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Studies of Lasing Without or with Inversion in Open Four-Level Atomic System*

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Abstract: The model of an open four-level atomic system is presented on the basis of the closed four-level atomic system. In the rotating wave, slowly varying envelope and mean field approximations, the density-matrix motion equations of the system are solved, and the linear analytical solution in the steady state is obtained. The numerical calculation from the steady-state solution of the open system is presented. It is shown that a change from LWI to lasing with inversion action can occur due to the Rabi frequency of the driving field increasing; another change from lasing with inversion to LWI action can also occur as the exit rate or ratio of injection rates increases; high refractive index without absorption can be obtained; the curve of $\rho_{33}-\rho_{11}$ (vs Δ_2) presents the shape of the rectangle wave.

Key words: Lasing without inversion; Open system; Four-level; Gain
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0 Introduction

Quantum coherence and interference have led to a number of important optical consequences such as coherence population trapping[1], electromagnetically induced transparency[2], and lasing without inversion (LWI)[3]. In particular, LWI has attracted more attentions[3-12] due to its important scientific sense and potential wide application. Many schemes for lasing without population inversion have been proposed and the dependence of the optical gain on various system parameters has been examined[3]. There are many ways to generate quantum coherence and interference. Generally, there are produced by a coherent driving field or by initial coherence. In this paper, an open four-level system is discussed, where only a single driving laser is required. The system with injected initially coherence can exhibit lasing with or without inversion. Furthermore, the system can also be used to generate a large refractive index along with zero absorption.

1 Motion equation and solutions

The system considered here is illustrated in Fig. 1. The transition $|1\rangle\leftrightarrow|2\rangle$ of frequency ω_{21} is driven by a laser of frequency ω_d with Rabi frequency 2Ω. A weak-probe laser of frequency ω_p with Rabi frequency $2g$ is applied to the transition $|1\rangle\leftrightarrow|3\rangle$. $2\gamma_{ij}$ is the spontaneous decay rate from state $|i\rangle$ to state $|j\rangle$. The transitions $|2\rangle\leftrightarrow|3\rangle$ and $|4\rangle\leftrightarrow|1\rangle$ are forbidden. If the probe laser is amplified through the system, lasing can be established on the transition $|1\rangle\leftrightarrow|3\rangle$. In the rotating wave, slowly varying envelope and mean field approximations, the density-matrix motion equations of the system can be derived as follows

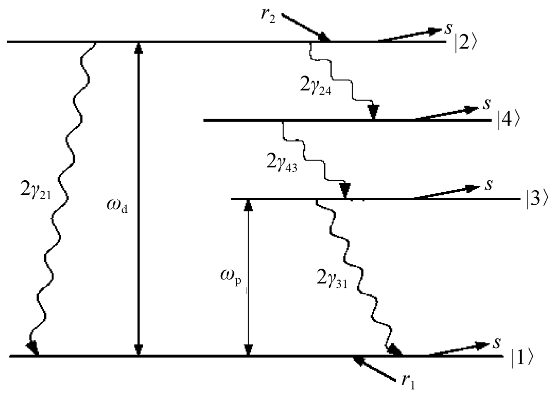

Fig. 1 An open four-level atomic system

$$\dot{\rho}_{11}=2\gamma_{31}\rho_{33}+2\gamma_{21}\rho_{22}+i\Omega(\rho_{21}-\rho_{12})+ ig(\rho_{31}-\rho_{13})-s\rho_{11}+r_1 \tag{1a}$$

$$\dot{\rho}_{22}=-(2\gamma_{21}+2\gamma_{24})\rho_{22}+i\Omega(\rho_{12}-\rho_{21})- s\rho_{22}+r_2 \tag{1b}$$

$$\dot{\rho}_{33}=2\gamma_{43}\rho_{44}-2\gamma_{31}\rho_{33}+ig(\rho_{13}-\rho_{31})-s\rho_{33} \tag{1c}$$

