Probabilistic Remote Preparation of Two-atom Entangled State*

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Abstract: A scheme for remote preparation of a two-atom entangled state is presented, which is based on the simultaneous nonresonant interaction of atoms with a single-mode cavity field. In view of the fact that two-particle entanglement is more easily generated than three-particle entanglement, the scheme prepares a two-atom entangled state via two pairs of two-atom entangled state as the quantum channel. Let two identical atoms Alice possesses interact with a single-mode cavity field simultaneously. Alice knows the state she wants to prepare. She selects a proper interaction time, measures the states of the atoms she possesses, and informs Bob of her measurement through a classical channel. Bob introduces an identical auxiliary atom and a single-mode cavity field in vacuum state to achieve the scheme. The scheme is insensitive to the cavity field states and cavity decay. The preparation can be achieved in a simple way. Based on current cavity quantum electrodynamics technique, the scheme can be realized experimentally.

Key words: Quantum information; Two-atom entangled state; Remote state preparation; Nonresonant interaction

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0 Introduction

In recent years, much attention has been paid to quantum entanglement.[1-3] The entanglement is a resource with which to perform quantum information processing, such as quantum computing,[4] quantum error correction,[5] dense coding[6] and quantum teleportation .[7] In particular, teleportation has generated a lot of interest since it was first proposed and demonstrated.[8-10]

Recently, Lo,[11] Pati[12] and Bennett *et al*.[13] have presented an interesting new application of multiparticle entanglement-remote state preparation (RSP), in which Alice wishes to help Bob in his laboratory to prepare a quantum state by means of prior shared entanglement and classical communication . In RSP, Alice is assumed to know the state which is to be prepared remotely, while in quantum teleportation, neither Alice nor Bob knows the identity of the teleported state. Because of the property of pre-knowledge of the quantum state that is to be prepared in RSP, the communication cost will be reduced in the course of entanglement and classical communication. A comparison of RSP with quantum teleportation shows that the former [11-13] to be the strong trade-off in cost between the required entanglement and the classical communication.

There has been number of theoretical protocols for generalization of RSP.[14-17] Peng *et al*. have reported an experimental implementation of RSP over interatomic distances using the technique of nuclear magnetic resonance .[18] Shi *et al*.[19] have proposed a scheme to remotely prepare a two-particle entangled state by a three-particle Greenberge-Horne-Zeilinger (GHZ) state . On the other hand, Liu *et al*.[20] and Zhang *et al*.[21] all have prepared a two-particle entangled state via two pairs of entangled particles as the quantum channel, in view of the fact that two-particle entanglement is more easily generated than three-particle entanglement . To our best knowledge, there is less discussion on remote preparation of atomic states.[21-22] In the scheme of Ref. [21], Zhang *et al*. prepared a two-atom entangled state remotely through interacting two atoms with a single-mode cavity field dispersively with the assistance of a strong classical field .

In this paper, we propose an alternative scheme for remote preparation of a two-atom entangled state via two pairs of entangled particles as the quantum channel, which is based on the simultaneous nonresonant interaction of atoms with a single-mode cavity field. The scheme is insensitive to both the cavity decay and the existence of thermal photons. In contrast to previous scheme [21], the successful probability of the present scheme is smaller. However, our

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scheme has the following advantages. First, it does not need any classical field to drive the atoms. Secondly, it needs only one single-mode cavity in vacuum state and one auxiliary atom for Bob to reconstruct the initial state . In a ward, the experimental device of our scheme is simplified, which should decrease the experimental errors.

1 Probabilistic remote preparation of a two-atom entangled state

Suppose that Alice wishes to help Bob remotely prepare a two-atom entangled state

$$|\phi\rangle = \alpha|eg\rangle + \beta|ge\rangle \quad (1)$$

where the parameters α and β are real numbers, they satisfy $|\alpha|^2+|\beta|^2=1$, and the state in Eq. (1) is known completely to Alice but unknown to Bob . We also suppose that Alice and Bob share the following two two-atom entangled states:

$$|\phi\rangle_{13} = a|eg\rangle_{13} + b|ge\rangle_{13} \quad (2)$$

$$|\phi\rangle_{24} = c|eg\rangle_{24} + d|ge\rangle_{24} \quad (3)$$

where coefficients a , b , c and d are real numbers, they satisfy $|a|^2+|b|^2=1$, and $|c|^2+|d|^2=1$. $|e\rangle_j$ and $|g\rangle_j$ $(j=1,2,3,4)$ are the excited and ground states of the jth atom, respectively . The atom 1 and 2 belong to Alice, and the atom 3 and 4 belong to Bob. Furthermore, the atoms 1, 2, 3 and 4 are identical.

