more students. Anecdotal evidence and formal evaluations of the CPM program indicate that the achievement of students from various ethnic/racial backgrounds is indeed enhanced. Teachers report that more students take algebra and pass it when CPM is adopted (Kysh, 1991). A formal evaluation of the program in 1992 found that on regular standardized tests the CPM students were equal to comparison classes (Sallee, 1992). More importantly, students of all ethnic groups in CPM classes scored significantly higher than their comparison peers on a specially designed test of problem solving. However, comparisons of groups taught with the CPM curriculum showed that differentials continued to exist between ethnic and racial groups that were comparable to those that exist when traditional curricula are used.

While these results are encouraging, a number of issues remain unresolved. The data reported by Sallee (1992) do not indicate how students with limited English proficiency perform in CPM, nor even if such students are receiving the CPM curriculum. There is also evidence from other evaluative studies of the program that students at high risk for academic failure do no better in CPM math than in traditional programs. At one school that adopted the program, only 23% of the Hispanic and African American students achieved grades in CPM algebra that were high enough (C- or better) for promotion to geometry (Risacher, 1994). This passing rate was no higher than with the traditional program at the school; this was particularly disappointing because many other support services had been established when the mathematics curriculum was changed.

There is also a lack of information about how innovative high school curricula, including CPM, have been implemented, particularly with regard to the needs of students with non-English-speaking backgrounds. Other studies suggest that language minority students do not receive appropriate language input in their mathematics instruction. Khisty's (1993, 1995) studies of bilingual elementary classrooms found that even with bilingual teachers, the students did not receive accurate training in the language of mathematics. Although fluent in Spanish, some of Khisty's teachers had not studied mathematics in Spanish and made errors in their mathematical terminology in Spanish. The teachers tended to use concurrent translation in their instruction (alternating between English and Spanish) but actually did less of the content instruction in Spanish and used the Spanish language more as a motivator. Khisty's work focused primarily on the teachers' use of language, but she also noted that the students sometimes used incorrect mathematical terms in English without correction from their teachers. Mestre and Gerace's (1986) study of Hispanic students in traditional algebra classes found that they were "extremely poor at verbalizing definitions of mathematical terms, even when they possessed a correct operational definition of the term," and they "could seldom verbalize the procedure they used in obtaining the solution" (p. 155). The authors found that the students seldom read the textbook even though this was one easily available source of mathematical language and vocabulary. Even with extensive modeling and bilingual instruction, becoming more effective mathematical communicators in two languages is a challenge for linguistic minority children. Thornburg and Karp (1992) found that over the course of one year in a project designed to promote more cooperative problem solving among linguistic minority students, the students greatly improved their mathematical communication in small groups when speaking their first language. Although they used more English over the course of the study, certain aspects of their native language never shifted to English. In addition, the students did not change the way they responded to teacher questions, continuing to answer briefly and unelaboratedly without demonstrating many of the skills they utilized in peer discussions.

CPM, like many other mathematics reform programs, emphasizes improved student communication in mathematics as a desired outcome. In support of this goal, CPM entails a number of pedagogical changes, some of which hold great promise for language minority students, others of which pose a challenge. In CPM, most learning occurs within small groups during problem solving sessions. The program incorporates both cooperative task structures and cooperative reward structures. The cooperative student groups have a major responsibility for learning because as the authors (Kasimatis & Sallee, 1993) put it, "knowledge will be generated within the group" (p. 6) rather than being directly transmitted from teachers to students. Early in the academic year, CPM gives the students tasks in which they must cooperate because each student is given a different piece of information necessary for solving a problem. Later in the year, the students are still expected to depend upon their group for most help.

For many language minority students in California, particularly Latino students, the small group format may prove to be a comfortable and culturally appropriate instructional format. Kagan (1986) and Losey (1995) find evidence in the literature that Mexican American children function well in cooperative situations as opposed to more individualistic or competitive settings. In addition, they may be more likely to participate in small group discussions than large group discussions, particularly when the language of large group interaction is English. There is also some evidence that the traditional large group recitation format that typically takes the form of Initiation-Response-Evaluation (Mehan, 1979; Tharp & Gallimore, 1988) is uncomfortable for Mexican American students because it conflicts with home styles of communication and places the individual student too much in the spotlight of class attention. Although small group formats may be comfortable and productive for Mexican American children, there is virtually no description of how students interact around subject matter content within peer groups. With the increased emphasis upon mathematical communication and higher expectations for what will be discussed in small groups, it is important to determine how teachers' instructional decisions affect the content of students' small group discussions.

In order to analyze what happened in the two classrooms, I used a number of theoretical constructs. The next section briefly reviews why mathematics education has begun to emphasize more discussion about mathematics. Then a framework for analyzing different kinds of mathematical communication is introduced. Finally, the construct of participant structures is described, with particular reference to its utility for understanding the classroom experiences of children from different cultural backgrounds.