$$\dot{\rho}_{44}=2\gamma_{24}\rho_{22}-2\gamma_{43}\rho_{44}-s\rho_{44} \tag{1d}$$

$$\dot{\rho}_{12}=i\Omega(\rho_{22}-\rho_{11})-(\gamma_{24}+\gamma_{21}+i\Delta_1)\rho_{12}+ ig\rho_{32} \tag{1e}$$

$$\dot{\rho}_{13}=ig(\rho_{33}-\rho_{11})-(\gamma_{31}+i\Delta_2)\rho_{13}+i\Omega\rho_{23} \tag{1f}$$

$$\dot{\rho}_{23}=i\Omega\rho_{13}-ig\rho_{21}-(\gamma_{31}+\gamma_{21}+\gamma_{24}+ i(\Delta_2-\Delta_1))\rho_{23} \tag{1g}$$

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along with the equations for the complex conjugates. Here ρ_{ij} is the atomic polarization between states $|i\rangle$ and $|j\rangle$, ρ_{jj} ($j=1-4$) is the population of the state $|j\rangle$. In Eqs. (1), r_1 (r_2) is the atomic injection rate for level $|1\rangle$ ($|2\rangle$), and s is the atomic exit rate from the cavity. Δ_1 ($=\omega_{21}-\omega_d$) and Δ_2 ($=\omega_{31}-\omega_p$) express the detuning of the driving and probe field from their relevant atomic transition, respectively. Both Rabi frequencies are assumed as real for convenience of calculation in the paper. The gain coefficient of the probe field is proportional to Im ρ_{13}. If Im $\rho_{13}>0$, the system exhibits gain for the probe field; if Im $\rho_{13}<0$, the probe field is attenuated. Furthermore, the dispersion is determined by Re ρ_{13}, and Re $\rho_{13}>0$ corresponds to the red shift of the frequency of the probe field; Re $\rho_{13}<0$ shows the blue shift[13]. The refractive index of medium is proportional to Re ρ_{13}.

In order to study the steady state behavior of the system, we set $d\rho_{ij}/dt=0$ and make always $r_1+r_2=s$ to keep $\langle\rho_{11}\rangle+\langle\rho_{22}\rangle+\langle\rho_{33}\rangle+\langle\rho_{44}\rangle=1$. Eqs. (1) reduce to a set of coupled algebraic equations. After splitting into real and imaginary parts, a system of 10×10 algebraic equations is obtained. These equations can be easily treated in all orders using the symbolic computation package *Maple*. However, the general analytical solutions are tedious. In the following, the numerical calculation result of the steady state for the open four-level system will be presented. The calculation result shows the effect of the coherence driving field such as the parameter Ω, injected initially coherence such as the exit rate s and ratio of injection rates $c=r_1/r_2$, and detuning of the probe field on the gain, dispersion and population difference in the open system.

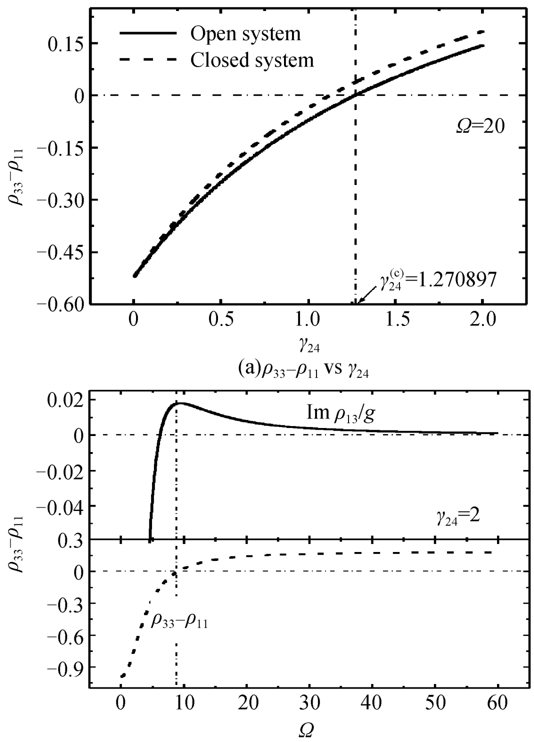
For $\Delta_1=\Delta_2=0$, Reρ_{13} always equals to zero, and in the limit of $\Omega\gg s$, r_1, r_2, γ_{ij} ($i,j=1-4$) and g, The population difference between states $|3\rangle$ and $|1\rangle$ is

