短文

广义正交多项式用于时变延时线性系统的参数辨识

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摘　　要

应用广义正交多项式（GOP）的展开式估计时变延时线性系统的参数. 其基本思想是状态函数和控制函数分别用有限多项广义正交多项式表示，利用 GOP 的运算矩阵，将时变延时微分方程转化为用展开系数表示的线性方程组，通过输入输出数据，参数能够辨识.

关键词：广义正交多项式，运算矩阵，延时，参数辨识.

1 引言

近年来，国内一些学者利用正交函数和正交多项式对线性系统进行分析 和 参 数 辨识[1,2]，得到了许多满意的结果. 然而，在国外一种"广义正交多项式"被提出[3,4]，并引起了广泛地注意. 它能表示各种个别的正交多项式，其特有的运算矩阵能包含所有正交多项式及非正交台劳级数的性质. 本文就是利用广义正交多项式对时变延时线性系统进行参数辨识，其优点是计算方法简单，节省计算时间，并能得到较精确的结果.

2 GOP 和它的运算矩阵

正交多项式 $\phi_i(t)$ 被定义为

$$\phi_i(t) = \sum_{j=0}^{i} f_{ij} t^j. \tag{1}$$

类似地，t^i 可表示为

$$t^i = \sum_{j=0}^{i} g_{ij} \phi_j(t). \tag{2}$$

写方程(1)和(2)为向量形式:

$$\underline{\phi} = [\phi_0(t)\phi_1(t)\cdots\phi_{m-1}(t)]^T = \underline{F}\underline{\theta}, \tag{3}$$

$$\underline{\theta} = [1\,t\ t^2\cdots t^{m-1}]^T = \underline{G}\underline{\phi} = \underline{G}\underline{F}\underline{\theta}. \tag{4}$$

其中 $\underline{F} = (f_{ij})_{m\times m}$ 和 $\underline{G} = (g_{ij})_{m\times m}$ 是展开系数的下三角矩阵，且有

$$\underline{G}\underline{F} = \underline{I}, \quad g_{ii} = \frac{1}{f_{ii}}, i = 0,1,2,\cdots,m-1. \tag{5}$$

E 可从下列递推公式得到：

$$\phi_{i+1}(t) = (a_i t + b_i)\phi_i(t) - c_i\phi_{i-1}(t). \tag{6}$$

在方程(6)中，f_{10} 和 f_{11} 已知，值 a_i、b_i 和 c_i 依赖于选取的正交多项式.

由文献[5]得

$$\int_0^t \underline{\theta}\mathrm{d}t = \underline{G}\int_0^t \underline{\phi}\mathrm{d}t \approx \underline{G}\underline{H}\underline{\phi} = \underline{G}\underline{H}\underline{F}\underline{\theta} = \underline{E}\underline{\theta}. \tag{7}$$

其中

$$\underline{E} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \dfrac{1}{2} & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \dfrac{1}{m-1} \\ e_0 & e_1 & e_2 & \cdots & e_{m-1} \end{bmatrix}, \tag{8}$$

$$e_i = -\frac{f_{mi}}{mf_{mm}}, i = 0,1,2,\cdots,m-1. \tag{9}$$

令 $e_i = 0, i = 0,1,\cdots,m-1$，方程(7)的近似可变为 Taylor 级数的近似.

当 $i \geqslant m$ 时，t^i 能被近似地表示为

$$t^i \approx \sum_{j=0}^{n-1} g_{ii}\phi_j \approx \sum_{j=0}^{m-1}\sum_{l=0}^{m-1} g_{ii}f_{il}t^l = \sum_{l=0}^{m-1}\left(\sum_{j=l}^{m-1} g_{ii}f_{il}\right)t^l. \tag{10}$$

令 $i = m + k$，则式(10)可写成

$$t^{m+k} \approx \sum_{l=0}^{m-1} r_{kl}t^l, \quad r_{kl} \triangleq \sum_{j=l}^{m-1} g_{m+k,i}f_{il}. \tag{11}$$

$$t^k\underline{\theta} = [t^k t^{k+1} t^{k+2}\cdots t^{k+m-1}]^T = \underline{R}_k\underline{\theta}. \tag{12}$$

