A Switching Method for a Model Reference Robust Control with Unknown High Frequency Gain Sign[1)]

YUAN Li-Na[1] LIN Yan[2]

[1](*Mechanics and Electricity Department, Wuyi University, Guangdong* 529020)

[2](*School of Automation, Beijing University of Aeronautics and Astronautics, Beijing* 100083)

(E-mail: yln@263.net, linyanee2@yahoo.com.cn)

Abstract The problem of controlling a single-input-single-output plant without prior knowledge of high frequency gain sign is addressed by using the model reference robust control approach. A switching method is proposed based on a monitoring function so that after a finite number of switchings the tracking error converges to zero exponentially. Furthermore, it is shown that if some initial states of the closed-loop system are zero, only one switching is needed.

Key words Model following, high-frequency gain, robust control, switching control

1 Introduction

Model reference robust control (MRRC) was introduced in [1,2] as a new means of I/O based controller design for linear time-invariant plants with nonlinear input disturbance and has been extended to MIMO and non-minimum phase systems[3,4]. To overcome the influence of the nonlinear input disturbance, the conventional parameter adaptive law in model reference adaptive control (MRAC) was abandoned in the MRRC. Instead, the concept of bounding function was introduced in the control law design. Like most of the model following schemes, one of the basic assumptions of the MRRC is that the high frequency gain sign is known a priori. The relaxation of the assumption of high frequency gain sign has long been an attractive topic in control community and can be traced back to the paper by Morse[5]. Several approaches have been proposed so far[6∼8] and most of them, however, are based on Nussbaum gain. The main disadvantage of Nussbausm-type gain methods lies in the fact that it lacks robustness to measurement noise. Furthermore, the transient behavior may be unacceptable. In this paper, a switching scheme is proposed for MRRC of plants with relative degree one and unknown high frequency gain sign. Based on the Comparison Lemma[9], we first construct a monitoring function to supervise the behavior of the tracking error, and then put forward a switching method for the control signal. We show that under the supervision of the monitoring function, only a finite number of switchings is needed and the tracking error will converge to zero exponentially.

2 Problem formulation

Consider the following single input/single output linear time-invariant plant

$$y = G_p(s)[u + d] = k_p(n_p(s)/s_p(s))[u + d] \tag{1}$$

where y and u are the system output and input, respectively, $G_p(s)$ is the plant transfer function with $d_p(s)$ and $n_p(s)$ being monic polynomials of degrees n and m, respectively, and d is an input disturbance. The reference model is given by

$$y_M = M(s)[r] = (k_M/d_M(s))[r],\ k_M > 0 \tag{2}$$

where $d_M(s)$ is a monic Hurwitz polynomial with $\deg(d_M(s)) = n - m := n^*$ and r is any piecewise continuous, uniformly bounded reference signal.

We make the following assumptions: A1) $G_p(s)$ is of minimum phase. The parameters of $G_p(s)$ are unknown but belong to a known compact set; A2) The degree n of $d_p(s)$ is a known constant; A3) The relative degree $n^* = 1$; A4) The sign of the high frequency gain $k_p(\neq 0)$ is unknown; A5) The lumped disturbance and uncertainty term $d(y,t)$ is bounded by a known continuous function $\rho(y,t)$ as, for all $(y,t) \in R \times R^+$, $|d(y,t)| \leqslant \rho(y,t)$, $\forall t \geqslant 0$, where the bonding function $\rho(y,t)$ is assumed

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to be continuous, uniformly bounded with respect to t and, locally uniformly bounded with respect to the system output y. The uncertainty $d(y,t)$ is not necessarily continuous but, if it is discontinuous, existence of the solution of y is assumed.