$$\rho_{33}-\rho_{11}=[4\gamma_{43}(\gamma_{24}-\gamma_{31})-sC]/(2D) \qquad (2)$$

here $C=2\gamma_{31}+2\gamma_{43}+s$, $D=C(\gamma_{24}+s)+4\gamma_{43}\gamma_{31}$. If the following inequalities are valid: $4\gamma_{43}(\gamma_{24}-\gamma_{31})>sC$ and Im $\rho_{13}>0$, the open system exhibits lasing without population inversion.

2 Discussions and results

All parameter values are normalized to γ_{31}. It is easy to see from Eq. (2) that the size of system parameter γ_{24} plays a *crucial role* in achieving laser action. The plots of $\rho_{33}-\rho_{11}$ versus γ_{24} are obtained for the open system and for the corresponding closed system, respectively, as shown in Fig. 2 (a). Values of parameters are $g=0.01$, $\gamma_{21}=6$, $\gamma_{43}=5$, $\gamma_{31}=1$, $\Delta_1=\Delta_2=0$, $s=0.2$ and $c=4$ in Fig. 2. The population difference $\rho_{33}-\rho_{11}$ of the open system is less than that of the closed system due to the existence of the exit rate and the ratio of injection rates in the open system. In the closed system, the point of γ_{24} being equal to 1 is not the critical point of change from $\rho_{33}-\rho_{11}<0$ to $\rho_{33}-\rho_{11}>0$ under the finite Ω, for example, $\Omega=20$[12].



(a)$\rho_{33}-\rho_{11}$ vs γ_{24}

(b)Im ρ_{13}/g and $\rho_{33}-\rho_{11}$ vs Ω

Fig. 2　$\rho_{33}-\rho_{11}$ vs γ_{24}, and Im ρ_{13}/g and $\rho_{33}-\rho_{11}$ vs Ω

Fig. 2(b) illustrates the gain Im ρ_{13}/g and population difference $\rho_{33}-\rho_{11}$ versus the Rabi frequency of the coherence driving field for the situation that the probe field and the coherent pump field are on resonance with the corresponding transitions. Fig. 2(b) shows that: 1) The open system exhibits gain for the probe laser if Ω is sufficiently large, for instance, $\Omega>6.18$. 2) As Rabi frequency Ω decreases, the open four-level system can induce a change from a conventional inversion laser (Im $\rho_{13}/g>0$ and $\rho_{33}-\rho_{11}>0$) to an inversionless laser (Im $\rho_{13}/g>0$ and $\rho_{33}-\rho_{11}<0$). 3) The open system presents inversion laser when the gain is peaked at a moderate Ω (≈9.4). 4) Gain without inversion is approximately equal to that with inversion within certain range of the Rabi frequency Ω.

To investigate the effect of the exit rate and injection rates on the gain, dispersion and population difference in the system with injected initially coherence, Im ρ_{13}/g, Re ρ_{13}/g and $\rho_{33}-\rho_{11}$ versus s and c have been plotted with the same parameters values as those in Fig. 2 except for $\Omega=20$, $\gamma_{24}=\gamma_{24}^{(c)}$, $\Delta_2=3$, respectively, as shown in Fig. 3. It is known from Fig. 3 that: 1) If the exit rate satisfies $s=0$, the model is only a closed system. Gain of the open system is smaller than that of the closed system, correspondingly. 2) With the exit rate s increasing, the open system can induce a change from a conventional laser to a noninversion laser. 3) As s increases within the certain range of exit rate ($s<3$), both the gain and population difference decrease monotonously, but the dispersion increases monotonously. The gain without inversion is smaller than that with inversion due to the exit rate of the atomic system with injected initially coherence. 4) With the ratio c of atomic injection rate increasing, the gain of the probe laser increases monotonously and is always positive, but $\rho_{33}-\rho_{11}$ decreases monotonously and changes from positive to negative. Hence the ratio c of atomic injection rate also can lead to a change from conventional inversion laser to noninversion laser action.