其中

$$\underline{R}_k = \begin{bmatrix} \underline{0}_{(m-k)\times k} & \vdots & \underline{I}_{(m-k)\times(m-k)} \\ \cdots & \cdots & \cdots \\ & \underline{M}_{k\times m} & \end{bmatrix}, \underline{M}_{k\times m} = (r_{ij})_{k\times m}(i = 0,1,2,\cdots,$$

$$k-1, j = 0,1,2,\cdots,m-1). \tag{13}$$

对于任意参数 $\tau, \underline{\theta}(t-\tau)$ 被表示为

$$\underline{\theta}(t-\tau) = [1(t-\tau)(t-\tau)^2\cdots(t-\tau)^{m-1}]^T = \underline{\omega}\underline{\theta}. \tag{14}$$

其中 $\underline{\omega} = (\omega_{ij})_{m\times m}, \omega_{ii} = 1, \omega_{i0} = (-\tau)^i, i = 0,1,2,\cdots,m-1;$

$$\omega_{i+1,j} = \omega_{i,j-1} + (-\tau)\omega_{ij}, j = 1,2,\cdots,i; \omega_{ij} = 0, j > i.$$

3 参数辨识

考虑下列时变延时微分方程：

$$\dot{x}(t) = \underline{A}(t)x(t) + \underline{B}(t)u(t) + \underline{C}(t)x(t - \tau) + \underline{D}(t)u(t - \tau), \tag{15}$$

$$x(0) = \underline{x}_0, x(t) = \underline{0}, t < 0.$$

其中 $x(t) = [x_1(t)x_2(t)\cdots x_n(t)]^T$; $u(t) = [u_1(t)u_2(t)\cdots u_r(t)]^T$; τ 是延时常数; $\underline{A}(t)_{n\times n}$, $\underline{B}(t)_{n\times r}$, $\underline{C}(t)_{n\times n}$, $\underline{D}(t)_{n\times r}$ 是时变系数矩阵; $\underline{A}(t)$, $\underline{B}(t)$, $\underline{C}(t)$, $\underline{D}(t)$, $x(t)$ 和 $u(t)$ 的第 k 行元素分别用 GOP 级数近似表示为

$$a_{kj}(t) = \sum_{l=0}^{m-1} a_{kjl}t^l, c_{kj}(t) = \sum_{l=0}^{m-1} c_{kjl}t^l, j = 1, 2, \cdots, n, \tag{16}$$

$$b_{kj}(t) = \sum_{l=0}^{m-1} b_{kjl}t^l, d_{kj}(t) = \sum_{l=0}^{m-1} d_{kjl}t^l \quad j = 1, 2, \cdots, r, \tag{17}$$

$$x_k(t) = \sum_{l=0}^{m-1} x_{kl}t^l = \underline{X}_k^T\theta, \underline{X}_k = [x_{k0}x_{k1}\cdots x_{km-1}]^T, \tag{18}$$

$$u_k(t) = \sum_{l=0}^{m-1} u_{kl}t^l = \underline{U}_k^T\theta, \underline{U}_k = [u_{k0}u_{k1}\cdots u_{km-1}]^T. \tag{19}$$

将方程(15)从 $t = 0$ 到 $t = t$ 对 t 积分, 得

$$x(t) - x(0) = \int_0^t \underline{A}(t)x(t)\mathrm{d}t + \int_0^t \underline{B}(t)u(t)\mathrm{d}t + \int_0^t \underline{C}(t)x(t - \tau)\mathrm{d}t$$

$$+ \int_0^t \underline{D}(t)u(t - \tau)\mathrm{d}t. \tag{20}$$

考虑式(20)的第 k 个子方程, 利用式(16—19), (14)和(12), $\underline{A}(t)x(t)$ 和 $\underline{C}(t)x(t - \tau)$ 的第 k 行元素表示为

$$\sum_{j=1}^n a_{kj}(t)x_j(t) = \sum_{j=1}^n \left(\sum_{l=0}^{m-1} a_{kjl}t^l\right)\underline{X}_j^T\theta = \sum_{j=1}^n \left(\sum_{l=0}^{m-1} a_{kjl}\underline{X}_j^T R_l\theta\right)$$

$$= \sum_{j=1}^n [a_{kj0}a_{kj1}\cdots a_{kjm-1}]\underline{Y}_j\theta = \underline{A}_k\underline{Y}\theta, \tag{21}$$

$$\sum_{j=1}^n c_{kj}(t)x_j(t - \tau) = \sum_{j=1}^n \left(\sum_{l=0}^{m-1} c_{kjl}t^l\right)\underline{X}_j^T\theta(t - \tau) = \sum_{j=1}^n \left(\sum_{l=0}^{m-1} c_{kjl}t^l\underline{X}_j^T\omega\theta\right)$$

$$= \sum_{j=1}^n \left(\sum_{l=0}^{m-1} c_{kjl}\underline{X}_j^T\omega R_l\theta\right) = \sum_{j=1}^n [c_{kj0}c_{kj1}\cdots c_{kjm-1}]\underline{Z}_j\theta = \underline{C}_k\underline{Z}\theta. \tag{22}$$

