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# 非球形粒子散射光的去偏振特性研究

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**摘 要**:目标对入射偏振光的散射特性反映了目标的属性信息。在 Rayleigh 散射理论的基础上, 通过单层非球形粒子对入射偏振光的散射数学模型,应用矢量传输方程来计算非球形粒子散射的 Mueller 矩阵元,求解散射介质的偏振度,讨论在不同介质层厚度、粒子半径和探测角的条件下, 非球形粒子散射光去偏振度的变化特性,给出了模拟仿真结果。该方法为研究目标的内部结构、厚 度和粗糙度等特征以及目标的探测和识别提供了一种新的途径。

 关键词:非球形Rayleigh 粒子;Mueller 矩阵;散射光;偏振度

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# Depolarization characteristics of scattered light emitted from aspherical particles

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Abstract: The scattering characteristics of the polarized light incident on the target represent the attributive information of the targets. Based on the Rayleigh scattering theory, the Mueller matrix elements of the aspherical particles were calculated by using the vector transmission equation and scattering mathematic model of the incident polarized light scattered by the singlelayer aspherical particles, and the polarization degree of the scattering medium was solved. Under the conditions of different thickness, particle radii and detecting angles, the variation characteristics of the depolarization degree of the light scattered by the aspherical particles are discussed, and the simulation result is given. A new way is provided by this method for characterizing the internal structure, thickness, roughness of a target, and for the object acquisition and identification.

Key words: Rayleigh aspherical particle; Mueller matrix; scattered light; polarization degree

#### 引言

激光作为主动成像的光源具有良好的偏振性。 由于其介质内部结构(即粒子的大小、形状、排列以 及材料等差异)和目标表面特征的差异,散射光表 现出不同的偏振特性,因此偏振度可以作为目标探 测和识别的重要特征。文献[1]以Mie 理论为基础 通过试验研究了微小粒子以及球形粒子的散射偏 振特性。非球形粒子的散射特性反映了众多的自然 目标和人造目标的重要特性,在军事遥感的许多领 域有重要的应用价值<sup>[2]</sup>。由于目标的偏振特性理论 研究比较复杂,目前散射光偏振度特性的研究仅有 一些实验上的探讨<sup>[2-3]</sup>,因此其理论研究还有待进

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一步深入。

本文通过应用矢量传输方程求解目标散射光 Mueller 矩阵的数值解,从理论上求得散射光的偏 振度,分析散射介质的参量与散射光偏振度之间的 关系。模拟结果说明散射光的偏振度与散射介质的 参量之间关系密切,这为散射光偏振特性的应用建 立了理论基础和数据参考。文章对散射光的偏振度 作为目标探测与识别的新方法提供了理论依据。

#### 1 非球形 Rayleigh 散射粒子模型

当粒子半径远小于光波波长,即r>0.1λ时, 可以用Rayleigh 粒子散射模型进行近似。对于非球 形粒子模型,必须考虑粒子本身几何坐标轴和散射 场坐标系之间的关系,通过Euler 角<sup>[4]</sup>实现二者之 间的转换。在远场近似条件下求得的小椭球的散射 场<sup>[4]</sup>为

$$\overline{E}^{\text{sca}}(\overline{r}) = \frac{\mathrm{e}^{ikr}k^2 v_0}{r} (\boldsymbol{\epsilon}_{sn} - 1) (\hat{v}_s \hat{v}_s + \hat{h}_s \hat{h}_s) \cdot \overline{A}_b g \hat{\mathbf{e}}_i E_0$$
(1)

式中极化张量

$$\bar{\bar{A}}_b = a_x \hat{x}_b \hat{x}_b + a_y \hat{y}_b \hat{y}_b + a_z \hat{z}_b \hat{z}_b$$
<sup>(2)</sup>

式中: $\hat{v}_s$ 和 $\hat{h}_s$ 分别为水平和垂直极化矢量;k为 传播波矢; $\hat{e}_i E_0$ 为外场作用。(1)式可进一步写为 散射振幅函数形式

$$\begin{bmatrix} E_v \\ E_h \end{bmatrix} = \frac{e^{ikr}}{r} \begin{bmatrix} f_{vv} & f_{vh} \\ f_{hv} & f_{hh} \end{bmatrix} \begin{bmatrix} E_{vi} \\ E_{hi} \end{bmatrix}$$
(3)