The control signal of the MRRC system is of the following form:

$$u = \hat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + u_R \tag{3}$$

where u_R is the nonlinear control to be designed to ensure that the tracking error

$$e := y - y_M \tag{4}$$

tends to be zero, the constant vector $\hat{\boldsymbol{\theta}} \in R^{2n}$ will be defined below and $\boldsymbol{\omega}$, the regressor vector, is defined as

$$\boldsymbol{\omega} := [\boldsymbol{\nu}_1^{\mathrm{T}} \quad y \quad \boldsymbol{\nu}_2^{\mathrm{T}} \quad r]^{\mathrm{T}} \tag{5}$$

where $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$ are generated by input/output filters according to

$$\dot{\boldsymbol{\nu}}_1 = \varLambda\boldsymbol{\nu}_1 + \boldsymbol{b}u,\ \dot{\boldsymbol{\nu}}_2 = \varLambda\boldsymbol{\nu}_2 + \boldsymbol{b}u,\ \boldsymbol{\nu}_1(0) = 0,\ \boldsymbol{\nu}_2(0) = 0,\ \varLambda \in R^{(n-1)\times(n-1)},\ \boldsymbol{b} \in R^{n-1} \tag{6}$$

where $\varLambda$ is a matrix such that $\det(sI - \varLambda)$ is a Hurwitz polynomial and $(\varLambda, \boldsymbol{b})$ is a controllable pair. It is well known[10] that under the above assumptions, there exits a unique constant vector $\boldsymbol{\theta}^* = [\boldsymbol{\theta}_1^{*\mathrm{T}} \quad \theta_0^* \quad \boldsymbol{\theta}_2^{*\mathrm{T}} \quad k^*]^{\mathrm{T}} \in R^{2n}$ such that, modulo exponentially decaying terms due to initial conditions, $y = G_p(s)[(\boldsymbol{\theta}^*)^{\mathrm{T}}\boldsymbol{\omega}] = M(s)[r] = y_M$. Since the plant parameters are assumed to be uncertain, the constant vector $\hat{\boldsymbol{\theta}}$ in (3) is then defined as $\hat{\boldsymbol{\theta}} = [\hat{\boldsymbol{\theta}}_1^{\mathrm{T}} \quad \hat{\theta}_0 \quad \hat{\boldsymbol{\theta}}_2^{\mathrm{T}} \quad \hat{k}]^{\mathrm{T}} \in R^{2n}$ which can be obtained from the nominal plant and is a rough estimate of $\boldsymbol{\theta}^*$. The tracking error can therefore be expressed from (1)∼(6) as

$$e = M(s)\kappa^*[\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + d_f + u_R] + \bar{\varepsilon} \tag{7}$$

where $\bar{\varepsilon}$ denotes a bounded, differentiable and exponentially decaying real function that represents non-zero initial conditions for all internal states of the MRRC system,

$$\tilde{\boldsymbol{\theta}} := \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*,\ \kappa^* := k_p/k_M = 1/k^*,\ d_f := (1 - d_1(s))[d],\ d_1(s) := \hat{\boldsymbol{\theta}}_1^{\mathrm{T}} adj(sI - \varLambda)\boldsymbol{b} \tag{8}$$

3 Control law design

We consider the control law design for plants with $n^* = 1$. From (2), $n^* = 1$ implies that we can write the reference model as $M(s) = k_M/(s+\lambda)$, where λ is a positive constant. Hence, from (7),

$$\dot{e} = -\lambda e + k_p(\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + d_f + u_R) + \varepsilon \tag{9}$$

where ε decays exponentially.

The following lemma summarizes the main result when the sign of k_p is known:

Lemma 1[1]**.** Let the MRRC system satisfy the assumptions A1), A2), A3) and A5). Suppose the sign of k_p is known. If the control signal is defined as

$$u_R := \begin{cases} -\dfrac{\mu|\mu|^{\tau}}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}g, & \text{if } k_p > 0 \\ \dfrac{\mu|\mu|^{\tau}}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}g, & \text{if } k_p < 0 \end{cases} \tag{10}$$

where $\beta \geqslant 0$, $\sigma > 0$ and $\tau \geqslant 0$ are design parameters, the functions g and μ are chosen such that

$$g = \mathrm{BND}(|\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + d_f|),\ \mu = eg \tag{11}$$

where BND(·) is the bounding function[1], then, e converges exponentially to either zero (if $\beta > 0$) or to a residual set (if $\beta = 0$) whose radius becomes zero in the limit as σ approaches zero.