Theoretical Background

The emphasis upon communication in the mathematics reform movement derives from a consensus that learning proceeds most effectively within a social context. This social constructivist perspective has led a number of authors to suggest that students need to go through a process of enculturation by participating in mathematics classrooms which are communities of mathematical practice (Bishop, 1991; Lampert, 1990; Schoenfeld, 1992). Through active discussion with their teacher and peers, students are expected to gain a greater understanding of the conceptual underpinnings of mathematics and become better problem-solvers. This approach is supported by the theoretical writings of Vygotsky. Vygotsky (1978) posited that learning takes place when the learner first collaborates with an adult or more competent peer to accomplish a task just beyond the learner's level of independent functioning, within the "zone of proximal development." What is accomplished in a social context is then internalized for individual mastery. When peers work together they must be able to describe their problem solving processes and also reflect upon these if they are to scaffold or otherwise support each others' problem solving performance. In addition to benefiting from peer interactions because peers can be a source of help, other work suggests that students benefit from hearing a variety of different perspectives about a problem solving situation. Sociocognitive conflict between peers of different levels of functioning can stimulate cognitive growth without the active peer tutoring implied in the Vygotskian model (Doise & Mugny, 1984). Growth can also occur when learners with different perspectives, but equal competency, "help each other incorporate new problem-attack and reasoning strategies into their repertoire" (Forman, 1989, p. 67). Cooperative collaboration of this sort enables students to accomplish tasks that may be beyond the competency of any individual participant.

Although cooperative group work is widely used to develop communities of mathematical practice in classrooms, surprisingly little research has been done that documents what actually occurs in student groups (Good, Mulryan, & McCaslin, 1992). Several reviews of the literature suggest that students who actively give explanations benefit the most from the small group experience (Webb, 1985, 1989). To better describe the kinds of explanations given by students and the language they use, I developed a Communications Framework for Mathematics (Brenner, 1994). Table 1 summarizes the framework.

Table 1

Communication Framework for Mathematics

Communication About Mathematics	Communication In Mathematics	Communication With Mathematics
1. Reflection on cognitive processes. Description of procedures, reasoning. Metacognition—giving reasons for procedural decisions.	1. Mathematical register. Special vocabulary. Particular definitions of everyday vocabulary. Modified uses of everyday vocabulary. Syntax, phrasing. Discourse.	1. Problem-solving tool. Investigations. Basis for meaningful action.
2. Communication with others about cognition. Giving point of view. Reconciling differences.	2. Representations. Symbolic. Verbal. Physical manipulatives. Diagrams, graphs. Geometric.	2. Alternative solutions. Interpretation of arguments using mathematics. Utilization of mathematical problem-solving in conjunction with other forms of analysis.

Within this framework, mathematical communication is seen as having three distinct aspects. Communication *about* mathematics entails the need for individuals to describe problem solving processes and their own thoughts about these processes. Given the current emphasis upon classroom discussion, students need to externalize processes that may not have even been consciously considered when working alone in traditional classrooms. This process of externalization may in itself contribute to high order reasoning as well as facilitating classroom communication. Communication *in* mathematics means using the language and symbols of mathematical conventions. This mathematical register (Halliday, 1978; Pimm, 1987) specifically refers to the special ways that language is used when discussing mathematics. The mathematical register encompasses special vocabulary, specialized usage of everyday vocabulary, and the syntax that is particular to the expression of mathematical relationships. A number of studies have described the particular problems that second language students face when learning the mathematical register (e.g., Cuevas, 1983; Cuevas, Mann, & McClung, 1986; Spanos & Crandall, 1990). This has traditionally been seen as the real content of mathematics instruction. However, placing this kind of knowledge within a communication framework stresses the interconnectedness of mathematical concepts, in contrast to skills-based approaches which see learning as mastery of discrete pieces. Communication *with* mathematics refers to the uses of mathematics which enables students to deal with meaningful problems. All three kinds of mathematical communication are needed for developing mathematical understanding, but this study focuses mainly on communication *about* and *in* mathematics.

While social constructivist theories of learning demonstrate the social interactionist basis of learning, anthropological studies of schooling strongly suggest that forms of communication in classrooms need to be changed to enable all children to participate effectively. Studies by Heath (1983), Jordan (1985), Au and Mason (1981), and Philips (1972) demonstrate the ways in which traditional styles of classroom organization have systematically blocked children from some cultures from participating in classroom interactions. Many children are uncomfortable in large group recitation formats because they are spotlighted or feel they have lost control over conversational rights that are important to them. This research base suggests that the increased communication demands of new mathematics programs will require corresponding changes in the organization of classrooms to promote the inclusion of children from diverse cultural backgrounds.

The instructional arrangements will be described in this study in terms of participant structures. Philips (1972) introduced the idea of a participant structure, which she defined as the way in which interactions are organized. Participant structures vary along dimensions of how many students participate, who has the right to set the topic, who has the right to determine the speaker, who the audience is, and so on (Au & Mason, 1981). When the participant structures from home and from school differ substantially, students become reluctant to participate in class discussions. Thus, it is important to describe the impact of new participant structures, such as peer collaboration, on students from diverse cultural groups when introducing an educational innovation such as the College Preparatory Math project. As the prior research on participant structures has shown, it is necessary to examine not just whether small group formats are used, but how they shape expectations for the nature of student and teacher participation in discussion.

Research Design

Samples

The two classes in this study were chosen after an extended process that included nominations from knowledgeable school and university personnel and some preliminary observations to ensure that group work was being used and that the classes were well managed.