式中 $f_{vv}$ ,  $f_{vh}$ ,  $f_{hh}$ 和 $f_{hv}$ 为散射振幅函数, 其表达式可 以通过 T-矩阵求得<sup>[4]</sup>.

$$t_{0} = \frac{2}{9} \cdot \frac{k^{3}ar^{2}cr(\epsilon/\epsilon_{0}-1)}{(1+v_{d}A_{c})}$$

$$t_{1} = \frac{2}{9} \cdot \frac{k^{3}ar^{2}cr(\epsilon/\epsilon_{0}-1)}{(1+v_{d}A_{c})}$$
(4)

$$T_{0} = it_{0} - t_{0}^{2}$$

$$T_{1} = it_{1} - t_{0}^{2}$$
(5)

对于长椭球粒子,有

$$br = cr < ar$$

$$A_{c} = \frac{2}{(ar^{2} - cr^{2})^{2/3}} \left[ \frac{\sqrt{ar^{2} - cr^{2}}}{cr} - \frac{\sqrt{ar^{2} - cr^{2}}}{cr} \right]$$

$$\arctan \frac{\sqrt{ar^{2} - cr^{2}}}{cr} \left[ \frac{\sqrt{ar^{2} - cr^{2}}}{cr} \right]$$
(6)

对于扁椭球粒子,有 br=cr>ar

$$A_{c} = -\frac{1}{cr^{3}e^{3}} [2e + \ln\frac{1-e}{1+e}]$$
(7)

式中: 
$$k$$
 为波数;  $e = \sqrt{1 - a^2/c^2}$ 为椭圆率;  $v_d = a^2 c(\frac{\varepsilon}{\varepsilon_0} - 1)/2$ ;  $A_a = \frac{1}{a^2 c} - A_c/2$ 。因此  
 $f_{vv}(\theta, \varphi; \theta', \varphi') = \frac{3}{2k} [T_1(\hat{\theta} \cdot \hat{\theta}') + (T_0 - T_1) \cdot (\hat{\theta} \cdot \hat{Z}_b) \cdot (\hat{Z}_b \cdot \hat{\theta})]$   
 $f_{vh}(\theta, \varphi; \theta', \varphi) = \frac{3}{2k} [T_1(\hat{\theta} \cdot \hat{\varphi}) + (T_0 - T_1) \cdot (\hat{\theta} \cdot \hat{Z}_b)(\hat{Z}_b \cdot \hat{\varphi}')]$   
 $f_{hv}(\theta, \varphi; \theta', \varphi) = \frac{3}{2k} [T_1(\hat{\varphi} \cdot \hat{\theta}') + (T_0 - T_1) \cdot (\hat{\varphi} \cdot \hat{Z}_b)(\hat{Z}_b \cdot \hat{\theta}')]$   
 $f_{hh}(\theta, \varphi; \theta', \varphi') = \frac{3}{2k} [T_1(\hat{\varphi} \cdot \hat{\varphi}) + (T_0 - T_1) \cdot (\hat{\varphi} \cdot \hat{Z}_b)(\hat{Z}_b \cdot \hat{\theta}')]$ 

### 2 非球形粒子Mueller 矩阵元及偏振 度的求解

 $(\hat{\varphi} \cdot \hat{Z}_b)(\hat{Z}_b \cdot \hat{\varphi}) ] \tag{8}$ 

单层散射介质的VRT 方程<sup>[6]</sup>

$$\cos\theta_{0} \frac{d}{dz} \overline{I}_{0}(\theta_{0},\varphi_{0},z) = -k_{e}(\theta_{0}) \overline{I}_{0}(\theta_{0},\varphi_{0},z) + \\ \overline{I}_{e}(\theta_{0},\varphi_{0}) + \int d\overline{\Omega}' \overline{P}(\theta_{0},\varphi_{0};\theta,\varphi) \cdot \overline{I}(\theta,\varphi,z)$$
(9)

