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The Influence of Imperfections Critical Loads of Pitchfork

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Abstract: By means of the theory of universal unfolding, the influence of multiimperfections upon the critical loads of pitchfork structures in engineering is analysed. The estimation formula for lower bounds on the increments of the critical loads that are caused by imperfections of the structures is given, and a simple and available numerical method for computing lower bounds is described.

Key words: imperfection; critical load; estimation of lower bound; universal unfolding; pitchfork

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1 Introduction

There are inevitabe imperfections in real engineering structures, and some important tests[4,12] have shown that the presence of imperfections of structures can decrease the load-bearing capacity. So the effects of imperfections on critical loads must be considered in designs according to the standard of stability. We describe the still equilibrum state of the perfect engineering structure and the structure with imperfections respectively as follows:

$$F(x,\lambda) = 0, \tag{1}$$

$$H(x,\lambda,\beta) = 0, \tag{2}$$

where $F: R^n \times R \to R^n$, $H: R^n \times R \times R^l \to R^n$ are smooth, $x \in R^n$ is a state variable vector, $\lambda \in R$ is the load bearing parameter, $\beta \in R^l$ is the imperfection parameter vector. Obviously, we have

$$H(x,\lambda,0) = F(x,\lambda), \tag{3}$$

Suppose that λ_p is the critical load of the perfect structure, and (x_p, λ_p) is the singular point of (1). Let $\lambda^*(\beta)$ be the critical load of structure (2) with imperfections. The important problem concerning the effects of imperfection upon critical load is studying the change law of $\lambda^*(\beta) - \lambda_p$, we are especialy interested in the estimation of the lower bound of the increments of critical loads as $\| \beta$

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$\| \leqslant \varepsilon$. Using imperfection sensitivity as $l = 1$, W. T. KOITER[6] gave a perfect explanation for most rapidly decline of critical load of structure with imperfections. For some simple singular point as $l > 1$, many authors (e.g. [5], [10]) have studied the effects of imperfections upon the loadbearing capacity by a given mode of the imperfection. Because of the randomness of the imperfections modes in real problem, a statistical method was presented to study the effects of random modes of imperfections by some authors (e.g. [7],[2]). Because statistical methods is expensive in real use, this methods sometimes they are not adopted in engineering designing. Another method of studying the randomness of imperfections mode is to determine the worst direction of the imperfection vector which causes the maximum change (decrease) of the load—bearing capacity. This method is simple and available. Though the critical load decreases most rapidly along the critical imperfection direction in perfect structure, for imperfection structures of strongly nonlinear problems the critical load does not decrease most rapidly along this direction. Thus, there are some problems needed to be further reserched for stronger nonlinear systems.

2 The case of regular unfolding

We discuss scale problem in this section and the next section. Suppose that $f(x,\lambda): R \times R \to R$ is a smooth function in a neighborhood of (0,0) , and (0,0) is a pitchfork point of the equation

$$f(x,\lambda) = 0. \tag{4}$$

On the basis of recognition conditions we refer to [3], $f(x,\lambda)$ has the form

$$f(x,\lambda) = ax^3 + bx\lambda + \theta_{40}(x,\lambda)x^4 + \theta_{21}(x,\lambda)x^2\lambda + \theta_{02}(x,\lambda)\lambda^2, \tag{5}$$

in a neighborhood of (0,0) , where a,b are constants and $ab < 0$, $\theta_{ij}(x,\lambda)$ is a smooth function. We call the item of the form $\theta_{40}(x,\lambda)x^4 + \theta_{21}(x,\lambda)x^2\lambda + \theta_{02}(x,\lambda)\lambda^2$ the high order item of $f(x,\lambda)$. Because of the simple transformation $x = (-\frac{b}{a})^{\frac{1}{2}}\bar{x}$, without loss of generality, we can always assume that $a = 1$, $b = -1$. Furthermore, using the theorem of universal unfolding in [3], we can get

$$g(x,\lambda,\alpha_1,\alpha_2) := f(x,\lambda) + \alpha_1 + \alpha_2 x^2, \tag{6}$$

