Robust Impulse Dissipative Control of Singular Systems with Uncertainties

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Abstract Based on the linear matrix inequality (LMI) method, this paper discusses the impulse dissipative problem for a class of singular systems with uncertainties under the description of state-space, and obtains the robust impulse dissipative state feedback controller and output feedback controller for singular systems with allowable uncertainties by solving the linear matrix inequalities. At last, we testify the feasibility of our results by a numerical example.

Key words Singular systems, linear matrix inequality, impulse, robust dissipative control

1 Introduction

The dynamic infinitude apices of singular systems decide whether the system is impulse free. If the system has impulse, the system can not work well, even it may be catastrophic, so it is important to eliminate the impulse of singular systems. For practical systems uncertainties always exist, thus many scholars are interested in the control problem of singular systems with uncertainties, and this problem has been extensively studied and applied in many areas[1~3]. Moreover, the dissipative theory has been of much interest to many scholars since it was put forward in 1972. Because dissipative control theory can not only solve the H_∞ control and positive real control problems but also open up new content, it is necessary to discuss the impulse dissipative control problem of singular system with uncertainties. [4] studied that how an interval linear impulsive system could be robustly dissipative with respect to the quadratic supply rate, and [5] illustrated the impulse robust control problem of generalizing uncertain time-vary systems with feed-forward control via static output feedback. However, the dissipative control problems of singular systems are almost under the assumption that the singular system is impulse free. In fact the impulse exists widely in singular systems, therefore, in order to study the dissipative control of singular system more widely, the paper discusses the impulse dissipative control problem with uncertainties under the description of state-space. We gain the robust impulse dissipative state feedback controller and output feedback controller for singular systems with the allowable uncertainties by solving the linear matrix inequalities. This kind of robust impulse dissipative controller can eliminate the impulse of singular systems to make the closed-loop system not only impulse-free but also robust dissipative.

The paper is structured as follows. In next section, we introduce notations and review the most important definitions and lemmas. And then, in Section 3, we prove two theorems to get the robust dissipative state feedback controller so that the closed-loop system is impulse free and robust dissipative. Section 4 contains a theorem by which we can obtain the robust dissipative output feedback controller to make the system impulse free and robust dissipative. Then in Section 5 we testify the feasibility of the theorems by a numerical example. Section 6 contains conclusions.

Received August 8, 2005; in revised form December 16, 2005
Supported by National Natural Science Foundation of P. R. China (60574011)
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DOI: 10.1360/aas-007-0554

2 System description and preliminaries

Consider the following uncertain singular systems

$$\left\{ \begin{array}{l} E\dot{x}(t) = (A + \Delta A)x(t) + Bu(t) \\ y(t)= Cx(t) + Du(t) \end{array} \right. \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ the input, $y(t) \in \mathbf{R}^m$ the output of the system, E, A, B, C and D are known constant matrices of appropriate dimensions. (E, A) is regular, where E is a singular matrix, and $\mathrm{rank}(E) = n_1 \leq n$. $\Delta A(t) \in U_\alpha$ is real-valued matrix functions representing time-varying norm-bounded parameter uncertainties satisfying $U_\alpha = \{H: \ \|H\| < \alpha, \ H \in \mathbf{R}^{n\times n}, \ \alpha > 0\}$, where $\|H\|$ stands for the spectrum norm of matrix H.

The nominal system of (1) is

$$\left\{ \begin{array}{l} E\dot{x}(t) = Ax(t) + Bu(t) \\ y(t)= Cx(t) + Du(t) \end{array} \right. \tag{2}$$

By the matrix theory, there exist orthogonal matrices U' and V such that

$$U'EV = \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix}$$

where Σ is $n_1 \times n_1$ diagonal positive matrix. So by transformation (U', V), system (2) is restricted system equivalent to the following form:

$$\begin{array}{l} \Sigma\dot{x}_1 = A'_{11}x_1 + A'_{12}x_2 + B'_1 u \\ 0 = A_{21}x_1 + A_{22}x_2 + B_2 u \\ y = C_1x_1 + C_2x_2 + Du \end{array} \tag{3}$$

then pre-multiplying both sides of the first equation of (3) by Σ^{-1}. And system (2) is restrictedly equivalent to the following form