式中:  $(\theta_0, \varphi_0)$ 为入射角,  $\overline{I}_0(\theta_0, \varphi_0, z)$ ,  $\overline{I}(\theta, \varphi, z)$ 分别为入射光和散射光的Stokes 矢量。VRT 方程 边界条件为<sup>[6]</sup>

$$\overline{I}(\pi-\theta,\varphi,z=0) = \overline{I}_0\delta(\cos\theta-\cos\theta_0)\delta(\varphi-\varphi_0)$$

$$\overline{I}(\theta,\varphi,z=-d) = \overline{R}(\theta) \cdot \overline{I}(\pi-\theta,\varphi,z=-d)$$
(10)

由以上公式可以看出,(9)式是线性方程,散射光的 Stocks 矢量与入射光的 Stocks 矢量成正比。其耦 合矩定义为 Mueller 矩阵,则

 $I_s(\theta, \varphi) = M(\theta, \varphi; \pi - \theta_0, \varphi_0) \cdot I_0$  (11) 式中:  $I_0$  为入射光的 Stocks 矢量;  $I_s(\theta, \varphi)$ 为散射 光的 Stocks 矢量;  $M(\theta, \varphi; \pi - \theta_0, \varphi_0)$ 为目标的 Mueller 矩阵。我们主要研究散射介质的去偏振特 性,因此只取 Mueller 矩阵的主对角元素:

$$M = \begin{pmatrix} m_{00} & & & \\ & m_{11} & & \\ & & m_{22} & \\ & & & m_{33} \end{pmatrix}$$
(12)

式中: $m_{00}$ 为总散射系数; $m_{11}$ 和 $m_{22}$ 为散射光的2 个正交方向的线偏振系数; $m_{33}$ 为散射光中圆偏振 光的偏振系数。散射偏转光的偏振度定义<sup>[5]</sup>为

$$pd = \frac{|m_{11}| + |m_{22}| + |m_{33}|}{3m_{00}} \tag{13}$$

其中散射粒子的消光系数表示散射粒子的吸收系数和散射系数之和,可以由散射振幅函数求得其数值解:

$$\bar{k}_{e}(\theta,\varphi) = \frac{2\pi}{k} n_{0} \begin{bmatrix} 2\operatorname{Im}\langle f_{vv}^{0}\rangle & 0 & \operatorname{Im}\langle f_{vh}^{0}\rangle & -\operatorname{Re}\langle f_{vh}^{0}\rangle \\ 0 & 2\operatorname{Im}\langle f_{hh}^{0}\rangle & \operatorname{Im}\langle f_{hv}^{0}\rangle & \operatorname{Re}\langle f_{hv}^{0}\rangle \\ 2\operatorname{Im}\langle f_{hv}^{0}\rangle & 2\operatorname{Im}\langle f_{vh}^{0}\rangle & \operatorname{Im}\langle f_{hh}^{0} + f_{vv}^{0}\rangle & \operatorname{Re}\langle f_{vv}^{0} - f_{hh}^{0}\rangle \\ 2\operatorname{Re}\langle f_{hv}^{0}\rangle & -2\operatorname{Re}\langle f_{vh}^{0}\rangle & \operatorname{Re}\langle f_{hv}^{0} - f_{vv}^{0}\rangle & \operatorname{Im}\langle f_{hh}^{0} + f_{vv}^{0}\rangle \end{bmatrix}$$
(14)

相矩阵 $ar{P}( heta_0,arphi_0; heta,arphi)$ 表示来自各个方向 $\int d\Omega'$ 的散射场和入射场的耦合,也可由散射振幅函数求得:

