A Petri Net Based Deadlock Prevention Approach for Flexible Manufacturing Systems[1)]

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Abstract A deadlock prevention strategy for flexible manufacturing systems is developed based on Petri nets and their structural analysis. The concept of elementary siphons is proposed, it is a class of SMS (strict minimal siphons) with a smaller cardinality, particularly in the Petri net models of large-scale systems. By adding a control place for each elementary siphon to make it never be emptied, deadlocks can be prevented for a special class of Petri nets, namely S^3PR. That means not all SMS need to be considered when ensuring no siphon loses its tokens. For S^3PR, An approach is proposed for finding elementary siphons and SMS. Compared with the existing methods that control all SMS in a Petri net, the deadlock prevention policy has at least three advantages: 1) only a smaller number of SMS need to be controlled, hence the deadlock-freeness or live Petri net model obtained has less additional places and arcs; 2) not need to compute the set of siphons beforehand; and 3) this policy is more suitable for large-scale Petri nets. These methods are illustrated with an example.

Key words Petri nets, deadlock prevention, elementary siphon, FMS

1 Introduction

Based on Petri nets, several methods have been developed to deal with deadlock problems in FMS context. The first one is to limit the number of processing parts entering an FMS. This method ensures the liveness of Petri net model although it is much conservative and hence deteriorates the system productivity and resource's utilization[1,2]. The second one is to avoid deadlocks by controlling requests for resources. The aim of this method is to forbid the request for a resource if permission of this request will lead to deadlocks[3]. In Petri net formalism, this method intentionally disables an enabled transition in order to avoid deadlock states to be reached. It is conservative as well. The third one is to ensure deadlock-free by modifying the structures of Petri net models[4,5], which is usually called deadlock prevention. A control place is added for each SMS such that every SMS becomes a controlled one and hence it cannot be cleared of tokens. The drawback of the method proposed in [4] is that the resultant Petri net model becomes more complex due to the numerous additional places and arcs. The last one is called deadlock detection and recovery[6,7]. Once deadlocks are detected in Petri net models, automatic or manually actions will be employed. High productivity and resource's utilization can generally be achieved by this method, which however requires some auxiliary devices. Moreover, some unlocking control software must be programmed when designing controllers for robots, machine tools, etc [8,9].

This paper proposes a deadlock prevention method based on elementary siphons. S^3PR[4] is used for modeling FMS in this paper. We verify that if elementary siphons are properly controlled, all SMS in a Petri net can possibly be marked and the controlled S^3PR is hence live. Because the number of elementary siphons is smaller than that of SMS, the controlled Petri net has a smaller number of additional control places and arcs. We assume

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the reader to be familiar with Petri nets[10].

2 Elementary siphons of Petri nets

In this section, we present the concept of elementary siphons. Elementary siphons are a unique type of SMS. The number of them is much smaller than that of SMS, particularly in a large Petri net structure. This concept is suitable not only for S^3PR class but also for more general Petri nets. This research reveals that a live controlled S^3PR can be obtained by just controlling its elementary siphons rather than all SMS in it. Π_{ES} is used to denote the set of elementary siphons in an S^3PR net system in the sequel.

Definition 1. Let $N=(P,T,F)$ be a Petri net (not necessarily an S^3PR) and $S\subseteq P$ be a siphon of N. A P-vector $\boldsymbol{\lambda}_S$ is called the characteristic P-vector of S iff $\forall p\in S, \boldsymbol{\lambda}_S(p)=1$; otherwise, $\boldsymbol{\lambda}_S(p)=0$.

Definition 2. Let $N=(P,T,F)$ be a Petri net, $S\subseteq P$ be a siphon of N, and λ_S be the characteristic P-vector of S. $\boldsymbol{\eta}_S$ is called the characteristic T-vector of S if $\boldsymbol{\eta}_S^T=\boldsymbol{\lambda}_S^T\cdot C$.

Theorem 1. The characteristic T-vector of the support of a P-invariant is **0**.

Proof. The characteristic P-vector of the support of a P-invariant I is exactly I itself. By definitions of P-invariants and characteristic T-vectors, this theorem apparently holds. □

Theorem 2. Let S be a siphon of net $N=(P, T, F)$ and $\boldsymbol{\eta}_S$ be its characteristic T-vector. We can conclude that $\{t\in T\mid \boldsymbol{\eta}_S(t)>0\}$, $\{t\in T\mid \boldsymbol{\eta}_S(t)=0\}$, and $\{t\in T\mid \boldsymbol{\eta}_S(t)<0\}$ are the sets of transitions whose firings will increase, maintain, and decrease the number of tokens marked in S, respectively.