$$\begin{pmatrix} U & 0 \\ 0 & I_r \end{pmatrix} \begin{pmatrix} sE - A & B \\ C & D \end{pmatrix} \begin{pmatrix} V & 0 \\ 0 & I_m \end{pmatrix} = \begin{pmatrix} sI_{n_1} - A_{11} & -A_{12} & B_1 \\ -A_{21} & -A_{22} & B_2 \\ C_1 & C_2 & D \end{pmatrix} \tag{4}$$

where $U = \begin{pmatrix} U_1^{\mathrm{T}} & U_2^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$, $V = \begin{pmatrix} V_1 & V_2 \end{pmatrix}$ are nonsingular matrices, $U_1 \in \mathbf{R}^{n_1\times n}$, $V_1 \in \mathbf{R}^{n\times n_1}$, and $U = \Sigma^{-1}U'$. Therefore, by the matrix theory we can obtain that the spectrum norm of U or V is unique.

In order to prove the main results, firstly we introduce the following lemmas.

Lemma 1[6]**.** Singular system (2) or the equivalent system (3) is impulse free if and only if A_{22} is nonsingular matrix, the sufficient and necessary condition of the impulse controllability is $\mathrm{rank}(A_{22}, B_{12}) = n_2$, and the sufficient and necessary condition of the impulse observability is $\mathrm{rank}(A_{22}^{\mathrm{T}}, C_2^{\mathrm{T}}) = n_2$.

Lemma 2[7]**.** Suppose that $S \in \mathbf{R}^{\,l\times l}$. If $\|S\| < q$, then $(qI_l \pm S)$ is nonsingular matrix.

Lemma 3[8]**.** If matrix Δ satisfies $\Delta^{\mathrm{T}}\Delta < I$, then for any $\varepsilon > 0$, the following matrix inequality holds for appropriate dimensions

$$X\Delta Y + (X\Delta Y)^{\mathrm{T}} \le \varepsilon XX^{\mathrm{T}} + \frac{1}{\varepsilon}Y^{\mathrm{T}}Y \tag{5}$$

Definition 1[9]**.** If there exists a function V : $X \to \mathbf{R}^+$(called storage function) such that for all $x_0 \in X,\ t \ge t_0$ and input u, the following inequality inequality

$$\mathrm{V}(x(t_1)) \le \mathrm{V}(x(t_0)) + \int_{t_0}^{t_1} r(u(t),\ y(t))\mathrm{d}t \tag{6}$$

holds, then state space is said to be dissipative via the supply rate $r(u(t),\ y(t))$, and inequality (6) is called dissipative inequality. If the inequality is inequality is strict, then state space is said to be strictly supply rate $r(u(t),\ y(t))$.

Definition 2. A controller is called a robust impulse dissipative controller if the closed-loop singular system is robustly dissipative and impulse free via the controller.

For system (1), the energy supply rate is chosen as

$$r(u,y) =< y, Qy >_{\mathrm{T}} +2 < y, Su >_{\mathrm{T}} + < u, Ru >_{\mathrm{T}}$$

where $< u,y >_{\mathrm{T}}= u^{\mathrm{T}}y$, $Q<0$, $R<0$ are given matrices, and S is given matrix of appropriate appropriate dimensions.

3 Robust impulse dissipative state feedback controller

Suppose that $u=Kx$ is the state feedback controller of system (1), then the restricted system equivalence of the closed-loop of system (1) is as follows

$$\begin{aligned}&\dot{x}_1 = (A_{11}+\Delta A_{11}+B_1K_1)x_1+(A_{12}+\Delta A_{12}+B_1K_2)x_2\\&0=(A_{21}+\Delta A_{21}+B_2K_1)x_1+(A_{22}+\Delta A_{22}+B_2K_2)x_2\\&y=C_1x_1+C_2x_2+Du\end{aligned} \tag{7}$$

where $K \in \mathbf{R}^{r\times n}$, and $K=\begin{pmatrix} K_1 & K_2\end{pmatrix}$.

The notation D^+ in the following theorems denotes the Moore-Penrose generalized inverse of matrix D.

Theorem 1. If $M=B_{12}D^+C_2$ is a nonsingular matrix, $D^{\mathrm{T}}S+S^{\mathrm{T}}D$ is a symmetric and nonsingular matrix, and the following LMIs hold (see (8) on bottom of this page) where

$$\begin{aligned}&\Omega_{11} = (A+d_0BD^+C)^{\mathrm{T}}P + P^{\mathrm{T}}(A+d_0BD^+C)+\varepsilon\alpha I-\\&C^{\mathrm{T}}QC - d_0C^{\mathrm{T}}QDD^+C - d_0(DD^+C)^{\mathrm{T}}QC-\\&d_0C^{\mathrm{T}}SD^+C - d_0(D^+C)^{\mathrm{T}}S^{\mathrm{T}}C\end{aligned}$$

then there exists robust impulse dissipative state feedback controller $u=Kx$ for system (1), where $K=d_0D^+C$.