$$\bar{P}(\theta,\varphi;\theta',\varphi') = n_0 \bar{L}(\theta,\varphi;\theta',\varphi') = \left\{ \begin{array}{ccc} \langle |f_{vv}|^2 \rangle & \langle |f_{vh}|^2 \rangle & \operatorname{Re}\langle f_{vv}f_{vh}^* \rangle & -\operatorname{Im}\langle f_{vv}f_{vh}^* \rangle \\ \langle |f_{hv}|^2 \rangle & \langle |f_{hh}|^2 \rangle & \operatorname{Re}\langle f_{hv}f_{hh}^* \rangle & -\operatorname{Im}\langle f_{hv}f_{hh}^* \rangle \\ 2\operatorname{Re}\langle f_{vv}f_{hv}^* \rangle & 2\operatorname{Re}\langle f_{vh}f_{hh}^* \rangle & \operatorname{Re}\langle f_{vv}f_{hh}^* + f_{vh}f_{hv}^* \rangle & -\operatorname{Im}\langle f_{vv}f_{hh}^* - f_{vh}f_{hv}^* \rangle \\ 2\operatorname{Im}\langle f, f^* \rangle & 2\operatorname{Im}\langle f, f^* \rangle & \operatorname{Im}\langle f, f^* + f, f^* \rangle & \operatorname{Re}\langle f, f^* - f, f^* \rangle \\ \end{array} \right\}$$
(15)

热辐射源 $\overline{I}_{\epsilon}(\theta, \varphi)$ 在主动遥感中可以忽略。将 VRT 方程进一步写为<sup>[6]</sup>

$$\cos\theta \frac{d}{dz} \overline{I}(\theta,\varphi,z) = -\overline{K}e(\theta,\varphi) \cdot \overline{I}(\theta,\varphi,z) + \overline{S}(\theta,\varphi,z) -$$

$$\cos\theta \frac{d}{dz} \overline{I}(\pi-\theta,\varphi,z) = -\overline{K}e(\theta,\varphi) \cdot \overline{I}(\pi-\theta,\varphi,z) + \overline{W}(\theta,\varphi,z) \qquad 0 \leqslant \theta \leqslant \pi/2, 0 \leqslant \varphi \leqslant 2\pi$$

$$(17)$$

$$(16)$$

$$(17)$$

$$\overline{I}(\pi - \theta, \varphi, z) = \overline{E}(\pi - \theta, \varphi)\overline{D}[\beta(\pi - \theta, \varphi)z\cos\theta]\overline{E}^{-1}(\pi - \theta, \varphi)\overline{I}_{0} \times \delta(\cos\theta - \cos\theta_{0})\delta(\varphi - \varphi_{0}) + \int_{z}^{0} dz' \left\{\overline{E}(\pi - \theta, \varphi)\overline{D}[\beta(\pi - \theta, \varphi)(z - z')\sec\theta]\overline{E}^{-1}(\pi - \theta, \varphi)\overline{W}(\theta, \varphi, z')\right\}$$
(18)

$$\overline{I}(\theta,\varphi,z) = \overline{E}(\theta,\varphi)\overline{D}[-\beta(\theta,\varphi)(z + d) \sec\theta]\overline{E}^{-1}(\theta,\varphi)\overline{R}(\theta)\overline{E}(\pi - \theta,\varphi)\overline{D}[-\beta(\pi - \theta,\varphi)\varphi] - \beta(\pi - \theta,\varphi) d\sec\theta]\overline{E}^{-1}(\pi - \theta,\varphi) \overline{I}^{0} \times \delta(\cos\theta - \cos\theta_{0})\delta(\varphi - \varphi_{0}) + \overline{E}(\theta,\varphi)\overline{D}[-\beta(\theta,\varphi)(z' + d)\sec\theta]\int_{-d}^{0} dz' \langle \overline{E}^{-1}(\theta,\varphi)\overline{R}(\theta)\overline{E}(\pi - \theta,\varphi)\overline{D}[-\beta(\pi - \theta,\varphi)(z' + d)\sec\theta] \cdot \overline{E}^{-1}(\pi - \theta,\varphi) \langle \overline{W}(\theta,\varphi,z) \rangle + \int_{-d}^{\pi} dz' \overline{E}(\theta,\varphi)\overline{D}[\beta(\theta,\varphi)(z' - z)\sec\theta]\overline{E}^{-1}(\theta,\varphi)\overline{S}(\theta,\varphi,z') \tag{19}$$