The two teachers in this study were both first-year teachers. Although this probably meant that they did not have some of the teaching skills of more experienced teachers, it was quite typical in this school district for the newest teachers to participate in the innovative mathematics programs and to be assigned the classes with the highest proportions of minority students. In addition, more recent graduates of teacher education programs are more likely to have been exposed to specialized teaching skills such as sheltered instruction than teachers who graduated in the past.

The school at which these teachers taught was located in a small urban school district in southern California which encompassed both very poor and quite wealthy neighborhoods. The school had about 2,000 students at the time of this study, of whom about 50% were minorities, primarily Latinos. The mathematics program at this school was quite clearly tracked. Both classes were considered to be college prep classes and served the middle range of students.

Information about the students in each class was collected through short surveys administered to all students. The students in Miss G's class were all fluent in Spanish and all but one of the students were of Mexican origin. Most of Miss G's students had begun their education outside of the United States and were fairly recent immigrants to the United States. Slightly more than half felt that Spanish was still their best language for studying mathematics. However, many students felt equally confident in their ability to learn mathematics in English. In contrast, although Miss Y's class was half Latino, approximately three-fourths of the students claimed to know only English. However, some students in Miss Y's class were indistinguishable from students in Miss G's class. They had begun school outside of the United States and felt comfortable studying mathematics in Spanish as well as in English. The students in Miss G's class were about one year older on the average than Miss Y's students. Both classes included students in grades nine through twelve.

As described more thoroughly in the "results" section, a mixture of Spanish and English was used in each class. Virtually all large-group discussion in both classes was done in English, primarily because neither teacher spoke Spanish. Spanish was used in varying amounts during small group discussions, depending on the context. Although the author and her assistants were usually nonparticipant observers, the author was occasionally asked questions about mathematics in Spanish in Miss G's class. She limited her interactions with the students to clarifications about the directions for tasks in order to minimize her impact upon the students' discussions. The students in Miss Y's class never addressed any questions to the observers although they occasionally inquired about the purpose of the videotaping, as did the students in Miss G's class.

Data Collection

The study was conducted during the final six weeks of the academic year. This time frame assured that the students and teachers were very familiar with the CPM program and that they were well settled into a familiar classroom routine. In addition, in accord with the teacher materials provided with CPM, the students were allowed to choose their own working groups for the last unit and this was considered conducive to more peer interaction. Each class was observed for several weeks before more active data collection through videotaping was begun. During the observation period, the students and teachers became accustomed to the researcher's presence.

Active data collection was begun at the beginning of the last unit in the program and continued until the end of the unit. Data were collected by videotaping and field notes. Each day two groups of students were videotaped, so there are data on four groups of students for each lesson. Only groups in which every member had returned a signed permission form were included.

Twenty hours of videotape were collected in each class. In addition, copies of the lessons and any other handouts such as quizzes were collected. Regular short discussions were held with the teachers to better understand their reasons for instructional choices on any given day.

Data Analysis

Videotapes were transcribed *verbatim* by graduate student research assistants. A draft transcript was prepared by one assistant and then carefully checked and edited by a second assistant or the principal investigator. Miss G's class tapes were transcribed by a native speaker of Spanish in the original Spanish.

Each transcription was chunked into mathematical incidents. These incidents varied in terms of length and content, but typical examples include students comparing answers to a problem, the teacher going over a homework problem, students asking an aide or the teacher for help, and the teacher giving an explanation to the whole class about how to solve a problem. Each of these incidents was then examined for the relevant participant structure, the kind of mathematical communication, and what language was used in the interaction (Spanish, English). In the presentation of results, an overall image of how each class functioned is presented as well as examples from specific incidents that exemplify the general patterns in the data.

Results

The Patterns of Large Group Instruction

Within CPM, large group instruction is minimized in favor of small group work. However, there is a role for large group meetings and these are used at the teacher's discretion. New ideas, concepts, and skills can be introduced during large group lessons. The teacher materials also suggest having the class discussions in which groups report back to the class as a whole *after* they have worked on a series of problems. Large group instruction may be particularly important for a class whose students are primarily speakers of English as a second language, because it gives them access to the mathematical register in a way that is probably unavailable with peers who are also developing their second language skills.

Miss G used large group instruction more than suggested in the CPM teacher materials and far more than Miss Y. She stated that this was in direct response to the needs of her limited English proficiency students. She felt the students in the sheltered instruction class needed more direct instruction or "they just didn't get it." Thus, every class began with large group review, warm-up, or explanation. Some days there was no small group work at all. Miss G had two other sections of CPM algebra and expressed satisfaction with the way in which those classes had progressed. However, with her sheltered instruction class she felt that things had not gone as smoothly and that the students needed more time to adequately cover the material. In addition, during this period she taught in a classroom organized into rows, a configuration not conducive to group work without rearranging desks. Combined with the perceived need for more teacher explanation, Miss G readily acknowledged that she had allowed much less time for student group work.

Miss G's large group instruction showed some sensitivity to the needs of her students and incorporated certain principles of sheltered instruction (Snow, 1990) that went beyond any activities suggested in the teacher materials supplied with CPM. She often brought visual and representational aids to introduce the topic of the day. For instance, in one lesson about the area that a goat can graze given the length of its leash and the proximity of the wall of an adjoining building, Miss G brought a physical representation of this situation using a string and a box. She then demonstrated to the students how the constraints of the string and box (as models for the leash and wall) allowed the goat to graze in a partial circle. Miss G also used game-like formats in her warm-up activities that evoked relatively high student involvement.