其中

$$\underline{A}_k = [a_{k10}\cdots a_{k1m-1}a_{k20}\cdots a_{k2m-1}\cdots a_{kn0}\cdots a_{knm-1}],$$

$$\underline{Y} = [\underline{Y}_1^T\underline{Y}_2^T\cdots \underline{Y}_n^T]^T, \underline{Y}_j = [\underline{X}_j^T R_0 \ \underline{X}_j^T R_1\cdots \underline{X}_j^T R_{m-1}]^T,$$

$$\underline{C}_k = [c_{k10}\cdots c_{k1m-1}c_{k20}\cdots c_{k2m-1}\cdots c_{kn0}\cdots c_{knm-1}],$$

$$\underline{Z} = [\underline{Z}_1^T\underline{Z}_2^T\cdots \underline{Z}_n^T]^T, \underline{Z}_j = [\underline{X}_j^T\omega R_0 \ \underline{X}_j^T\omega R_1\cdots \underline{X}_j^T\omega R_{m-1}]^T.$$

类似地, 可得

$$\sum_{j=1}^r b_{kj}(t)u_j(t) = \underline{B}_k\underline{P}\theta, \tag{23}$$

$$\sum_{j=1}^r d_{kj}(t)u_j(t - \tau) = \underline{D}_k\underline{Q}\theta, \tag{24}$$

$$\underline{B}_k = [b_{k10} \cdots b_{k1m-1} b_{k20} \cdots b_{k2m-1} \cdots b_{kr0} \cdots b_{krm-1}],$$

$$\underline{P} = [[\underline{U}_1^T \underline{R}_0 \cdots \underline{U}_1^T \underline{R}_{m-1}][\underline{U}_2^T \underline{R}_0 \cdots \underline{U}_2^T \underline{R}_{m-1}] \cdots [\underline{U}_r^T \underline{R}_0 \cdots \underline{U}_r^T \underline{R}_{m-1}]]^T,$$

$$\underline{D}_k = [d_{k10} \cdots d_{k1m-1} d_{k20} \cdots d_{k2m-1} \cdots d_{kr0} \cdots d_{krm-1}],$$

$$\underline{Q} = [[\underline{U}_1^T \omega \underline{R}_0 \cdots \underline{U}_1^T \omega \underline{R}_{m-1}][\underline{U}_2^T \omega \underline{R}_0 \cdots \underline{U}_2^T \omega \underline{R}_{m-1}] \cdots [\underline{U}_r^T \omega \underline{R}_0 \cdots \underline{U}_r^T \omega \underline{R}_{m-1}]]^T.$$

将初始条件及方程(18),(21)-(24)代入方程(20)的第 k 个子方程,并利用方程(7),得

$$\underline{X}_k^T - \underline{X}_k^T(0) = \underline{A}_k \underline{Y} \underline{E} + \underline{B}_k \underline{P} \underline{E} + \underline{C}_k \underline{Z} \underline{E} + \underline{D}_k \underline{Q} \underline{E}, \underline{X}_k^T(0) = [x_k(0) 0 \cdots 0]. \tag{25}$$

定义参数向量 $\underline{s}_k$,已知向量 $\underline{v}_k$ 和 $\underline{w}_k$ 分别为

$$\underline{s}_k = [\underline{A}_k \underline{B}_k \underline{C}_k \underline{D}_k]^T, \tag{26}$$

$$\underline{v}_k = [(\underline{Y}\underline{E})^T (\underline{P}\underline{E})^T (\underline{Z}\underline{E}^T)(\underline{Q}\underline{E})^T], \tag{27}$$

$$\underline{w}_k^T = \underline{X}_k^T - \underline{X}_k^T(0). \tag{28}$$

则方程(25)可写成

$$\underline{v}_k \underline{s}_k = \underline{w}_k. \tag{29}$$

式(29)中含有 $m(2n + 2r)$ 个未知数,对于一组输入只能得到 m 个方程,因此在输入组数不小于 $2(n + r)$ 和相应的输出条件下,参数 $\underline{s}_k$ 用最小二乘法估计为

$$\hat{\underline{s}}_k = (\hat{\underline{v}}_k^T \hat{\underline{v}}_k)^{-1} \hat{\underline{v}}_k^T \hat{\underline{w}}_k, \tag{30}$$

$$\hat{\underline{v}}_k = \begin{bmatrix} \underline{v}_{k1} \\ \underline{v}_{k2} \\ \vdots \\ \underline{v}_{k2(n+r)} \\ \vdots \end{bmatrix}, \quad \hat{\underline{w}}_k = \begin{bmatrix} \underline{w}_{k1} \\ \underline{w}_{k2} \\ \vdots \\ \underline{w}_{k2(n+r)} \\ \vdots \end{bmatrix}. \tag{31}$$