To help Bob prepare a two-atom entangled state in his laboratory, Alice will send the two identical atoms 1 and 2 into an optical cavity simultaneously, in which the two atoms interact with a single-mode cavity field dispersively. The interaction Hamiltonian of the system can be expressed as[23-24]

$$H = \omega a^+ a + \sum_{j=1}^{2}\omega_0 S_{jZ} + \sum_{j=1}^{2} g(aS_j^+ + a^+ S_j^-) \quad (4)$$

where $S_{jz} = (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)/2$, $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ $(j=1,2)$ being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operator for the cavity field frequency, ω_0 is the atomic transition frequency, ω is the cavity frequency, and g is the atom-cavity coupling strength . In the case $\Delta = \omega_0 - \omega \gg g\sqrt{\bar{n}+1}$, with $\bar{n}$ being the mean photon number of the cavity field, there is no energy exchange between the atomic system and the cavity . The energy-conversing transitions are between $|e_1 g_2 n\rangle$ and $|g_1 e_2 n\rangle$. The Rabi frequency λ for the transitions between these states, mediated by $|g_1 g_2 n+1\rangle$ and $|e_1 e_2 n-1\rangle$, is given by[23]

$$\lambda = (\langle e_1 g_2 n|H_i|g_1 g_2 n+1\rangle\langle g_1 g_2 n+1|H_i|g_1 e_2 n\rangle)/\Delta + (\langle e_1 g_2 n|H_i|e_1 e_2 n-1\rangle\langle e_1 e_2 n-1|H_i|g_1 e_2 n\rangle)/\Delta = g^2/\Delta \quad (5)$$

Since the two transition paths interfere destructively, the Rabi frequency is independent of the photon number of the cavity mode. Then the effective Hamiltonian is

$$H_e = \lambda[\sum_{j=1,2}(|e_j\rangle\langle e_j|aa^+ - |g_j\rangle\langle g_j|a^+ a) + (S_1^+ S_2^- + S_1^- S_2^+)] \quad (6)$$

where $\lambda = g^2/\Delta$. The first and second terms describe the photon-number-dependent Start shifts, the third and the fourth terms describe the dipole coupling between the two atoms induced by the cavity mode . The effective Hamiltonian of the system can be rewritten as

$$H_e = H_{e1} + H_{e2} \quad (7)$$

where

$$H_{e1} = \lambda\sum_{j=1,2}(|e_j\rangle\langle e_j|aa^+ - |g_j\rangle\langle g_j|a^+ a) \quad (8)$$

$$H_{e2} = \lambda(S_1^+ S_2^- + S_1^- S_2^+) \quad (9)$$

If define the number of excited atoms as

$$N_e = \sum_{j=1}^{2}|e_j\rangle\langle e_j| \quad (10)$$

then $[H_e, a^+ a] = 0$ and $[H_e, N_e] = 0$, so the number of photons and the number of excited atoms remain constant during the interaction between the atoms and the cavity . For the sake of simplicity we first suppose that the cavity field is initially in the Fock state $|n\rangle$. After an interaction time t , we have

$$|eg\rangle_{12}|n\rangle \to e^{-i\lambda t}[\cos(\lambda t)|eg\rangle_{12} - i\sin(\lambda t)|ge\rangle_{12}]|n\rangle \quad (11)$$

$$|ge\rangle_{12}|n\rangle \to e^{-i\lambda t}[\cos(\lambda t)|ge\rangle_{12} - i\sin(\lambda t)|eg\rangle_{12}]|n\rangle \quad (12)$$

$$|ee\rangle_{12}|n\rangle \to e^{-i2\lambda(n+1)t}|ee\rangle_{12}|n\rangle \quad (13)$$

$$|gg\rangle_{12}|n\rangle \to e^{i2\lambda nt}|gg\rangle_{12}|n\rangle \quad (14)$$