Proof. If $M=B_2D^+C_2$ is inverse, and $K=d_0D^+C$, then $K_2=d_0D^+C_2$, and

$$A_{22}+\Delta A_{22}+B_2K_2 = M\left(M^{-1}A_{22}+M^{-1}\Delta A_{22}+d_0I\right)$$

Therefore,

$$\begin{aligned}&\left\|M^{-1}A_{22}+M^{-1}\Delta A_{22}\right\| \le\\&\qquad \left\|M^{-1}\right\|\cdot(\|A_{22}\|+\alpha\|U_2\|\,\|V_2\|) = k < d_0\end{aligned} \tag{9}$$

By Lemma 2, we can obtain that $(A_{22}+\Delta A_{22}+B_2K_2)$ is a nonsingular matrix. Then by Lemma 1, the closed-loop system is impulse free.

Suppose that the storage function of system (1) is $\mathrm{V} = x^{\mathrm{T}}E^{\mathrm{T}}Px$. Then by Lemma 3, we can obtain that

$$\begin{aligned}&\dot{V} - r(u,y) < x^{\mathrm{T}}[(A+d_0BD^+C)^{\mathrm{T}}P + P^{\mathrm{T}}(A+\\&d_0BD^+C)+\varepsilon\alpha I + \tfrac{1}{\varepsilon}P^{\mathrm{T}}P - (C+d_0DD^+C)^{\mathrm{T}}Q(C+\\&d_0DD^+C) - d_0^2(D^+C)^{\mathrm{T}}RD^+C - d_0C^{\mathrm{T}}SD^+C-\\&d_0(D^+C)^{\mathrm{T}}S^{\mathrm{T}}C - d_0^2(D^+C)^{\mathrm{T}}D^{\mathrm{T}}SD^+C-\\&d_0^2(D^+C)^{\mathrm{T}}S^{\mathrm{T}}DD^+C]x\end{aligned} \tag{10}$$

If $D^{\mathrm{T}}S+S^{\mathrm{T}}D$ is a symmetric and nonsingular matrix, by Schur Complement[10] we can get that if (8) holds, then

$$\dot{V} < r(u(t),\ y(t))$$

In a word, the closed-loop system (7) is robustly dissipative and impulse free, and the controller we have get is a robust impulse dissipative state feedback controller. □

Theorem 2. If matrix $M=B_2D^+C_2$ is nonsingular, $D^{\mathrm{T}}S$ is a symmetric and nonsingular matrix, $\left\|D^{\mathrm{T}}S\right\| < \frac{1}{2}\|R\|$, and the following linear matrix inequalities hold

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \ge 0$$

$$\tilde{\Omega}=\begin{pmatrix}\Omega_{11} & d_0(DD^+C)^{\mathrm{T}} & (d_0D^+C)^{\mathrm{T}} & P^{\mathrm{T}}\\ d_0DD^+C & Q^{-1} & 0 & 0\\ d_0D^+C & 0 & (D^{\mathrm{T}}S+S^{\mathrm{T}}D+R)^{-1} & 0\\ P & 0 & 0 & -\varepsilon I\end{pmatrix}<0 \tag{11}$$

$$k = (\|A_{22}\| + \alpha\,\|U_2\|\,\|V_2\|)\left\|M^{-1}\right\| < d_0$$

then there exists robust impulse dissipative state feedback $u=Kx$ for system (1), where $K=d_0D^+C$, and Ω_{11} is refer to Theorem 1.

Proof. Refer to Lemma 2 and the proof of Theorem 1. □

4 Robust impulse dissipative output feedback controller

Consider the following uncertain singular system

$$\begin{cases} E\dot{x}(t) = (A+\Delta A)x(t) + Bu(t)\\ y(t)= Cx(t)\end{cases} \tag{12}$$

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \ge 0$$

$$\Omega = \begin{pmatrix}\Omega_{11} & d_0(DD^+C)^{\mathrm{T}} & (d_0D^+C)^{\mathrm{T}} & (d_0D^+C)^{\mathrm{T}} & P^{\mathrm{T}}\\ d_0DD^+C & Q^{-1} & 0 & 0 & 0\\ d_0D^+C & 0 & (D^{\mathrm{T}}S+S^{\mathrm{T}}D)^{-1} & 0 & 0\\ d_0D^+C & 0 & 0 & R^{-1} & 0\\ P & 0 & 0 & 0 & -\varepsilon I\end{pmatrix} < 0 \tag{8}$$

$$k = (\|A_{22}\| + \alpha\,\|U_2\|\,\|V_2\|)\left\|M^{-1}\right\| < d_0$$

Design the output feedback controller of system (12) to be $u = Fy$, $F \in \mathbf{R}^{m\times r}$, such that the closed-loop of system (12) is impulse-free and dissipative.