Corollary 1. The MRRC system is stable if and only if the tracking error e is uniformly bounded.

Remark 1. The bounding function of a signal f, say, $\mathrm{BND}(|f|)$ is a known, continuous, nonnegative function that bounds the magnitude (or Euclidean norm) of f[1].

Since, however, the sign of k_p is unknown, we have to redefine the control as

$$u_R := \begin{cases} u_R^+ = -\dfrac{\mu|\mu|^\tau}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}g, & \text{if } t \in T^+ \\ u_R^- = \dfrac{\mu|\mu|^\tau}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}g, & \text{if } t \in T^- \end{cases} \tag{12}$$

and design a monitoring function to decide when u_R should be switched from u_R^+ to u_R^- and vice versa, where both the sets T^+ and T^- are the union of some intervals like $[t_k, t_{k+1})$, over which u_R^+ and u_R^- are applied, respectively, and

$$T^+ \cup T^- = [0, \infty), \ T^+ \cap T^- = \phi \tag{13}$$

The difference between (10) and (12) is that if the sign of k_p is known, we only need one control signal while if the sign of k_p is unknown, two control signals, u_R^+ and u_R^-, are needed, where u_R^+ and u_R^- correspond to $\text{sgn}(k_p) > 0$ and $\text{sgn}(k_p) < 0$, respectively.

To design the monitoring function, we consider Lyapunov function $V = e^2/2$. The time derivative of V along the trajectory of (9) yields

$$\dot{V} = -\lambda e^2 + k_p[(\tilde{\boldsymbol{\theta}}^{\text{T}}\boldsymbol{\omega} + d_f)e + eu_R] + e\varepsilon \tag{14}$$

Suppose we have correctly estimated the sign of k_p for some $t \geqslant \bar{t}_0 \geqslant 0$ where $\bar{t}_0$ is any finite initial time, then, by replacing (10) in (14) it follows that

$$\begin{aligned} \dot{V} \leqslant & -\bar{\lambda}e^2 + |k_p|\{|(\tilde{\boldsymbol{\theta}}^{\text{T}}\boldsymbol{\omega} + d_f)e| - \frac{\mu|\mu|^\tau}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}eg\} + \frac{1}{2c_\varepsilon}\varepsilon^2 \leqslant \\ & -\bar{\lambda}e^2 + |k_p|\{|\mu| - \frac{\mu|\mu|^{\tau+1}}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}\} + \frac{1}{2c_\varepsilon}\varepsilon^2 \leqslant \\ & -\bar{\lambda}e^2 + |k_p|\{\frac{|\mu|\sigma^\tau\exp(-\beta\tau t)}{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]}\}\sigma\exp(-\beta t) + \frac{1}{2c_\varepsilon}\varepsilon^2 \leqslant \\ & -2\bar{\lambda}V + |k_p|\sigma\exp(-\beta t) + \frac{1}{2c_\varepsilon}\varepsilon^2 \to \begin{cases} 0, & \text{if } \beta > 0 \\ |k_p|\sigma/2\bar{\lambda}, & \text{if } \beta = 0 \end{cases}, \ t \geqslant \bar{t}_0 \end{aligned} \tag{15}$$

where we have used the triangle inequality $\varepsilon e \leqslant (c_\varepsilon e^2 + \varepsilon^2/c_\varepsilon)/2$ with c_ε being any positive constant, and the positive constant $\bar{\lambda}$ is defined as $\bar{\lambda} := \lambda - c_\varepsilon/2$ and is given by designer in advance by properly choosing c_ε. Also, note that from (11), $|(\tilde{\boldsymbol{\theta}}^{\text{T}}\boldsymbol{\omega} + d_f)e| \leqslant |\mu|$, and the following inequality has been used[1]:

$$|\mu|\sigma\exp(-\beta\tau t)/\{|\mu|^{\tau+1} + \sigma^{\tau+1}\exp[-\beta(\tau+1)t]\} \leqslant 1 \tag{16}$$