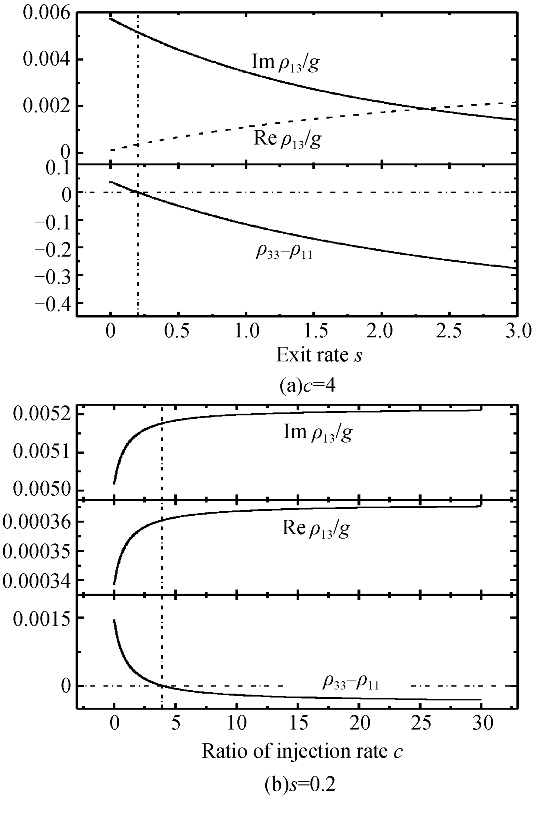


Fig. 3 Im ρ_{13}/g, Re ρ_{13}/g and $\rho_{33}-\rho_{11}$ vs s and vs c, respectively

Fig. 4 shows the numerical calculations for the gain, dispersion, and population difference when the driving laser with Rabi frequency $\Omega=20$ is on resonance with the transition between state $|1\rangle$ and $|2\rangle$ with the same parameters values as those in Fig. 2. From Fig. 4, it is found that: 1) With γ_{24} increasing, Im ρ_{13}/g increases correspondingly. If γ_{24} is very small, Im ρ_{13}/g is always negative and the probe field can not be amplified. The two negative peaks at about $\pm\Omega$ are due to absorption at the transitions from the Antler-Townes' doublet ($|1\rangle\pm|2\rangle$) to state $|3\rangle$. 2) When $\gamma_{24}=0.5$, the open system exhibits gain without inversion and the maximum gain occurs at $\Delta_2=0$. Im ρ_{13}/g crosses the dash dot curve (zero gain) at two frequencies symmetrically located around $\Delta_2=0$, and Re ρ_{13}/g has appreciable values at the two frequencies where Im $\rho_{13}/g=0$. An ensemble of such open four-level atoms system will have a large refractive index and zero absorption. The characteristic is useful in a number of applications[14]. 3) When $\gamma_{24}=\gamma_{24}^{(c)}$, the open system approaches highly the boundary between gain without inversion and gain with inversion. The line profile of Im ρ_{13}/g evolves from a double-peaked absorptive shape into a dispersive shape (only shape!). The dispersion shows the evolution from a purely dispersive shape at small γ_{24} into an