Despite her efforts to make large group instruction more interesting and accessible to her students, Miss G's large group instruction did not succeed in stimulating two-way communication with the students. Apart from game-like activities, students were reluctant to speak in the large group format. Students seldom asked questions and were reluctant to answer the teacher's questions. Those questions that got responses were those that required simple one-word answers. A typical example is given in Excerpt 1. In this excerpt the teacher was going over a quiz which had been handed back to the students. Thus the students knew when they had a correct answer. Nonetheless, with only one exception, the students responded only with answers to a computation or identification of a number—they did not talk about procedural aspects of the problem. Many of Miss G's questions required only one-word answers, but she also asked several questions that called for more elaborate answers (in bold face); she received few student responses.

Excerpt 1 (Tape 1 5/31/94)

Miss G: Let's look at number three. We are trying to find the perimeter of a trapezoid. And we know this is four, this is nine, and this is seven. What's this number right here, Maria?

M: Two.

Miss G: Good and **how do you know that, Maria?** (Maria does not respond.) So you took nine, minus, seven. And then what's the height of this little right triangle?

Student: Four.

Miss G: Four. OK, so for the perimeter we are going to take the seven, plus the four, plus the nine. And then we have to add this. But we don't know what that is. How would you figure it out?

A: Pythagorean Theorem

Miss G: Pythagorean Theorem, right. So we've got 4 square, plus 2 square, equals x square. That gives you 16 plus 4. Now, some of you got this far but you didn't remember how to simplify it. When you square root both sides, what's the biggest perfect square that divides evenly into twenty?

A: Four.

Miss G: Remember perfect squares? One, four . . .

A: Four

Miss G: Good. What are some more? Nine, sixteen. So four is the biggest one that divides into twenty. This is the same thing as the square root of four times five. What is the square root of four? What is the square root of four?

Student: Two.

Miss G: So the two can come outside of the square root sign but the five has to stay inside. So this is going to be added into our perimeter. Now the answer I gave you, I added these up, twenty and then I put plus 2 square root of five. Now, if you were to round about to the whole number you could have gotten two point four. So if you got twenty-four . . . I also gave you credit. OK, but the real answer was the exact answer of letter D. Number two, for the area you just do four times seven is twenty-eight, right? **And how do you find the area of a triangle?** What's the formula?. OK, so what's our base? And what's our height?

In contrast to Miss G, Miss Y was content with the functioning of all of her CPM algebra classes. She felt that the observed class was very similar to her other two classes in terms of management and about mid-way between her other two classes in terms of achievement. Although Miss Y also used large group time to introduce new concepts to the students and to review homework, her students spent more time in the small group format. Miss Y felt that the largest benefit from the small group arrangement was that the students were able to experience more social interactions with their peers, something that was necessary for students of this age. Excerpt 2 shows how Miss Y conducted a large group discussion that was similar in content to Excerpt 1 from Miss G's class. In this case the structure of the interaction was very similar, but the students were expected to give procedural answers and they did so. In addition, the students always responded to teacher questions, even if with an answer such as J's in which he simply said he did not know the answer. Miss Y also used the technique of re-phrasing a question so that students could respond with the expected kind of answer, as shown in the bold faced example. Although Miss Y's class did not deal with mathematical problems in a very open-ended manner during large group instruction, the students practiced with a larger number of aspects of the mathematical register. Unlike Miss G's class, the Spanish-speaking students were equally likely as the native speakers of English to participate in the whole class discussions.

Excerpt 2 (Tape 3 5/31/94)

Miss Y: -2 1/3 or -2.33, if you use your calculator. So now if I want to know what my slope is, what's the easiest thing to do first? If I'm looking for my slope, what form should I put my equation in?

N: y form.

Miss Y: Let's put it in y form, so y=mx + b. So we want to solve for y. This is my equation, 2x - 3y =7. S., what is my first step?

S: Subtract 2x.

Miss Y: Subtract 2x from both sides, good, I end up with -3y = -2x + 7. And J., my last step?

J: I don't know.

Miss Y: Divide by -3, and each term gets divided by -3. So I end up with y= -2/-2, is the same as . . .

N: 2/3.

[A few minutes later in the same lesson ...]

Excerpt 2 continued:

Miss Y: **Remember, what do I do with these parentheses? It's a property, what's the property called? It starts with a 'd.' It's a property we learned, what's one of the properties we learned way back, last semester?**

S: Oh, distributive.

Miss Y: Thank you, S. OK, so what does this distributive property mean? What am I gonna do?

S.: Get rid of the parentheses.

Although both teachers were using an Initiation-Response-Evaluation sequence to run the large group discussions, they seemed to have different expectations about the student participation. Like Miss Y, Miss G asked questions that could be answered with descriptions of procedures, but she didn't really seem to expect to receive such answers. Unlike Miss Y, she did not rephrase questions until she received an answer. In addition, she allowed very little time for student responses and often answered her own questions.