The construction of the monitoring function is as follows. We consider the following differential equation motivated from (15):

$$\dot{\xi} = -2\bar{\lambda}\xi + |k_p|\sigma\exp(-\beta t) + \varepsilon^2/2c_\varepsilon, \ \xi(\bar{t}_0) = V(\bar{t}_0), \ t \geqslant \bar{t}_0 \tag{17}$$

Comparing (17) with (15), it follows that

$$\dot{V}(t) \leqslant \dot{\xi}(t), \ \forall t \geqslant \bar{t}_0 \tag{18}$$

Note that $\xi(\bar{t}_0) = V(\bar{t}_0)$; hence, by applying the Comparison Lemma[9] to (14) and (17), we have

$$V(t) < \xi(t), \ \forall t > \bar{t}_0 \tag{19}$$

provided that a correct sign of k_p has been estimated for all $t \geqslant \bar{t}_0$.

We therefore consider the solution of (17). Since ε decays exponentially, there exist constants $\delta > 0$ and $c > 0$, such that

$$|\varepsilon(t)| \leqslant c\exp(-\delta t), \ t \geqslant 0 \tag{20}$$

Hence, the solution to (17) satisfies

$$\xi(t) \leqslant \exp[-2\bar{\lambda}(t - \bar{t}_0)]V(\bar{t}_0) + c_\beta\exp[-\beta_0(t - \bar{t}_0)]\exp(-\beta\bar{t}_0) +$$

$$c_\delta \exp[-2\delta_0(t-\bar{t}_0)]\exp(-2\beta\bar{t}_0) \tag{21}$$

where

$$\beta_0=\min\{2\bar\lambda,\beta\},\ \beta_0=\min\{\bar\lambda,\delta\},\ c_\beta=2|\bar{k}_p|\sigma/|2\bar\lambda-\beta|,\ c_\delta=c^2(2|\bar\lambda-\delta|) \tag{22}$$

where $|\bar{k}_p|$ is an upper bound of $|k_p|$ which, from the assumption A1), can be obtained a priori. Since β is a design parameter, we can choose β such that $\beta<2\bar\lambda$; also, we can let $\delta<\bar\lambda$ due to the fact that a less δ can only make (20) more conservative. As a result,

$$\beta_0=\beta,\ \delta_0=\delta \tag{23}$$

Taking into account (23), the inequality (21) can be rewritten as

$$\xi(t)\leqslant \exp[-2\bar\lambda(t-\bar{t}_0)]V(\bar{t}_0)+c_\beta\exp(-\beta t)+c_\delta\exp(-2\delta t),\ t\geqslant \bar{t}_0 \tag{24}$$

Recalling that the inequality (19) holds if the sign of k_p is correctly estimated, it seems natural to use ξ as a benchmark to decide whether a switching of u_R is needed, *i.e.*, the switching occurs only when (19) is violated. The motivation behind the introduction of the monitoring function is that ε is not available for measurement, which implies that c, δ as well as δ given by (22) and (20), respectively, are unknown. Hence, we have to introduce a monitoring function, say, φ_k, to replace ξ and invoke the switching of φ_k:

$$\begin{aligned}&\varphi_k(t)=\exp[-2\bar\lambda(t-t_k)]V(t_k)+c_\beta\exp(-\beta t)+(k+1)\exp(-2\delta_k t)\\&t\in\begin{cases}[t_k,t_{k+1}), & \text{if } k=0,1,2,\cdots,;\ t_0:=0\\ [t_k,+\infty), & \text{if no new switching occurs for } t>t_k\end{cases}\end{aligned} \tag{25}$$

where t_k is the switching time to be defined and δ is any monotonically decreasing sequence satisfying

$$\delta_k\to 0 \text{ as } k\to\infty \tag{26}$$

It is clear that we obtain φ_k from (24) mainly by replacing both c_δ and δ by $k+1$ and δ_k, respectively, and by introducing the switching of them. Note that the value of k increases as the switching proceeds while δ_k satisfies (26).

Remark 2. The use of the sequence $\{(k+1)\}$ in (25) is just for the sake of simplicity. In fact, it may be replaced by any monotonically increasing sequence $\{z_k\}$ that tends to infinity.