Now consider the evolution of the total system

$$(a|eg\rangle_{13} + b|ge\rangle_{13})(c|eg\rangle_{24} + d|ge\rangle_{24})|n\rangle \to ad\,e^{-i\lambda t}|ge\rangle_{34}[\cos(\lambda t)|eg\rangle_{12} - i\sin(\lambda t)|ge\rangle_{12}]|n\rangle + bc\,e^{-i\lambda t}|eg\rangle_{34}[\cos(\lambda t)|ge\rangle_{12} - i\sin(\lambda t)|eg\rangle_{12}]|n\rangle + ac\,e^{-i2\lambda(n+1)t}|ee\rangle_{12}|gg\rangle_{34}|n\rangle + bd\,e^{i2\lambda nt}|gg\rangle_{12}|ee\rangle_{34}|n\rangle \quad (15)$$

Since Alice has known the parameters α and β , she can select the atomic velocity to satisfy the condition $\cos\lambda t = \alpha$, $\sin\lambda t = \beta$. Then she will measure the states of atoms 1 and 2, and inform Bob of her measurement through a classical channel . If the result of Alice ′ s measurement is $|ge\rangle_{12}$, the state of atoms 3 and 4 on Bob′s side will become

$$|\Psi\rangle_{34} = bc\alpha|eg\rangle_{34} - iad\beta|ge\rangle_{34} \quad (16)$$

The common phase factor is discarded. We can see that the state of atoms 3 and 4 on Bob′s side is independent of the photon number of the cavity field. Thus the scheme allows the cavity field to be in any state with a few photons, e. g. , a thermal state. After a rotation operation, the state in Eq. (16) can be written as

$$|\Psi\rangle'_{34}=bc\alpha\ |eg\rangle_{34}+ad\beta\ |ge\rangle_{34} \quad (17)$$

In order to achieve the scheme of remote preparation of a two-atom entangled state, Bob must introduce an identical auxiliary atom 5 in the initial state $|g\rangle_5$ and a single-mode cavity field in vacuum state. If $|ad|>|bc|$, Bob will send atom 4 and atom 5 into the single-mode cavity simultaneously. The effective Hamiltonian of the system has the same form as Eq. (6)

$$H_e=\lambda'[\sum_{j=4,5}(|e_j\rangle\langle e_j|aa^+-|g_j\rangle\langle g_j|a^+a)+(S_4^+S_5^-+S_4^-S_5^+)] \quad (18)$$

where $\lambda'=g'^2/\Delta'$, $\Delta'=\omega_0{}'-\omega'$, g' is the atom-cavity coupling strength, $\omega_0{}'$ is the atomic transition frequency, ω' is the cavity frequency, $S_j^+=|e_j\rangle\langle g_j|$ and $S_j^-=|g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ $(j=4,5)$ being the excited and ground states of the jth atom. After an interaction time t', the system evolves into

$$|\Psi\rangle_{345}=bc\alpha\ |egg\rangle_{345}+\mathrm{e}^{-\mathrm{i}\lambda't'}[ad\beta\cos(\lambda't')\ |geg\rangle_{345}-\mathrm{i}ad\beta\sin(\lambda't')\ |gge\rangle_{345}] \quad (19)$$

Bob can select the atomic velocity to satisfy $\cos(\lambda't')=bc/ad$, the state in Eq. (19) will become

$$|\Psi\rangle'_{345}=bc(\alpha\ |eg\rangle_{34}+\mathrm{e}^{-\mathrm{i}\lambda't'}\beta\ |ge\rangle_{34})\ |g\rangle_5-\mathrm{i}\mathrm{e}^{-\mathrm{i}\lambda't'}\beta\sqrt{a^2d^2-b^2c^2}\ |gge\rangle_{345} \quad (20)$$

Then Bob detects the state of the auxiliary atom 5. If the result of the measurement is $|g\rangle_5$, after a rotation operation, Bob will obtain the state

$$|\Psi\rangle_f=bc(\alpha\ |eg\rangle_{34}+\beta\ |ge\rangle_{34}) \quad (21)$$

The remote preparation of the state $\alpha|eg\rangle+\beta|ge\rangle$ is successful with the probability of $|bc|^2$.