式中 $\{\underline{v}_{ki}, \underline{w}_{ki}\}$ 是由第 i 组输入产生的第 k 个子系统的数据. 设第 $q(q \geqslant 2(n + r))$ 次观测数据所得的参数最小二乘法估计值为

$$\hat{\underline{s}}_{kq} = (\hat{\underline{v}}_{kq}^T \hat{\underline{v}}_{kq})^{-1} \hat{\underline{v}}_{kq}^T \hat{\underline{w}}_{kq}, \hat{\underline{g}}_{kq} \triangleq (\hat{\underline{v}}_{kq}^T \hat{\underline{v}}_{kq})^{-1}. \tag{32}$$

令 $q + 1$ 次观测的数据为

$$\hat{\underline{v}}_{kq+1} = \begin{bmatrix} \hat{\underline{v}}_{kq} \\ \cdots \cdots \\ \underline{v}_{kq+1} \end{bmatrix}, \hat{\underline{w}}_{kq+} = \begin{bmatrix} \hat{\underline{w}}_{kq} \\ \cdots \cdots \\ \underline{w}_{kq+1} \end{bmatrix}. \tag{33}$$

则有

$$\hat{\underline{g}}_{kq+1} = (\hat{\underline{v}}_{kq+1}^T \hat{\underline{v}}_{kq+1})^{-1} = (\hat{\underline{g}}_{kq}^{-1} + \underline{v}_{kq+1}^T \underline{v}_{kq+1})^{-1},$$

$$\hat{\underline{s}}_{kq+1} = \hat{\underline{g}}_{kq+1} \hat{\underline{v}}_{kq+1}^T \hat{\underline{w}}_{kq+1} = \hat{\underline{s}}_{kq} + \hat{\underline{g}}_{kq+1} \underline{v}_{kq+1}^T (\underline{w}_{kq+1} - \underline{v}_{kq+1} \hat{\underline{s}}_{kq}). \tag{34}$$

利用方程(34)可以对系统(15)进行递推参数估计[1].

4　算例

例 1. 考虑延时线性系统:

$$\dot{x}(t) = ax\left(t - \frac{1}{4}\right), x(0) = 1, x(t) = 0, -\frac{1}{4} \leqslant t < 0.$$

取 $a = 4, m = 6$. 用 10 组输入输出信息得表 1 结果.

例 2. 考虑延时线性系统:

$$\dot{x}(t) = ax(t) + bx(t - 1) + cu(t) + du(t - 1),$$

$$x(0) = 1, x(t) = 0, t < 0, u(t) = t, 0 \leqslant t \leqslant 1.$$

表 1

序 列	$\hat{a}$
移位的勒让德多项式	4.0000614
第一类移位的切比雪夫多项式	3.9993412
第二类移位的切比雪夫多项式	3.9999825

取 $a = -1, b = 2, c = 0$ 和 $d = 1$. 对于 $m = 5$ 和 $m = 10$,用 20 组测试数据,利用台劳级数辨识,得

$$m = 5, \hat{s}^T = [-1.20032, 2.26823, 0.00319, 1.08469];$$

$$m = 10, \hat{s}^T = [-1.00539, 2.00895, -0.00043, 1.00298].$$

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PARAMETER IDENTIFICATION OF TIME-VARYING DELAY LINEAR SYSTEMS VIA GENERALIZED ORTHOGONAL POLYNOMIALS

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ABSTRACT

In this paper, generalized orthogonal polynomials (GOP) expansion is applied to estimating the parameters of the time-varying delay linear systems. The basic concept is that the state and control functions are expressed in terms of the GOP. The time-varying delay differential equations by the operation matrices of the GOP are transformed into a series of linear equations using expansion coefficients. The parameters can be identified by input and output data.

Key words: Generalized orthogonal polynomial, operational matrices, time-delay, parameter identification.