Without loss of generality, we suppose that B_2 is of row full rank and C_2 is of column full rank.

By the matrix theory we can obtain that there exist nonsingular matrices X, Y, Z and W such that

$$B_2 = X\begin{pmatrix} B_{21} \\ 0 \end{pmatrix} Y, \quad C_2 = Z\begin{pmatrix} C_{21} & 0 \end{pmatrix} W$$

where B_{21} and C_{21} are respectively nonsingular matrices.

Theorem 3. For uncertain singular system (12), the symbol is as the above, and $N = XW$ is a nonsingular matrix. If the following linear matrix inequalities hold

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \geq 0$$

$$\Xi = \begin{pmatrix} \Xi_{11} & d_1(\tilde{F}C)^{\mathrm{T}} & P^{\mathrm{T}} \\ d_1\tilde{F}C & R^{-1} & 0 \\ P & 0 & -\varepsilon I \end{pmatrix} < 0 \tag{13}$$

$$l = (\|A_{22}\| + \alpha\,\|U_2\|\,\|V_2\|)\cdot\left\|N^{-1}\right\| < d_1$$

where

$$\tilde{F} = Y^{-1}\begin{pmatrix} B_{21}^{-1} & 0 \end{pmatrix}\begin{pmatrix} C_{21}^{-1} \\ 0 \end{pmatrix} Z^{-1}$$

$$\begin{aligned}\Xi_{11}=&(A+d_1B\tilde{F}C)^{\mathrm{T}}P + P^{\mathrm{T}}(A+d_1B\tilde{F}C) + \varepsilon\alpha I - \\ &C^{\mathrm{T}}QC - d_1C^{\mathrm{T}}S\tilde{F}C - d_1C^{\mathrm{T}}\tilde{F}^{\mathrm{T}}S^{\mathrm{T}}C\end{aligned} \tag{14}$$

then there exists robust impulse dissipative output feedback controller $u = Fy$ for system (12), where $F = d_1\tilde{F}$.

Proof. Refer to the proof of Theorem 1. □

5 Example

Example. The parameters in system (1) are as follows

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} -12 & 1 & 2 \\ 0.5 & -9 & 1 \\ -10 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} -8 & 0 \\ 1 & -0.9 \\ 0 & 0.5 \end{pmatrix}, \quad C = \begin{pmatrix} -0.2 & 0 & 1 \\ 0 & -0.3 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} -3 & 1 \\ 1 & -2 \end{pmatrix}, \ \mathit{\Delta} A(t) = 0.09\sin t * I, \ \alpha = 0.1$$

$$Q = R = -I, \quad S = \begin{pmatrix} 2 & 1 \\ 1 & 1.5 \end{pmatrix}$$

Using Matlab, we can have feasible solutions for inequality (8)

$$d_0 = 75.9367 > k = 75.8367, \quad \varepsilon = 25632$$

$$P = \begin{pmatrix} 144.6 & 0 & 0 \\ 2230.6 & -668.9 & 1705.6 \\ 2099.5 & -3800.4 & -1170.3 \end{pmatrix}$$

Therefore, the state feedback controller of system (1) is

$$u = \begin{pmatrix} 6.0785 & 4.5589 & -30.3927 \\ 3.0393 & 13.6767 & -15.1963 \end{pmatrix} x$$

So we have that, for any t, the closed-loop system (7) is impulse free and dissipative, therefore the state feedback controller solved is robust impulse dissipative.

6 Conclusion

The paper discussed the robust impulse dissipative control problem for singular systems with uncertainties by the method of LMI, and obtained the robust impulse dissipative state feedback controller and the output feedback controller. The method also fits the system with uncertain control matrix and uncertain output matrix, but the result will be more complex. The nonlinear systems exist widely, therefore robust dissipative control and impulse control are our future work.

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