The Patterns of Small Group Interactions

Despite a constant flood of chatter, almost exclusively in Spanish, very few kinds of mathematical communication actually took place in the small groups in Miss G's class. Most frequently, students asked for an answer from another student. Infrequently the students actually described a procedure used to solve a problem. Often these procedural statements were just a phrase indicating perhaps a key step in a solution. Although this type of discussion could be considered communication *about* mathematics, as given in the *Mathematics Communication Framework*, it had little of the richness of speech that is called for in documents such as the *California Mathematics Framework* or the national standards. There was no metacognitive content and students were never observed to make conjectures, to generate generalizations, or to compare alternative solutions. Even more striking was the overall scarcity of student talk about mathematics. On the videotapes there were examples of groups of students working steadily on problems and conversing the entire time, but in fact, none of the discussion was about mathematics. Within this classroom context, the small groups were clearly not a community of mathematical practice.

Miss Y's class also hummed with a constant flow of student talk in a combination of Spanish and English. And although Miss Y's students also talked about rock and roll, their friends, and the general state of the world, talk about mathematics was interwoven with the discussion of other topics. Excerpt 3 was taken from the first day of the new unit and the four girls were just beginning to work on the first problem, EP1. Two of the girls were native English speakers and two were native Spanish speakers. Within this excerpt the girls were communicating in a variety of ways about mathematics. In the first pair of boldfaced quotes, S1 and S2 offered alternative ways of approaching the problem. In the following line S1 began to describe a procedure for finding an answer and S2 continued the description of the procedure being followed. In the next boldfaced quote, S4 evaluated the procedure that had just been described by S1 and S2 by pointing out that it resulted in an impossible answer. Two lines later S2 suggested another way to approach the problem. As classified in the Communication Framework for Mathematics, these girls were engaged in communication *about* mathematics. They were also fluently using the mathematical register to describe their procedures. As will be shown later in Excerpt 6, the students in Miss G's class only reluctantly cooperated and had great trouble explaining procedures to each other.

Excerpt 3 (Tape 2 5/23/94)

S2: I have no clue how to do it.

S1: You could do -x, so that would be 4x, for the first one I guess.

S2: Or divide it by x.

S1: and then cuz like you have a number and you want to get it. So you go factor it, so you minus that number, and then –

S2: it's divided by x, and then we get 3 equals x over x

S4: **and then x over x equals 1, but 3 won't equal 1.**

S1: I don't know.

S2: So maybe what we do is subtract it? So it's 2x equals zero?

S1: square it, so the x equals 0?

Participant Structures

The differences in how the student groups discussed mathematics in Miss G's and Miss Y's classes arose in part from the different participant structures that the teachers created. Miss G did not really turn over the responsibility for mathematical learning to the small groups. Within the CPM program, it is explicitly expected that the groups are the site for the generation of most mathematical knowledge within the class. According to the teacher's materials, the teacher is expected occasionally to only provide direct explanations about how to do the problems. More often the students are given a series of problems that lead them to explore key concepts. For instance, in the unit observed in this study, the teacher materials explicitly tell the teachers *not* to explain the meaning of '$\pm$' when it is used in a quadratic equation (Holm, Kasimatis, & Petersen, n.d.). The students in their groups are expected to explore what this symbol can mean and reach a group consensus. The group rules in CPM further reinforce the necessity for the students to work together to figure out most of the problems without outside help. One of the group rules is that all four students in the group must have the same question for the teacher before they can ask for help. However, in Miss G's class, the groups were not really expected to have any of these functions, as shown by the directions Miss G gave in Excerpt 4 as the students started to do their own work after a demonstration by the teacher.

The phrases in bold face demonstrate what Miss G expected as the students did their work. She stated two times that the goal was to get the exercises right. There was no mention that students should try different ways to get their answers or to understand the process. In addition, she explicitly mentioned two sources of help outside of the group (the answer key and the three adults in the classroom) that could be consulted but made no reference to consulting the other students in the group. It seems that Miss G did not really expect that the groups would be able to determine the answers to the problems so she had provided an answer key that was not part of the regular course materials. Through these arrangements Miss G had taken away the authority of the group to develop mathematical knowledge and delegated it to the traditional authorities—teachers and printed materials.

Excerpt 4 (Tape 8 6/9/94)

Miss G: OK, this is actually the example. Go ahead and do the book examples. They have example two and example three. Do those before you do 9 A through D, to **make sure you get the right answers**, all right? You have almost the whole period to work on this and **make sure you get them right,** check your answers on the **answer key** and also . . . M. is here and Miss S.[the two aides] and myself, so if you don't get the right answer, raise your hand and **we will help you**.

The student interactions recorded on the videotape mirror the directions given by Miss G as shown in Excerpt 5. This excerpt directly follows Excerpt 4 and focuses on two male students, Ju and H, who were sitting next to each other.

Excerpt 5 (Tape 8 6/9/94)

[After Miss G finishes her directions in Excerpt 4, the boys spend the next eight minutes eating candy and teasing each other. They do not move their desks together. They finally begin to work on the assignment and the next sentences are their first mathematical communication].

Ju: Ya cuatro . . . ¿Qué te salió en la A? H ¿qué te entró en la A?
(Already four. What did you get for A? H, what did you put for A?)

[H keeps working without answering.]

Ju: ¿Nada? ¿Qué te salió en la A?
(Nothing? What did you get for A?)

[Ju starts working. Within a few seconds he calls out in a loud voice to attract the teacher's attention.] Miss G!

[She does not hear him so he then continues working. About 40 seconds later Miss G walks toward them.]