$$\overline{M}_{lj}^{(1)}(\theta,\varphi;\pi-\theta_0,\varphi_0) = \sum_{k,i} \sec\theta \{\overline{\overline{E}}(\theta,\varphi)\overline{D}[-\beta(\theta,\varphi) \operatorname{dsec}\theta]\overline{\overline{E}}^{-1}(\theta,\varphi) \cdot \overline{R}(\theta)\overline{\overline{E}}(\pi-\theta,\varphi)\}_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{D}[-\beta(\theta,\varphi) \operatorname{dsec}\theta]\overline{\overline{E}}^{-1}(\theta,\varphi) \cdot \overline{R}(\theta)\overline{\overline{E}}(\pi-\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{D}[-\beta(\theta,\varphi) \operatorname{dsec}\theta]\overline{\overline{E}}^{-1}(\theta,\varphi) \cdot \overline{R}(\theta)\overline{\overline{E}}(\pi-\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \operatorname{dsec}\theta \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi)\overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \cdot \overline{\overline{E}}(\theta,\varphi) \right]_{lk} \cdot \left[ \sum_{k,i} \overline{\overline{E}}(\theta,\varphi) \right]_{$$

$$\begin{split} & \{\overline{E}^{-1}(\pi-\theta,\varphi)\overline{P}(\pi-\theta,\varphi;\theta_{0},\varphi_{0})\overline{E}(\theta_{0},\varphi_{0})\}_{ki} \cdot \frac{1-\operatorname{exp}\left[-p_{k}(\pi-\theta,\varphi)\operatorname{dscc}\theta - p_{i}(\theta_{0},\varphi_{0})\operatorname{dscc}\theta_{0}\right]}{\beta_{k}(\pi-\theta,\varphi)\operatorname{sec}\theta + \beta_{i}(\theta_{0},\varphi_{0})\operatorname{sec}\theta_{0}} \cdot \\ & \{\overline{E}^{-1}(\theta_{0},\varphi_{0}) \cdot \overline{R}(\theta_{0})\overline{E}(\pi-\theta_{0},\varphi_{0})\operatorname{dscc}\theta_{0}]\overline{E}^{-1}(\pi-\theta_{0},\varphi_{0})\}_{ij} + \\ & \sum_{k,i}\operatorname{sec}\left\{\overline{E}(\theta,\varphi)\overline{D}\left[-\beta(\theta,\varphi)\operatorname{dscc}\theta\right]\overline{E}^{-1}(\theta,\varphi)\overline{R}(\theta) \cdot \overline{E}(\pi-\theta,\varphi)\right\}_{l,k}\left\{\overline{E}^{-1}(\pi-\theta,\varphi)\overline{P}(\pi-\theta,\varphi;\pi-\theta_{0},\varphi_{0})\right\}_{ij} + \\ & \varphi_{0}(\Phi_{0})\overline{E}(\pi-\theta_{0},\varphi_{0})\right\}_{kj} \cdot \frac{\operatorname{exp}\left[-\beta_{k}(\pi-\theta,\varphi)\operatorname{dscc}\theta - \beta_{i}(\pi-\theta_{0},\varphi_{0})\operatorname{dscc}\theta_{0}\right]}{\beta_{i}(\pi-\theta_{0},\varphi_{0})\operatorname{sec}\theta_{0} - \beta_{k}(\pi-\theta,\varphi)\operatorname{sec}\theta}\left\{\overline{E}^{-1}(\pi-\theta_{0},\varphi_{0})\right\}_{ij} + \\ & \operatorname{sec}\theta\sum_{k,i}E_{lk}(\theta,\varphi)\left\{\overline{E}^{-1}(\theta,\varphi)\overline{P}(\theta,\varphi;\theta_{0},\varphi_{0})\overline{E}(\theta_{0},\varphi_{0})\right\}_{ki} \cdot \frac{\operatorname{exp}\left[-\beta_{k}(\theta,\varphi)\operatorname{dscc}\theta - \beta_{i}(\theta,\varphi)\operatorname{dscc}\theta_{0}\right]}{\beta_{i}(\theta_{0},\varphi_{0})\operatorname{sec}\theta_{0} - \beta_{k}(\theta,\varphi)\operatorname{dscc}\theta_{0}}\right] \\ & \left\{\overline{E}^{-1}(\theta_{0},\varphi_{0})\overline{R}(\theta_{0}) \cdot \overline{E}(\pi-\theta_{0},\varphi_{0})\overline{D}\left[-\beta(\pi-\theta_{0},\varphi_{0})\operatorname{dscc}\theta_{0}\right]\overline{E}^{-1}(\pi-\theta_{0},\varphi_{0})\right\}_{ki} \cdot \\ & \operatorname{sec}\theta\sum_{k,i}\overline{E}_{lk}(\theta,\varphi)\left\{\overline{E}^{-1}(\theta,\varphi)\overline{P}(\theta,\varphi;\pi-\theta_{0},\varphi_{0})\overline{E}(\pi-\theta_{0},\varphi_{0})\operatorname{dscc}\theta_{0}\right]\overline{E}^{-1}(\pi-\theta_{0},\varphi_{0})\right\}_{ki} \cdot \\ & \operatorname{sec}\theta\sum_{k,i}\overline{E}_{lk}(\theta,\varphi)\left\{\overline{E}^{-1}(\theta,\varphi)\overline{P}(\theta,\varphi;\pi-\theta_{0},\varphi_{0})\overline{E}(\pi-\theta_{0},\varphi_{0})\right\}_{ki} \cdot \\ & \operatorname{sec}\theta\sum_{k,i}\overline{E}_{ki}(\theta,\varphi)\left\{\overline{E}^{-1}(\theta,\varphi)\overline{P}(\theta,\varphi;\pi-\theta_{0},\varphi_{0})\overline{E}(\pi-\theta_{0},\varphi_{0})\right\}_{ki} \cdot \\ & \operatorname{sec}\theta\sum$$