Since both the control signals u_R^+ and u_R^- are continuous, for any finite number of switchings, u_R is piece-wise continuous and therefore, the solution of (9) exists and is continuous[11]. From $V=e^2/2$, (25) and the continuity of e, we always have $V(t_k)<\varphi_k(t_k)$ for each switching instant. We thus define the switching time for u_R (from u_R^- to u_R^+ or u_R^+ to u_R^-) and φ_k as follows:

$$t_{k+1}=\begin{cases}\min\{t: t>t_k,\ V(t)=\varphi_k(t)\}, & \text{if the minimum exists}\\ +\infty, & \text{otherwise}\end{cases} \tag{27}$$

That is, the switching occurs only when the condition $V(t)<\varphi_k(t)$ is violated.

We have the following main result of this section.

Theorem. Suppose the MRRC system given by equations (1), (2) and (9) satisfies the assumptions A1)~A5). Let the control signal u_R be defined by (12) and the switching time of u_R (from u_R^+ to u_R^- and vice versa) be defined by (27). Then, the switching will stop after at most finite number of switchings, and the tracking error will converge to zero exponentially.

Proof. By contradiction, suppose u_R switches between u_R^+ and u_R^- without stopping. Note that c_δ and δ defined by (22) and (20), respectively, are constant, and from (12), u_R only has two choices, $u_R=u_R^+$ or $u_R=u_R^-$, and changes its sign alternately for each switching; hence, after finite k switchings, must have a correct sign on $[t_k,t_{k+1})$, *i.e.*, $u_R=u_R^+$ if $k_p>0$ or $u_R=u_R^-$ if $k_p<0$ and, at the same time, from (25) and (26),

$$c_\delta<(k+1),\ exp(-2\delta t)<\exp(-2\delta_k t),\ \forall t>t_k \tag{28}$$

where t_k is the k-th switching time. Comparing (24) with (25), and noting (28), it follows that

$$\xi(t) < \varphi_k(t),\ \forall t \in [t_k, t_{k+1}) \tag{29}$$

where we have replaced $\bar{t}_0$ by t_k in (24). However, since for a correct estimate of the sign of k_p, V satisfies (19), the above inequality implies that

$$V(t) < \varphi_k(t),\ \forall t \in [t_k, +\infty) \tag{30}$$

From (27), no new switching will occur again. In fact, if this is not the case, from (27), a finite time switching implies that the condition

$$V(t_{k+1}) = \varphi_t(t_{k+1}) \tag{31}$$

would be satisfied at some time instant $t = t_{k+1}$. Since, by the assumption, the sign of k_p has been correctly estimated on $[t_k, t_{k+1})$, the inequality (19) holds on the interval, which, together with (29), implies that

$$V(t_{k+1}) \leqslant \xi(t_{k+1}) < \varphi_k(t_{k+1}) \tag{32}$$

a contradiction; hence, no switching will occur again. It should be pointed out that from (25), the inequality (29) also holds on $[t_k, +\infty)$ if no new switching occurs after $t = t_k$. Further, the continuity of e and the finite switchings imply that $\varphi_k(t_k)$ is bounded and, by (25), converges to zero exponentially as $t \to \infty$. Therefore, from (30) and that $V = e^2/2$, V as well as e will also converge to zero exponentially. Finally, by invoking the Corollary 1, the system is stable. This completes the proof. □

The following corollary shows a more interesting (probably surprising) fact for the relative degree one MRRC system.

Corollary. If $\varepsilon = 0$, then at most one switching of u_R is needed.

Proof. From (20), $\varepsilon = 0$ implies that the term $c_\delta \exp(-2\delta t)$ in (24) should be canceled, *i.e.*,

$$\xi(t) \leqslant \exp[-2\bar{\lambda}(t - \bar{t}_0)]V(\bar{t}_0) + c_\beta \exp(-\beta t),\ t \geqslant \bar{t}_0 \tag{33}$$