H: I don't, I don't–I can't get the right answer for D.

Miss G: For D? OK, show me how you put it in. Probably you are forgetting the parentheses [she watches him punch in the equation] and then do the invisible parenthesis. **So do 3 divided by** [she pauses as he starts to enter numbers] **open 2 parenthesis minus 5 square root and close your parenthesis, right?**

[Ju. has intently watched this interaction and keeps looking at his classmates as they work. Then he borrows a calculator and starts working again.]

In this excerpt the only mathematical interaction between the boys was a request to find out a correct answer by Ju. H apparently felt no obligation to answer and ignored the request. Ju then tried to get the teacher's attention. When he didn't get her attention he proceeded to do the problem on his own. Shortly thereafter when the teacher came close, it was the other boy, H, who took the opportunity to ask a question. As promised in her directions the teacher provided explicit directions for finding a correct answer, without asking for any verbal explanation from H. A few minutes later Ju started the same question. Instead of asking H for help, he also asked the teacher for help as shown in Excerpt 6.

Excerpt 6 (Tape 8 6/9/94)

Ju: Miss G, I can't get the right answer.

Miss G: For D? You know what? I just showed H how to do it. [To H.] Can you show him how to do it with the parentheses?

[H nods in a funny way, mocking her]

Ju: En la pelicula van a decir, "este niño mal educado".
(In the movie they're going to say, "this ill-mannered boy.")

H: ¿Cuál es la D? Fíjate.
(Which is D? Look.)

[Ju reaches for another candy and looks at J. behind him instead of at H.)

H: !Fíjate, que te fijes! tres, fíjate, que te fuiste. ¿Tres dividido, qué?
(Look, look! Three, look, where did you go. Three divided, what?)

[H. stands by Ju's desk and pushes the keys on the calculator as they speak.]

Ju: close parenthesis.

H: **Ves que aquí tiene como dice ella unos invisibles? Open el**
(Do you see that here are what she calls the invisible ones? Open the
paréntesis, dos minus five close paréntesis, 12.7 negativo
parenthesis, two minus five close parenthesis, negative 12.7)

[H returns to his seat. They start hitting each other in a teasing way.]

In this excerpt Miss G did ask one student to get help from another, but she simply expected the designated helper to convey what she herself had already explained. In all cases observed in this class' transcripts, students were referred to their peers when the teacher already knew that the potential helper had the right answer or had had the procedure explained by a teacher. Students were not expected to negotiate different solutions or to work together to determine an answer. In this particular excerpt, the boys expressed some discomfort with the helper/helpee roles assigned by the teacher although they did carry out the task. H acted rude at first, and Ju ignored his efforts to help. At the end of the explanation, the boys started fooling around in a manner very similar to how they began the seatwork part of the lesson.

As shown in Excerpt 3, the students in Miss Y's class automatically worked together with little prompting from the teacher. When asked for help Miss Y was much more likely to direct questions back to the group unless she saw evidence that the group had exhausted its resources. And when she did decide to respond to student requests for help, Miss Y was more likely to give a series of clues to redirect the students' efforts in the correct direction. In contrast, Miss G often explained exactly how to solve a problem, as she did in Excerpt 5.

In addition to exercises at the beginning of the year, CPM incorporated one type of lesson throughout the year with a participant structure designed to facilitate small group interaction: group quizzes. Although students have individual worksheets in a group quiz, interdependence is created among the students because only one student's paper is graded for each group. The grade on this paper applies to all members of the group. However, during the group quiz observed in Miss G's class, the students did not do more than the usual amount or type of mathematical communication with each other. Rather, they called upon the teacher, the aides, and even the observers for more help than usual. Once again, the interdependence of students in small groups was thwarted by dependence upon external resources.

In one circumstance, the students in Miss G's class engaged in extensive mathematical communication. This was when they went to the computer lab. In the lab setting students were paired up to work on the limited number of computers. In this case there was real interdependence because there was no way that both students could work simultaneously and independently. In addition, the students had to rely more upon their own resources rather than external help. With the students working in pairs rather than groups of four, the ratio of students to adults was greatly increased and it was difficult to attract a teacher's or aide's attention. The computer itself seemed to be engaging for the students. Unlike most class days, the students talked about mathematics for the entire class period with little irrelevant talk interspersed.

The Mathematical Register

Difficulties with the mathematical register was a second factor that seemed to cause a paucity of mathematical communication in Miss G's class. The language input received by these students in both English and Spanish seemed insufficient to adequately develop students' competence in mathematical vocabulary and syntax. The lack of practice using mathematical language in both large and small group settings and small group settings as described above may have further exacerbated the situation for the students.

Although Miss G used sheltered instruction, her procedural explanations were explicit and sound. As Excerpt 4 shows, she used standard mathematical vocabulary and syntax in a way that resembled what any mathematics teacher might use. Given the emphasis upon large group instruction, it was clear that the students heard a lot of mathematical language in English. What may be less apparent is what was missing in any lesson. During this lesson, Miss G referred to invisible parentheses a number of times as in the fifth line of Excerpt 4. She was talking about a larger issue of order of operations, but did not refer to this by name. Thus, the rationale for the invisible parentheses was never mentioned during the observational period and the nonstandard term "invisible parentheses" was used as a procedural description without a conceptual linkage. This contrasts with how Miss Y reminded her students of the distributive property in the bold faced portion of Excerpt 2. Although Miss G's students heard her talk in detail about the sequence of steps needed to solve problems, they heard less connection of general principles to procedures than Miss Y's students heard. It is these connections that may facilitate students' problem solving when they confront new problems.