which is a universal unfolding of $f(x,\lambda)$. In this section, we consider the unfolding problem of g perturbed by high order item with the following form (simply called the regular unfolding problem)

$$h(x,\lambda,\alpha_1,\alpha_2,\beta) := x^3 - x\lambda + \psi_{40}x^4 + \psi_{21}x^2\lambda + \psi_{02}\lambda^2 + \alpha_1 + \alpha_2 x^2 = 0, \tag{7}$$

where $\psi_{ij}: R \times R \times R^l \to R$ are smooth functions such that

$$\psi_{ij}(x,\lambda,0) = \theta_{ij}(x,\lambda). \tag{8}$$

In order to get the critical load of h , consider a system of equations satisfied by the singular point

$$h(x,\lambda,\alpha_1,\alpha_2,\beta) = x^3 - x\lambda + \psi_{40}x^4 + \psi_{21}x^2\lambda + \psi_{02}\lambda^2 + \alpha_1 + \alpha_2 x^2 = 0, \tag{9}$$

$$h_x(x,\lambda,\alpha_1,\alpha_2,\beta) = 3x^2 - \lambda + 2\alpha_2 x + \frac{\partial}{\partial x}(\psi_{40}x^4 + \psi_{21}x^2\lambda + \psi_{02}\lambda^2) = 0. \tag{10}$$

Because $h_x(x,\lambda,\alpha_1,\alpha_2,\beta)$ does not include α_1 , and

$$h_x(0,0,\alpha_1,0,\beta) = 0, \qquad h_{x\lambda}(0,0,\alpha_1,0,\beta) = -1 \neq 0,$$

by applying the implicit function theorem and (10), we can get a smooth function $\lambda = \bar{\lambda}(x,\alpha_2,\beta)$ with $\bar{\lambda}(0,0,\beta) = 0$ in a neighborhood of (0,0) . Expanding $\bar{\lambda}(x,\alpha_2,\beta)$ at $(x,\alpha_2) = (0,0)$, we get

$$\bar{\lambda}(x,\alpha_2,\beta) := 3x^2 + 2\alpha_2 x + \gamma_{30}x^3 + \gamma_{21}x^2\alpha_2 + \gamma_{12}x\alpha_2^2 + \gamma_{03}\alpha_2^3, \tag{11}$$

where $\gamma_{ij} = \gamma_{ij}(x, \alpha_2, \beta)$ are smooth functions. Substituing (11) into (9), we can obtain the equation satisfied by the x-coordinate at the singular point of h

$$\varphi(x, \alpha_1, \alpha_2, \beta) := 2x^3 + \alpha_2 x^2 - \alpha_1 + \bar{\psi}_{40}x^4 + \bar{\psi}_{31}x^3\alpha_2 + \bar{\psi}_{22}x^2\alpha_2^2 + \bar{\psi}_{13}x\alpha_2^3 = 0, \tag{12}$$

where $\bar{\psi}_{ij} = \psi_{ij}(x, \alpha_2, \beta)$ are smooth functions. Now using the method in general bifurcation theory[11] to discuss the solution of (12), we have the priori estimation of (12) as follows:

Lemma 2.1 There exist a neighborhood V of $(x, \alpha_1, \alpha_2, \beta)$ at the origin and a constant $\kappa > 0$, such that every solution $(x, \alpha_1, \alpha_2, \beta)$ of (12) in V satisfies

$$|x| \leqslant \kappa(|\alpha_1|^{\frac{1}{3}} + |\alpha_2|). \tag{13}$$

Proof Suppose that the conclusion does not hold. Then there exists $(x_n, \alpha_{1n}, \alpha_{2n}, \beta_n)$ which satisfy $(x_n, \alpha_{1n}, \alpha_{2n}, \beta_n) \to 0$ as $n \to \infty$, and $\frac{|\alpha_{1n}|^{\frac{1}{3}}}{|x_n|}$, $\frac{|\alpha_{2n}|}{|x_n|} \to 0$ as $n \to \infty$. Dividing the two sides of (12) by x_n^3, we have

$$0 = \frac{\varphi(x_n, \alpha_{1n}, \alpha_{2n}, \beta_n)}{x_n^3} = 2 + O\left(|x_n| + \left|\frac{\alpha_{2n}}{x_n}\right| + \left|\frac{\alpha_{1n}}{x_n^3}\right|\right) \to 2.$$

This contradiction shows that the conclusion of Lemma 2.1 is true.