Proof. $\forall t\in\{t\in T\mid \boldsymbol{\eta}_S(t)>0\}$ there exist D_1, D_2, D_3 such that $S=D_1\cup D_2\cup D_3$ (note that D_1, D_2, and D_3 can not be empty sets simultaneously), and $\forall p\in D_1$, $p\in t^{\bullet}$; $\forall p\in D_2$, $p\in {}^{\bullet}t$; $\forall p\in D_3$, $p\notin t^{\bullet}\cup{}^{\bullet}t$. According to the firing rules of Petri nets, the firing of t will increase the number of tokens in D_1, decrease that of tokens in D_2, but maintain that in D_3. By Definition 2, if $t\in\{t\in T\mid \boldsymbol{\eta}_S(t)>0\}$ then one will have $|D_1|>|D_2|$. Therefore, $\forall t\in\{t\in T\mid \boldsymbol{\eta}_S(t)>0\}$, *the token increments in* S *will be* $|D_1|-|D_2|(\boldsymbol{\eta}_S(t))$ by the firing of t. The two other cases can be similarly proved. □

Definition 3. Let S_0, S_1, S_2, $\cdots$, and $S_n (n\in IN/\{0,1\})$ be SMS of a net N and $\boldsymbol{\eta}_{S_i}$ be the characteristic T-vector of S_i, $i=0,1,2,\cdots,n$. S_0 is called a redundant SMS (denoted by RS) with respect to S_1, S_2, $\cdots$, and S_n if $\boldsymbol{\eta}_{S0}=\boldsymbol{\eta}_{S1}+\boldsymbol{\eta}_{S2}+\cdots+\boldsymbol{\eta}_{S_n}$ holds. We denote the set of redundant SMS in a net by Π_{RS}.

Definition 4. Let Π be the set of SMS of N. $\forall S\in\Pi$, if $\neg\exists S_1$, S_2, $\cdots$, and $S_n\in\Pi\backslash S$ such that $\boldsymbol{\eta}_{S1}+\boldsymbol{\eta}_{S2}+\cdots+\boldsymbol{\eta}_{S_n}=\boldsymbol{\eta}_S$ holds, S is called an elementary siphon. Obviously, we can get $\Pi_{ES}\cup\Pi_{RS}=\Pi$.

3 Solution to the elementary siphons in S^3PR

The computation for SMS is a necessity for the approach in [4]. It is well known that theoretically, the time complexity of traditional algorithms of siphon computation for general Petir nets is exponential with respect to the size of a Petri net although it is not the case practically. We develop a method to obtain SMS in an S^3PR net based on its structural analysis. The relevant notations, definitions, and results about S^3PR can be referred to [4].

Definition 5. Let $\{r_1,r_2,\cdots,r_m\}\subseteq P_R (m\in IN/\{0,1\})$ be a set of resources in an S^3PR (N,M_0). $\{r_1,r_2,\cdots,r_m\}$ is called a resource circuit, denoted by L, if ${}^{\bullet}r_1\cap r_2^{\bullet}\neq\emptyset$, ${}^{\bullet}r_2\cap r_3^{\bullet}\neq\emptyset,\cdots$, and ${}^{\bullet}r_m\cap r_1^{\bullet}\neq\emptyset$ hold. Note that the number of expressions may be larger than m due to the fact that a resource can be shared by two or more processes.

Theorem 3. Let $L=\{r_1,r_2,\cdots,r_m\}$ be a resource circuit of an S^3PR N, $N=(P\cup P^0\cup P_R,T,F)$. $S=\{r_1,r_2,\cdots,r_m\}\cup\{p\mid p\in\bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\not\subseteq\bigcup_{r\in L}H(r)\}$ is a minimal siphon in N. And if S does not contain the support of any P-invariant, then S is an SMS.

Proof. We first claim that S is a siphon. For this, we have to prove $\forall t\in {}^\bullet S$, $t\in S^\bullet$. $\forall t\in {}^\bullet S$, either $t\in {}^\bullet\{r_1, r_2,\cdots,r_m\}$ or $t\in {}^\bullet\{p\mid p\in \bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\nsubseteq \bigcup_{r\in L}H(r)\}$ holds. We accordingly have the following two cases.