Although Miss G's students regularly heard mathematics explained in English, they had almost no exposure to the mathematical register in Spanish. Miss G spoke no Spanish and the aides were expected to provide this dimension for the students. Unfortunately, the two aides who were observed during the course of this study did not demonstrate much knowledge of algebra and they used little of the mathematical register in Spanish. One of the aides did not even try to deal with mathematical issues. She simply translated task instructions from English to Spanish for the students and otherwise encouraged them to keep trying and to consult with their peers. The other aide, Miss S, made more of an effort to work with the students on the mathematics as shown in Excerpt 7.

Excerpt 7 (Tape 7 6/9/94)

Miss S: Bueno, primero haz esta parte . . . **negative tres square**, haz esa parte

(Good, first do this part negative 3 square, do that part)

M: Pongo paréntesis, ¿verdad?

(I put parenthesis, right?)

Miss S: No, haz esta parte y lo que te salga usa.

(No, do this part and what you get, use it.)

¿Ya lo hiciste?

(Did you do it?)

M: **Porque tiene el paréntesis esta**, no.

(Because this one has the parenthesis, no.)

Miss S: Oh, sí, esa sí

(Oh, yes, that's right.)

M: cuatro . . .

(four)

Miss S: así . . . **square**, esa no es, es esta

(so . . . square, it's not that one, it's this one.)

M: Oh

Miss S: **square, más menos** . . . ¿Por qué te sale la E? **Lo haces aquí, square, hmm**

(square, more less . . . Why did you get an E? You do it here, square.)

In this excerpt the aide was working with two girls on the same series of problems that were being discussed in Excerpts 4 to 6. Her attempts to use the mathematical register are shown in bold face. As is characteristic of her speech throughout this unit, Miss S mixed English and Spanish whenever she needed to use mathematical terms. The Spanish and English were co-mingled within phrases, which suggests that Miss S needed to use English because she did not know a term in Spanish. While this kind of code switching or lexical borrowing may in fact be quite meaningful to the students who have already heard the explanation in English, the aide made numerous mistakes when using mathematical terminology in both languages. In the first bold faced phrase in this section the aide should be saying “squared” but simply says “square.” In the last bold faced phrase in this excerpt, the aide once again uses the word “square” in English but this time she should be saying “square root.” As the final turn in this excerpt shows, doing the wrong operation at this point in the problem caused the calculator to give an E (i.e., error) message and the aide went to consult the teacher. Although we cannot tell from this excerpt whether the aide did not know the mathematical language or was mixed up about procedures, her imprecise use of the word “square” coupled with a mistake in the process reduced the comprehensibility of her mathematical language.

As this lesson continued, Miss S returned to help M and MC after consulting with the teacher. She was again stymied after a few more steps into the problem and went again to consult with the teacher. She returned to work with the students for a third round on this problem and together they were able to solve it successfully. In some ways, Miss S provided needed help to the students. She was very conscientious about getting help from the teacher when she was unable to carry through on a solution with the students. She essentially acted as a translator for explanations that the teacher might give in English. However, she was not a very good role model of how to do mathematical communication in Spanish.

The students’ efforts to do mathematical explanations were very similar to Miss S’s. As shown in the bold faced phrase in Excerpt 6, H also did codeswitching or lexical borrowing between English and Spanish, at times practically alternating every other word between languages. In this case the directions given by H were accurate up until the word five. At this point he should have said “square root” as did Miss G when describing the same procedure in Excerpt 5 (bold faced section). However he totally skipped saying anything at all for this step in the procedure and concluded the procedural directions by telling Ju to enter his second parenthesis. Apparently the procedure was done correctly despite H’s lack of verbalization about the square root because the correct answer was achieved.

The students in this class spoke almost exclusively in Spanish. The only times they used English were when they talked to the teacher or when they interjected mathematical terms in English into their discussions with their peers. H was actually quite competent in English—he was observed to talk fluently with the teacher and to ask questions of her quite often in English. On the survey he also indicated that English was his best language for mathematics class. However, his use of the mathematical register in both English and Spanish seemed very tentative and he often avoided using mathematical terms altogether as shown in the example above.

Discussion and Conclusions

Educational equity dictates that students who are culturally and linguistically different should be included in schools' efforts to change their mathematics curriculum and that appropriate instructional methods be used to meet the range of these students' needs. This paper describes the efforts of two teachers to teach a new mathematics program to classes with linguistic minority students. Ironically, the teacher whose class was designated to meet the needs of linguistic minority students was unsuccessful at stimulating mathematical discussion in either Spanish or English. The other teacher created a classroom climate that led to more mathematical discussion for both her native speakers of English and her language minority students.