From (13) we can introduce the relation between α_1 and α_2

$$\alpha_2 = k\alpha_1^{\frac{1}{3}}, \text{ or } \alpha_1 = k'\alpha_2^3. \tag{14}$$

Let $\bar{\Omega}_1 := \{(\alpha_1, \alpha_2): \alpha_2 = k\alpha_1^{\frac{1}{3}}, |k| \leqslant d_0, |\alpha_1| \leqslant \delta\}$ and $\bar{\Omega}_2 := \{(\alpha_1, \alpha_2): \alpha_1 = k'\alpha_2^3, |k'| \leqslant d'_0, |\alpha_2| \leqslant \delta\}$, $\bar{\Omega}_1 \cup \bar{\Omega}_2$ will cover a neighborhood of the origin in the $\alpha_1 - \alpha_2$ plane as $d_0 > (d'_0)^{-\frac{1}{3}}$, where δ is a positive constant. Consider the element $(\alpha_1, \alpha_2) \in \bar{\Omega}_1$. Suppose

$$\alpha_2 = k\alpha_1^{\frac{1}{3}}, \ -d_0 \leqslant k \leqslant d_0. \tag{15}$$

Let $t = \alpha_1^{\frac{1}{3}}$. Substituting (15) into (12). Then

$$\varphi_1(x, t, \beta, k) := 2x^3 + kx^2t - t^3 + O(|x|^4 + |x^3t| + |x^2t^2| + |xt^3|) = 0. \tag{16}$$

In terms of the method of Newton polygon and the tightness of $[-d_0, d_0]$, it is easy to obtain the small solution of (16) which has the expansion

$$x(t, \beta, k) = y(k)t(1 + o(1)), \tag{17}$$

where $o(1)$ is a uniform infinitesimal with β in the neighborhood of the origin and $|k| \leqslant d_0$ as $t \to 0$, and $y(k)$ is the real root of equation

$$2y^3 + ky^2 - 1 = 0. \tag{18}$$

(18) has three real roots as $k > 3$; two real roots (one of these roots is a double root) as $k = 3$; and only one real root as $k < 3$. We denote these real roots by $y(k)$. Substituting (17) into (11), we may get the λ-coordinate of the singular point of h:

$$\lambda(\alpha_1, \alpha_2, \beta) = \xi(k)(\alpha_1^{\frac{2}{3}} + \alpha_2^2)(1 + o(1)), \tag{19}$$

where

$$\xi(k) = (3y^2(k) + 2ky(k))(1 + k^2)^{-1}, \tag{20}$$

$$k = \begin{cases} 0, & \alpha_1 = 0, \\ \alpha_2\alpha_1^{-\frac{1}{3}}, & \alpha_1 \neq 0. \end{cases} \tag{21}$$

Using numerical methods, we get the minimum value $\xi^* = -0.3$ of $\xi(k)$ as $|k| < \infty$. For the regular problem (7), we have

$$\lambda^*(\alpha_1, \alpha_2, \beta) \geqslant \xi^*(\alpha_1^{\frac{2}{3}} + \alpha_2^2)(1 + o(1)). \tag{22}$$

The same analysis for the set $\bar{\Omega}_2$ can also lead to the estimation form (22).

3 Arbitrarily scale unfolding

Suppose $h(x,\lambda,\beta)$ is an arbitrary l-parameter unfolding of $f(x,\lambda)$, where f is defined by (5), $a=1$, $b=-1$. We may get the equation by expanding h:

$$\begin{aligned} h(x,\lambda,\beta) = x^3 - x\lambda + \{\varphi_{00} + \varphi_{10}x + \varphi_{01}\lambda + \varphi_{20}x^2 + \varphi_{11}x\lambda + \varphi_{30}x^3\} + \\ \{\psi_{40}(x,\lambda,\beta)x^4 + \psi_{21}(x,\lambda,\beta)x^2\lambda + \psi_{02}(x,\lambda,\beta)\lambda^2\} = 0, \end{aligned} \tag{23}$$