其中特征向量  $\beta(\theta,\varphi)$ 均是散射振幅函数的  $4 \times 4$  矩 阵。特征矩阵  $\overline{E}(\theta,\varphi)$ ,对角阵  $\overline{D}[\beta(\theta,\varphi)d\sec\theta]$ 也 是由散射振幅函数求得<sup>[6]</sup>的。取其中的 Mueller 矩 阵元  $m_{00}, m_{11}, m_{22}$ 和  $m_{33}$ ,可以根据(13)式进一步求 得非球形粒子的偏振度 pd。

## 3 非球形粒子偏振度的数值仿真计 算及分析

3.1 偏振度随散射介质层厚度变化的情况

首先讨论散射介质层的厚度对后向散射光去 偏振度的影响。设数值模拟参量中波长为 532.8 nm的连续线偏振光,入射角 $\theta_0$ =30°,探测角 取后向散射方向,即探测角等于入射角( $\theta = \theta_0$ = 30°)。设非球形粒子的2个短轴半径相等,即*br*= *cr*, *ar*=2.5 nm, *cr*=*ar*/5,散射介质的介电常数 $\epsilon$ = $\epsilon_0$ (1.2+*i*×0.001),散射介质的厚度分别取3,3. 5,4,4.5,5,5.5,6,6.5(单位为 $\mu$ m)等8组参量。求 解(10)式和(11)式,对Mueller 矩阵进行数值计算, 并应用(8)式求解散射介质的偏振度,结果见表1。

表1 偏振度随散射介质层厚度的变化

# Table 1The variation of the polarization ratiowith the thickness of the medium

$d/\mu{ m m}$	3	3.5	4	4.5
рd	0.6456	0.5025	0.4288	0.4047
$d/\mu{ m m}$	5	5.5	6	6.5

由表1可以看出,介质层厚度会影响散射光在 介质中的散射次数,即发生多次散射,介质层厚度 的变化对散射光偏振度的变化影响很大。通过这8 组数据可以发现,虽然每2组相邻数据的介质层厚 度相差仅为0.5 $\mu$ m,而后向散射光的偏振度的差 异却很明显,最小为0.0185,最大为0.2427,平均 差异为0.1009。当散射介质层厚度分别为6.5 $\mu$ m 和4.5 $\mu$ m时,后向散射光的偏振度差别最大可达 到0.4653;当散射介质层厚度为4 $\mu$ m和5 $\mu$ m时, 后向散射光的偏振度差别最小为0.0056。因此,通 过对散射光偏振度的探测,可以有效检测散射介质 层厚度的微小变化。