Active student participation in classroom discussions, whether in a large group or small group format, serves many functions. The community of practice perspective adopted in this paper has emphasized the benefits of discussion for the students, particularly as a potentially powerful way for students to achieve the new goals outlined in the math reform movement. For the teacher, classroom discussion is also useful because it provides information about how well the students understand the content of the day's lesson (Pimm, 1987; Secada & De la Cruz,1996). In Miss G's class, she was missing this important information because her students avoided participation in the large group discussions and Miss G could not understand the discussions in the small groups. Thus, Miss G's decision early in the school year to emphasize direct instruction and large group recitations continued to seem valid throughout the school year based upon the information Miss G had available to her about the students' comprehension of each day's lesson. The students' persistent requests for help and reluctance to rely on their peers in doing their seat work created a clear impression that they could not figure out how to do the work on their own. This finding is in accord with other research that has found that teachers who have a transmissionist philosophy, i.e., the belief that students need to be told academic content in lieu of constructing it, are unlikely to use small groups in their classes (Cohen & Tellez, 1994).

However, for language minority students in particular, the opportunity to discuss mathematics in a small group may precede competent participation in large group discussion. Studies comparing students' communication in their two languages, in large group discussion and in small groups, have found that language minority students display the lowest level of competency when talking in English during large group discussions, frequently leading to underestimation of children's academic competency (Secada & De La Cruz, 1996; Thornburg & Karp, 1992). In both of these studies, the researchers observed competent mathematical discussions occurring in small groups that were not witnessed by school personnel. Secada and De La Cruz (1996) described the example of a girl who made mistakes when explaining a mathematical concept to her teacher in English immediately after having explained it correctly to peers in Spanish. In the Thornburg and Karp study, the students acquired competency in mathematical communication in their small group setting initially in Spanish and subsequently in English over the course of the year, but did not always display this competency in their direct interactions with the teachers. The small groups in Miss Y's class also used more sophisticated communication than was demonstrated in larger group discussions. It is not surprising, given their lesser experience speaking in small groups, that Miss G's students resisted large group discussions in English.

Although the observations in Miss G's class present a dismal picture of how the small groups functioned, there are several illuminating points that emerge from this study. The observations in Miss Y's classroom demonstrated that emergent speakers of English can learn to become active participants in classroom discussions about mathematics by the end of one academic year. Although with the data collected in this study it was not possible to determine if the Spanish speaking students in Miss Y's class differed systematically from those in Miss G's class (e.g., prior mathematical preparation, personality factors), they may have had two advantages in Miss Y's class. From the beginning of the year they had had the opportunity to work in small groups with their peers. As the authors of CPM stress (Kasimatis & Sallee, in press; Kysh, 1991), it is a challenge for all students to learn to work effectively in small groups. Miss G's students may have lacked time to develop this expertise because of their more limited amount of time in small groups, as well as the reduced need to talk within the groups. Working with English-speaking peers may also have helped to develop the English language skills and the confidence of the Spanish-speaking students in Miss Y's class. Although some discussions were observed in Spanish in Miss Y's class, it seemed that the presence of even one English-speaking student in a group tended to shift language usage towards English. In contrast, although at least some of the students in Miss G's class had good English fluency, easily matching that of students in Miss Y's class, they were never observed to use it within their small groups. Thus, they never got a chance to practice their mathematical English within the safer environment of the small group.

A second encouraging trend was discerned in these data. The students in Miss G's class demonstrated their willingness and ability to discuss mathematics when they worked in the computer lab as did the students in Miss Y's class. Other authors have also noted that working on the computer tends to increase the quantity and quality of student discussion (Hoyles, Sutherland, & Healy, 1991). It has already been suggested in this paper that the altered participant structure in the computer lab made inter-student communication more necessary since there were fewer sources of help from outside the student group. More speculatively, it is possible that the computer also provides support for students who are just beginning to communicate *about* mathematics, as described in the *Communication Framework for Mathematics* (Brenner, 1994). One challenge for students who are learning to talk about mathematics is that they must learn to externalize cognitive processes which they previously accomplished as solitary activities while doing individual seatwork. The computer makes more of this visible as students monitor each others' keystrokes and the subsequent outcomes on the computer screen. The resultant discussion may also seem like less of a discussion about an individual's private thoughts, since the computer appears to do some of the 'cognitive' work. Dixon (1995**)** has shown that strategic use of the computer in mathematics classes can also enhance the achievement of LEP students in classes with English-speaking peers.

Many of the questions that arise from this research project call for a longer term view of the development of mathematical communication. The data reported here focused on instructional arrangements that were the culmination of a year of interaction between the teachers and the students. Although this study focused on Miss G's instructional decisions, the students also played a role in determining what kind of instruction became the negotiated and persistent pattern. Even during these lessons at the end of the school year, it was apparent that the students *chose* not to participate in certain kinds of large group interactions with the teacher. Miss G may have tried to stimulate more varied mathematical communication earlier in the year, but gave up the effort in the face of student resistance. A longitudinal study from early in the year would give more insight as to why particular accommodations are reached by the end of the school year. In addition, further study is needed to determine what the optimal mixture of first and second language speakers in classes when students are developing their second language skills while studying complex subject material. Although the Spanish-speaking students in the classroom with a majority of English speakers showed more active communication in this study, the limitations of the study (small sample size, other differences between the classes) does not warrant strong conclusions for policy.

Although this study was modest in scope, it is one of the few studies that has actually examined what is happening in mathematics reform classrooms with language minority children. Such work should be continued so that we can better meet the educational needs of the many children with limited English proficiency who now live in the United States.

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