where φ_{ij} are smooth functions with β, φ_{ij} and ψ_{ij} satisfy

$$\varphi_{ij}(0) = 0, \quad \psi_{ij}(x,\lambda,0) = \theta_{ij}(x,\lambda).$$

By primary method, it is easy to get

$$X(x,\beta) := (1+\varphi_{30})^{\frac{1}{3}}x - \varphi_{01}(1+\varphi_{30})^{\frac{1}{3}}(1-\varphi_{11})^{-1}, \tag{24}$$

$$\begin{aligned} \Lambda(\lambda,\beta) := (1-\varphi_{11})(1+\varphi_{30})^{-\frac{1}{3}}\lambda - 3\varphi_{01}^2(1+\varphi_{20})^{\frac{2}{3}}(1-\varphi_{11})^{-2} - \\ 2\varphi_{01}\varphi_{20}(1-\varphi_{11})^{-1}(1+\varphi_{30})^{-\frac{1}{3}} - \varphi_{10}(1+\varphi_{30})^{-\frac{1}{3}}, \end{aligned} \tag{25}$$

$$\alpha_1(\beta) := \varphi_{00} + \varphi_{10}\varphi_{01}(1-\varphi_{11})^{-1} + \varphi_{01}^2\varphi_{20}(1-\varphi_{11})^{-2} + \varphi_{01}^3(1+\varphi_{30})(1-\varphi_{11})^{-3}, \tag{26}$$

$$\alpha_2(\beta) := \varphi_{20}(1+\varphi_{30})^{-\frac{2}{3}} + 3\varphi_{01}(1+\varphi_{30})^{\frac{1}{3}}(1-\varphi_{11})^{-1}, \tag{27}$$

such that

$$x^3 - x\lambda + \varphi_{00} + \varphi_{10}x + \varphi_{01}\lambda + \varphi_{20}x^2 + \varphi_{11}x\lambda + \varphi_{30}x^3 = X^3 - \Lambda X + \alpha_1(\beta) + \alpha_2(\beta)X^2. \tag{28}$$

Obviously, we have from (24)～(27)

$$X(x,0) = x, \quad \Lambda(\lambda,0) = \lambda, \quad \alpha_1(0) = \alpha_2(0) = 0, \tag{29}$$

and these show that the left of (28) factors through $x^3 - \lambda x + \alpha_1 + \alpha_2 x^2$, (24)～(27) are their factor transformation. Substituting (28) into (23), we obtain

$$h(x,\lambda,\beta) = X^3 - \Lambda X + \alpha_1 + \alpha_2 X^2 + \psi_{40}x^4 + \psi_{21}x^2\lambda + \psi_{02}\lambda^2 = 0. \tag{30}$$

Now let

$$z = X(x,\beta), \quad \mu = \Lambda(\lambda,\beta), \tag{31}$$

and its inverse transformation be denoted by

$$x = Z(z,\beta), \quad \lambda = M(\mu,\beta). \tag{32}$$

Now (30) can be rewriten as

$$\bar{h}(z,\mu,\beta) := z^3 - \mu z + \alpha_1 + \alpha_2 z^2 + \bar{\psi}_{40}z^4 + \bar{\psi}_{21}z^2\mu + \bar{\psi}_{02}\mu^2 = 0, \tag{33}$$

where $\bar{\psi}_{ij}$ are smooth functions of (z,μ,β). If (z,μ,β) is a singular point of (33) in a neighborhood of origin, then $(Z(z,\beta), M(\mu,\beta),\beta)$ is a singular point of (23) in a neighborhood of the origin; and the converse proposition is also true. Suppose that $(\bar{z},\bar{\mu},\beta)$ is a singular point of (33). Then $(\bar{x},\bar{\lambda},\beta)$ is a correspondingly singular point of (23). From $M(\bar{\mu},0) = \bar{\mu}$, we have

$$\begin{aligned} \bar{\lambda} = M(\bar{\mu},\beta) = M(\bar{\mu},0) + M_\beta(\bar{\mu},0)\beta + \frac{1}{2}\beta^T M_{\beta\beta}(\bar{\mu},0)\beta + O(\|\beta\|^3) = \\ \bar{\mu}(1+O(\|\beta\|)) + M_\beta(0,0)\beta + \frac{1}{2}\beta^T M_{\beta\beta}(0,0)\beta + O(\|\beta\|^3). \end{aligned} \tag{34}$$