If $|ad|<|bc|$, Bob must send atom 3 and atom 5 into the single-mode cavity simultaneously. Under this condition, Bob must select the atomic velocity to satisfy $\cos(\lambda't')=ad/bc$. Then Bob measures the state of the auxiliary atom 5. If the result of the measurement is $|g\rangle_5$, after a rotation operation, Bob will obtain the state

$$|\Psi\rangle_f{}'=ad(\alpha\ |eg\rangle_{34}+\beta\ |ge\rangle_{34}) \quad (22)$$

In this case, the successful probability of the remote preparation is $|ad|^2$. We can see that the successful probability is only dependent on the smaller product of coefficients.

From eq. (15), we know that if the result of Alice's measurement is $|eg\rangle_{12}$ the state of atoms 3 and 4 on Bob's side will become

$$|\Psi'\rangle_{34}=ad\alpha\ |ge\rangle_{34}-\mathrm{i}bc\beta\ |eg\rangle_{34} \quad (23)$$

The common phase factor is discarded. After a rotation operation and Pauli operations, the state in Eq. (23) can be written as

$$|\Psi'\rangle'_{34}=ad\alpha\ |eg\rangle_{34}+bc\beta\ |ge\rangle_{34} \quad (24)$$

If $|ad|<|bc|$, Bob sends atom 4 and atom 5 initially in the state $|g\rangle_5$ into the single-mode cavity simultaneously and selects the atomic velocity to satisfy $\cos(\lambda't')=ad/bc$. If Bob detects the atom 5 in the state $|g\rangle_5$, after a rotation operation, he obtains the state

$$|\Psi'\rangle_f=ad(\alpha\ |eg\rangle_{34}+\beta\ |ge\rangle_{34}) \quad (25)$$

If $|ad|>|bc|$, Bob must send atom 3 and atom 5 into the cavity simultaneously and select the atomic velocity to satisfy $\cos(\lambda't')=bc/ad$. After detecting the atom 5 in the state $|g\rangle_5$, and making a rotation operation, he obtains the state

$$|\Psi'\rangle'_f=bc(\alpha\ |eg\rangle_{34}+\beta\ |ge\rangle_{34}) \quad (26)$$

Nevertheless, If Alice's measurement result is $|ee\rangle_{12}$ or $|gg\rangle_{12}$, the scheme fails.

In our scheme, if $a=b=c=d=1/\sqrt{2}$, Alice and Bob share two two-atom maximally entangled states. In this case, it is unnecessary for Bob to introduce any single-mode cavity and auxiliary atom to realize the remote preparation. What Bob must do is to perform proper operations on the states of atom 3 and atom 4.

2 Conclusion

In summary, we have proposed a scheme for remote preparation of a two-atom entangled state. In our scheme, Alice interacts dispersively the two atoms she possesses with a single-mode cavity field in thermal states. The scheme is insensitive to both the cavity decay and the existence of thermal photons. The successful probability of the present scheme is half of that of the scheme of Ref. [21]. However, our scheme has the following advantages. First, it does not need any classical field to drive the atoms. Secondly, it needs only one single-mode cavity in vacuum state and one auxiliary atom for Bob to reconstruct the initial state . In a ward, the experimental device of our scheme is simplified, which should decrease the experimental errors. Following the scheme by Zheng *et al*.[25] two-atom entangled state has been realized experimentally in cavity QED[26]. We believe that our scheme might also be realized in practice.

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远程制备双原子纠缠态

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摘　要：提出一种远程制备双原子纠缠态的方案，该方案基于两个原子与单模腔场的同时非共振相互作用．由于双粒子纠缠态比三粒子纠缠态容易制备，方案用两对双原子纠缠态作为量子通道．Alice 拥有的两个相同原子同时与一单模腔场非共振相互作用．Alice 已知她要制备的纠缠态，她选择适当的相互作用时间、测量她所拥有的两个原子并通过经典通道通知 Bob．Bob 引入一个相同的辅助原子和一个单模腔场来实现方案．方案对腔场状态和腔损耗不敏感，基于当前的腔 QED 技术，方案能在实验上实现．该方案有望在量子信息过程中有重要的应用价值．

关键词：量子信息；双原子纠缠态；远程态制备；非共振相互作用

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