Therefore, once the correct sign of k_p is chosen, from (19), (33) and with the same arguments given by the proof of Theorem, the following inequality holds for any finite k:

$$V(t) \leqslant \xi(t) \leqslant \exp[-2\bar{\lambda}(t - t_k)]V(t_k) + c_\beta \exp(-\beta t) < \varphi_k(t),\ \forall t \in [t_k, +\infty) \tag{34}$$

where we have replaced $\bar{t}_0$ by t_k. Taking (27) into account, the inequality (34) shows that if the correct sign of k_p is chosen at $t_0 = 0$ ($k = 0$), no switching occurs; whereas, one switching is enough. □

4 Simulation results

Let the plant $G_p(s) := -(s+1)/(s^2 - s - 1)$, $x(0) = [0.5, 0.5]^{\mathrm{T}}$, where x is the state vector of a controllable canonical form of the plant. Note that $\mathrm{sgn}(k_p) < 0$. The reference model is $M(s) = 1/(s+2)$. The parameters of the input/output filters are $\Lambda = -2$ and $b = 1$. The reference signal $r = \sin(2t)$ and the disturbance $d(y,t) = 0.2\sin t + 0.5\sin y + y^2\cos t$. The design parameters defined by (12) are $\tau = 0$, $\sigma = 0.15$ and $\beta = 1$, respectively. To obtain the bounding function of (11), similar to [1], we write $\mathrm{BND}(\tilde{k})|r| + \mathrm{BND}(\tilde{\theta}_0)|y| + \mathrm{BND}(\tilde{\theta}_1)|\nu_1| + \mathrm{BND}(\tilde{\theta}_2)|\nu_2| + d_1(s)[\rho] + \rho := BND(|\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\omega} + d_f|)$ and choose $\hat{\boldsymbol{\theta}} = \mathbf{0}$, $\mathrm{BND}(\tilde{k}) = 4$, $\mathrm{BND}(\tilde{\theta}_0) = \mathrm{BND}(\tilde{\theta}_1) = \mathrm{BND}(\tilde{\theta}_2) = 5$ and $\rho(y,t) = 1 + y^2$. We choose at $t = 0$ the control signal of (12) to be $u_R(0) = u_R^+(0)$, that is, an incorrect control is given at the beginning of the simulation since $\mathrm{sgn}(k_p) < 0$. The monitoring function φ_k is given by (25) where δ_k is chosen as $\delta_k = 1/(k+1)$ and $|\bar{k}_p|$ in (22) is chosen as $|\bar{k}_p| = 5$. The simulation results are shown in figures 1 and 2, from which we can see that after one switching of u_R from u_R^+ to u_R^-, the plant output y soon follows y_M and the tracking error converges to zero exponentially.

5 Conclusion

In this paper, we have introduced a switching methodology for the controller design of MRRC systems without the knowledge of the high frequency gain sign. We have shown that for plants with relative degree one, our approach can guarantee the tracking error to converge to zero exponentially. Furthermore, if some of the initial states of the closed-loop system are zero, we have shown that at most one switching is needed.

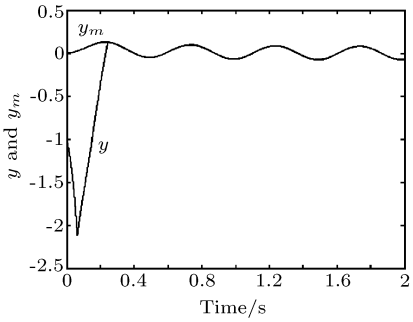

Fig. 1 Tracking error

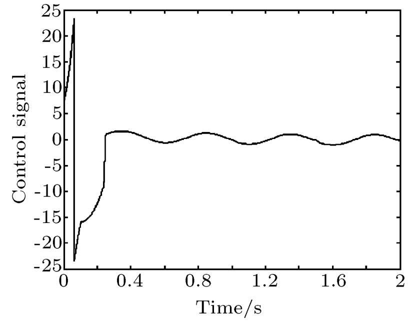

Fig. 2 Control signal

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YUAN Li-Na Received her master degree from the Nanjing Institute of Technology in 1986. She is currently an associate professor of Wuyi University and also a Ph. D. candidate at the Beijing University of Aeronautics and Astronautics (BUAA). Her research interests include product quality control and robust control.

LIN Yan Professor in the School of Automation, BUAA. His research interests include robust control and adaptive control.