Furthermore, from (25) and $\Lambda(M(\mu,\beta),\beta) \equiv \mu$, we can obtain

$$M_\beta(0,0)\beta = (\varphi_{10})_\beta^0\beta, \tag{35}$$

$$\frac{1}{2}\beta^T M_{\beta\beta}(0,0)\beta = [(\varphi_{10})_\beta^0\beta][(\varphi_{11})_\beta^0\beta] + 3[(\varphi_{01})_\beta^0\beta]^2 + 2[(\varphi_{01})_\beta^0\beta][(\varphi_{20})_\beta^0\beta] + \frac{1}{2}\beta^T(\varphi_{10})_{\beta\beta}^0\beta. \tag{36}$$

where the superscript 0 represents the function value at $\beta = 0$. Because $(\bar{x}, \bar{\mu}, \beta)$ is a singular point of (33), we can get by (22)

$$\bar{\mu} \geqslant \xi^*(\alpha_1^{\frac{2}{3}}(\beta) + \alpha_2^2(\beta))(1 + o(1)). \tag{37}$$

If we approximately regard $(1 + o(1))$ as 1 and omit $O(\|\beta\|^3)$, then it is easy to obtain the estimation of lower bound of the critical loads for arbitrary unfolding (23) by using (34)~(37), (26), (27):

$$\begin{aligned} \lambda^*(\beta) \geqslant & -0.3[(\varphi_{00})_\beta^0\beta]^{\frac{2}{3}} + (\varphi_{10})_\beta^0\beta + \{0.3[(\varphi_{01})_\beta^0\beta]^2 + \\ & 0.2[(\varphi_{01})_\beta^0\beta][(\varphi_{20})_\beta^0\beta] - 0.3[(\varphi_{20})_\beta^0\beta]^2 + \frac{1}{2}\beta^T(\varphi_{10})_{\beta\beta}^0\beta\}, \end{aligned} \tag{38}$$

where

$$\begin{cases} (\varphi_{00})_\beta^0 = h_\beta(0,0,0), \quad (\varphi_{10})_\beta^0 = h_{x\beta}(0,0,0), \quad (\varphi_{01})_\beta^0 = h_{\lambda\beta}(0,0,0), \\ (\varphi_{20})_\beta^0 = \frac{1}{2}h_{xx\beta}(0,0,0), \quad (\varphi_{10})_{\beta\beta}^0 = h_{x\beta\beta}(0,0,0). \end{cases} \tag{39}$$

If we further omit $O(\|\beta\|^2)$ in (38), then we have a simple estimation of the lower bound:

$$\lambda^*(\beta) \geqslant -0.3[h_\beta(0,0,0)\beta]^{\frac{2}{3}} + h_{x\beta}(0,0,0)\beta. \tag{40}$$

When we omit $O(\|\beta\|^2)$, (40) shows that two projections of β on the directions $h_\beta(0,0,0)$ and $h_{x\beta}(0,0,0)$ are the primary items in the influences of imperfection parameters $\beta = (\beta_1, \beta_2, \cdots, \beta_l)^T$ upon critical loads. In the reduction mentioned above, we suppose that $a = 1, b = -1$ in (5). For the general case, (38) (40) can be respectively modified as

$$\begin{aligned} \lambda^*(\beta) \geqslant & \ 0.3a^{\frac{1}{3}}b^{-1}[(\varphi_{00})_\beta^0\beta]^{\frac{2}{3}} - b^{-1}(\varphi_{10})_\beta^0\beta + \{-0.3ab^{-3}[(\varphi_{01})_\beta^0\beta]^2 + \\ & 0.2b^{-2}[(\varphi_{01})_\beta^0\beta][(\varphi_{20})_\beta^0\beta] + 0.3a^{-1}b^{-1}[(\varphi_{20})_\beta^0\beta]^2 + \frac{1}{2}b^{-2}\beta^T(\varphi_{10})_{\beta\beta}^0\beta\}, \end{aligned} \tag{41}$$