3.2 偏振度随粒子半径的变化情况

数值模拟参量中波长  $\lambda = 632.8 \text{ nm}$ ,入射角  $\theta_0$ 

 $= 30^{\circ}$ ,探测角取后向散射方向( $\theta = \theta_0 = 30^{\circ}$ ),半径 br = cr,cr = ar/5,介电常数 $\epsilon = \epsilon_0(1.2 + i \times 0.001)$ , 介质层厚度 $d = 5 \mu m_o$ .通过求解Mueller 矩阵的数 值解,在粒子半径范围为 2.5~9 nm 的条件下,求 解的偏振度见表 2。

表2 偏振度随粒子半径的变化

Table 2The variation of polarization ratio with<br/>the radius of the particle

ar	2.5	3	3.5	4	4.5	5
pd	0.4232	0.6821	0.7027	0.7019	0.6899	0.6674
ar	6	7	7.5	8	8.5	9
pd	0.5686	0.3438	0.4778	0.6468	0.7922	0.8925

由表2可以看出,散射光的偏振度对粒子半径 的变化很敏感(表2中半径较大粒子的散射光偏振 度的变化更为明显),相邻每2组数据的粒子半径 仅相差 0.5 nm, 然而散射光的偏振度的差别比较 明显,半径相对较小粒子的后向散射光的偏振度的 差异最小为 0.0008,最大为 0.2589, 平均为 0.0630。当半径分别为2.5nm 和3.5 nm 时, 后向 散射光的偏振度差异最大为0.2795。表2中半径较 大粒子的后向散射光的偏振度差异最小为 0.1003, 最大为 0.303, 平均为 0.1815。当半径分别为 9 nm 和 7 nm 时, 后向散射光的偏振度差异最大为 0.5487。显然,半径较大的粒子对后向散射光的偏 振度的影响要大于半径相对较小的粒子。尽管半径 仅有几个纳米的差异,但散射光的偏振度的变化比 较敏感。偏振度的这一性质可应用于粒子半径的探 测。

#### 3.3 偏振度随探测角的变化情况

下面讨论散射光的偏振度随探测角的变化情况。仍采用上述参量作为仿真计算的数据:非球形粒 子半径ar=2.5 nm,br=cr=ar/5,散射介质的介电 常数为 $\varepsilon = \varepsilon_0 (1.2+i \times 0.001)$ ,散射介质的厚度 $d = 5 \mu m$ ,入射角 $\theta_0 = 55^\circ$ ,探测角 $\theta$ 的变化范围为 $0^\circ \sim 180^\circ$ ,变化间隔为 $1^\circ$ 。用Matlab软件对Mueller 矩阵 进行数值求解,并应用(8)式求偏振度,仿真结果如 图1 所示。从图1 可以看出,随着探测角的变化,散射 光偏振度的变化也十分明显。在 $0^\circ \sim 90^\circ$ 范围内变化 比较剧烈,出现几个很明显的峰值。当探测角在 $90^\circ$  $\sim 180^\circ$ 范围内时,偏振度为一条变化比较规律的曲 线,此时只有1 个峰值,对应的探测角为 $125^\circ$ 。这个 探测角度正是镜面反射方向的角度,镜面反射的去 偏振度为最小<sup>[2]</sup>。



图1 散射光偏振度随探测角的变化曲线

Fig. 1 Variation of the polarization degree of scattered light with the detection angle

#### 4 结论

通过对 Rayleigh 非球形粒子散射模型的模拟 计算,得到了目标散射光的偏振度随散射介质的厚 度、粒子半径以及探测角的变化特性。散射光的偏振 度随探测角的变化比较明显,这一结果为偏振度探 测的实际应用提供了参考。通过数值仿真结果可以 看到,偏振度可以明显表征散射介质的厚度以及粒 子半径的微小变化。偏振度的这一特性为研究目标 的内部结构、厚度和粗糙度等特征以及目标的探测 和识别提供了新的途径,并且为偏振度探测的实际 应用提供了理论基础。

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