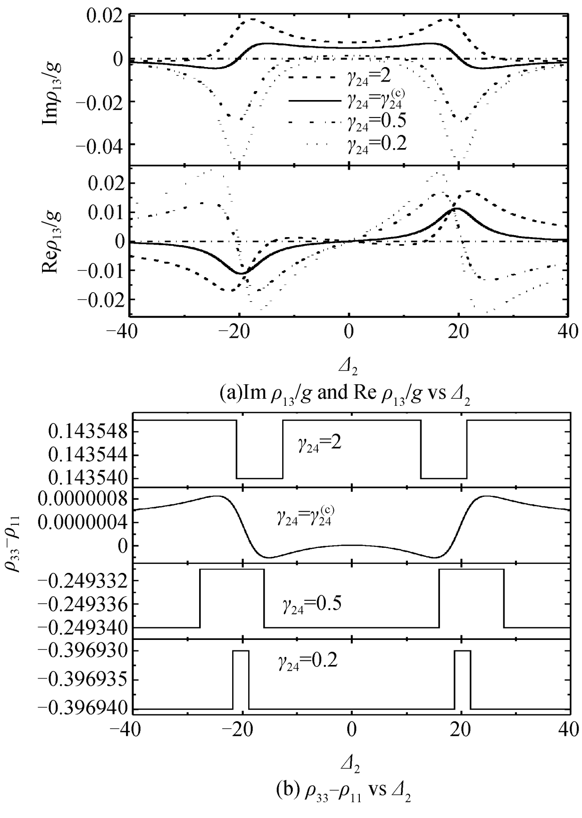


Fig. 4 Im ρ_{13}/g, Re ρ_{13}/g and $\rho_{33}-\rho_{11}$ vs Δ_2

absorption shape at $\gamma_{24}=\gamma_{24}^{(c)}$, then back into a dispersive shape again at large γ_{24}. 4) When $\gamma_{24}=2$, the open atomic system exhibits gain with inversion and also possesses the characteristic of *large refractive index with zero absorption*. The probe gain profile presents two peaks which correspond to the transitions from state $|3\rangle$ to the Antler-Townes′ doublet. 5) From the nonlinear dynamics viewpoint, the lasing arising always corresponds to a loss of stability of the nonlasing stationary solution. The population difference $\rho_{33}-\rho_{11}$ (vs Δ_2) is a continuous curve only when $\gamma_{24}=\gamma_{24}^{(c)}$. The shape of the population difference curve is similar to the negative shape of the curve about corresponding Im ρ_{13}/g. The variation of the strength of the transition between $|1\rangle$ and $|3\rangle$ is closely related to the gain value. When the value γ_{24} deviates from $\gamma_{24}^{(c)}$, it is more interesting that the curve of $\rho_{33}-\rho_{11}$ presents the shape of the rectangle wave and the nonlinear property, and $\rho_{33}-\rho_{11}$ only shows record in the region near the dominant absorption or gain peaks.

3 Conclusions

In conclusion, a model and density matrix equations of motion for an open four-level system have been derived under the electric dipole and rotating wave approximations. Then using numerical calculation result of the steady state, the effect of the coherence driving field is investigated such as Ω, injected initially coherence such as the exit rate and ratio of injection rates, and detuning of the probe field on the gain, dispersion and population difference of the system. The study results show that a change from LWI to lasing with inversion action can occur due to the Rabi frequency of the driving field increasing; another change from lasing with inversion to LWI action can also occur as the exit rate or ratio of injection rates increases; high refractive index without absorption can be obtained; the curve of $\rho_{33}-\rho_{11}$ (vs Δ_2) presents the shape of the rectangle wave.

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开放四能级原子系统粒子数反转或无反转激光研究

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摘　要:在封闭的简单四能级原子系统模型的基础上提出了开放的四能级原子系统模型.通过在电偶极和旋转波近似下,解此系统的半经典密度矩阵运动方程得其稳态线性解析解.对该稳态解的数值模拟显示:随着驱动场的Rabi频率的连续增加系统会发生由输出的无粒子数反转激光到输出粒子数反转激光的转变;随着系统退出速率或者注入速率比的连续增大,系统会发生由输出的粒子数反转激光到输出无粒子数反转激光的转变;该系统能获得无吸收高色散;在稳态,探测激光上下能级粒子数差随探测场失谐的变化曲线呈弱矩形波.
关键词:无反转激光;开放系统;四能级;增益

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