$$\lambda^*(\beta) \geqslant 0.3a^{\frac{1}{3}}b^{-1}[h_\beta(0,0,0)\beta]^{\frac{2}{3}} - b^{-1}h_{x\beta}(0,0,0)\beta, \tag{42}$$

where

$$a = \frac{1}{6}h_{xxx}(0,0,0), \quad b = h_{x\lambda}(0,0,0). \tag{43}$$

Example In figure 1 we illustrate a simple physical system which exhibits a pitchfork bifurcation. The system consists of two rigid rods of unit length connected by pins which permit rotation in a plane, it is subjected to a compressive force λ which is resisted by a torsional spring of unit strength. The state of the system is described by the angle x measuring the deviation of the rods from the horizontal. One natural perturbation to consider is a small vertical force ε applied to the pin. This force models the weight of the structure. Another such perturbation comes from imagining that the torsional spring is slightly asymmetric, exerting zero torque when $x = \delta$ rather than when $x = 0$. The potential function with the presence of these two perturbations is

$$V(x,\lambda,\varepsilon,\delta) = (x - \delta)^2/2 + 2\lambda(\cos(x) - 1) + \varepsilon\sin(x),$$

and the equilibrium equation is

$$h(x,\lambda,\varepsilon,\delta) = x - \delta - 2\lambda\sin(x) + \varepsilon\cos(x).$$

Expanding $h(x,\lambda,\varepsilon,\delta)$ with x, λ at $(x,\lambda) = (0,0)$, we have

$$\begin{aligned} h(x,\lambda,\varepsilon,\delta) = & \ ax^3 + bx\lambda + \varphi_{00}(\varepsilon,\delta) + \varphi_{10}(\varepsilon,\delta)x + \varphi_{01}(\varepsilon,\delta)\lambda + \\ & \varphi_{11}(\varepsilon,\delta)x\lambda + \varphi_{20}(\varepsilon,\delta)x^2 + \varphi_{30}(\varepsilon,\delta)x^3 + h.o.t., \end{aligned}$$

where $a = 1/6, b = -2, \varphi_{00} = \varepsilon - \delta, \varphi_{10} = 0, \varphi_{20} = -1/2, \varphi_{10} = \varphi_{11} = \varphi_{30} = 0$, and h. o. t. represents higher-order terms. Now according to our method mentioned in this paper, we can obtain

$$\alpha_1(\varepsilon,\delta) = \frac{\sqrt{3}}{12}(\varepsilon - \delta), \quad \alpha_2(\varepsilon,\delta) = -\frac{\sqrt{3}}{2}\varepsilon, \quad M(\mu,\varepsilon,\delta) = \mu.$$

So we immediately get the estimation formula for the increments of the critical loads that are caused by imperfections of the structure

$$\lambda^*(\epsilon,\delta) \geqslant \frac{1}{2} + \xi^*\left[\frac{1}{2\sqrt[3]{6}}(\epsilon-\delta)^{2/3} + \frac{3}{4}\epsilon^2\right] + h.o.t.,$$

here $\xi^* = -0.3$.

Remark Because the imperfection parameters ϵ,δ are independent, we do not omit the term $\frac{3}{4}\epsilon^2$ in this example.

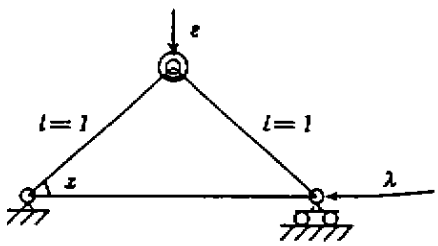

Figure 1

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缺陷对音叉型结构临界载荷的影响

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摘要：利用普适开折定理研究了多重缺陷对音叉型结构临界载荷的影响，给出了由缺陷引起的结构临界载荷改变量的下界估计公式及有效的数值计算公式.

关键词：缺陷；临界载荷；普适开折；下界估计；音叉