1) $t\in {}^\bullet\{r_1,r_2,\cdots,r_m\}$ means $\exists i\in\{1,2,\cdots,m\}$, $t\in {}^\bullet r_i$. By the definition of a resource circuit, two subcases are considered.

a) There exists $j\in\{1,2,\cdots,m\}$ such that ${}^\bullet r_i\cap r_j^\bullet\neq\varnothing$. If $t\in {}^\bullet r_i\cap r_j^\bullet$, $t\in\{r_1,r_2,\cdots,r_m\}^\bullet$ holds. b) There does not exist $j\in\{1,2,\cdots,m\}$ such that $t\in {}^\bullet r_i\cap r_j^\bullet$. From the definitions of S^{3}PR[4], there must exist a state place $p\in H(r_i)$ such that $t\in p^\bullet$ holds. Therefore, one needs to prove $p\in S$. Since $S=\{r_1,r_2,\cdots,r_m\}\cup\{p\mid p\in\bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)\}$, one equivalently has to prove $(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)$. By contradiction, assume $(p^{\bullet\bullet}\cap P)\subseteq\bigcup_{r\in L}H(r)$. Hence we have $\exists j\in\{1,2,\cdots,m\}$, $(p^{\bullet\bullet}\cap P)\subseteq H(r_j)$ and $t\in {}^\bullet r_i\cap r_j^\bullet$ by definitions of S^{3}PR. This is clearly contradictory to the condition that there does not exist $j\in\{1,2,\cdots,m\}$ such that $t\in {}^\bullet r_i\cap r_j^\bullet$. Consequently, if there exists a transition $t\in {}^\bullet\{r_1,r_2,\cdots,r_m\}$ and does not exist $j\in\{1,2,\cdots,m\}$ such that $t\in {}^\bullet r_i\cap r_j^\bullet$, there certainly exists a place $p\in H(r_i)$ such that $t\in p^\bullet$ and $p\in S$, i. e., $t\in S^\bullet$. Therefore, $\forall t\in {}^\bullet\{r_1, r_2,\cdots,r_m\}$, $t\in S^\bullet$ holds.

2) $t\in {}^\bullet\{p\mid p\in\bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)\}$ means $\exists r_i\in L(i\in\{1,2,\cdots,k\})$, $p\in H(r_i)$ such that $t\in {}^\bullet p$. By definitions of S^{3}PR, $t\in r_i^\bullet$ holds. Thus we can conclude that $\forall t\in {}^\bullet\{p\mid p\in\bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)\}$ means $t\in S^\bullet$.

By the proofs for cases 1) and 2), one can get $\forall t\in {}^\bullet(\{r_1,r_2,\cdots,r_m\}\cup\{p\mid p\in\bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)\})$, $t\in S^\bullet$ holds, i. e., ${}^\bullet S\subseteq S^\bullet$. Trivially, S is a siphon.

Next we prove S is a minimal siphon. By contradiction. Assume that S is not a minimal siphon. That is to say, there exists a siphon S_X such that $S_X\subset S$ holds. Let $S=S_R\cup S_P$, $S_R=S\cap P_R$, $S_P=S\backslash S_R$, $S_X=S_{XR}\cup S_{XP}$, $S_{XR}=S_X\cap P_R$, and $S_{XP}=S\backslash S_{XR}$. If $S_X\subset S$ is true, then one of the following three cases holds: a) $S_{XR}=S_R$, $S_{XP}\subset S_P$; b) $S_{XR}\subset S_R$, $S_{XP}=S_P$; and c) $S_{XR}\subset S_R$, $S_{XP}\subset S_P$. We first deal with case a). By $S_{XP}\subset S_P$, there exists at least a place $p\in H(r)$ such that $r\in S_R(S_{XR})$, $p\in S_P$ and $p\notin S_{XP}$ are ture. From the definitions of S^{3}PR, we know that there exists a transition $t\in p^\bullet\cap {}^\bullet r_i$ such that $t\in {}^\bullet S_{XR}$ and $t\in {}^\bullet S_X$, as shown in Fig. 1. Owing to $p\notin S_{XP}$, there does not exist a place $p_X\in S_{XP}$ such that $\exists t\in p^\bullet\cap p_X^\bullet$ and hence $t\in S_{XP}^\bullet$ holds. We can see that $t\notin S_{XP}^\bullet$. Moreover, there impossibly exists a place $r_j\in S_{XR}(S_R)$ such that $t\in r_j^\bullet$. Otherwise, it leads to $(p^{\bullet\bullet}\cap P)\subseteq\bigcup_{r\in L}H(r)$ which is contradictory to $(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)$ defined in S. Hence $t\notin S_{XR}^\bullet$ holds. As a result, we can conclude that S_X is not a siphon since there exists a transition t such that both $t\in {}^\bullet S_X$ and $t\notin S_{XR}^\bullet\cup S_{XP}^\bullet$ $(t\notin S_X^\bullet)$ hold, which is clearly contradictory to the assumption. Therefore, S is a minimal siphon. Case b) and c) can be similarly proved.

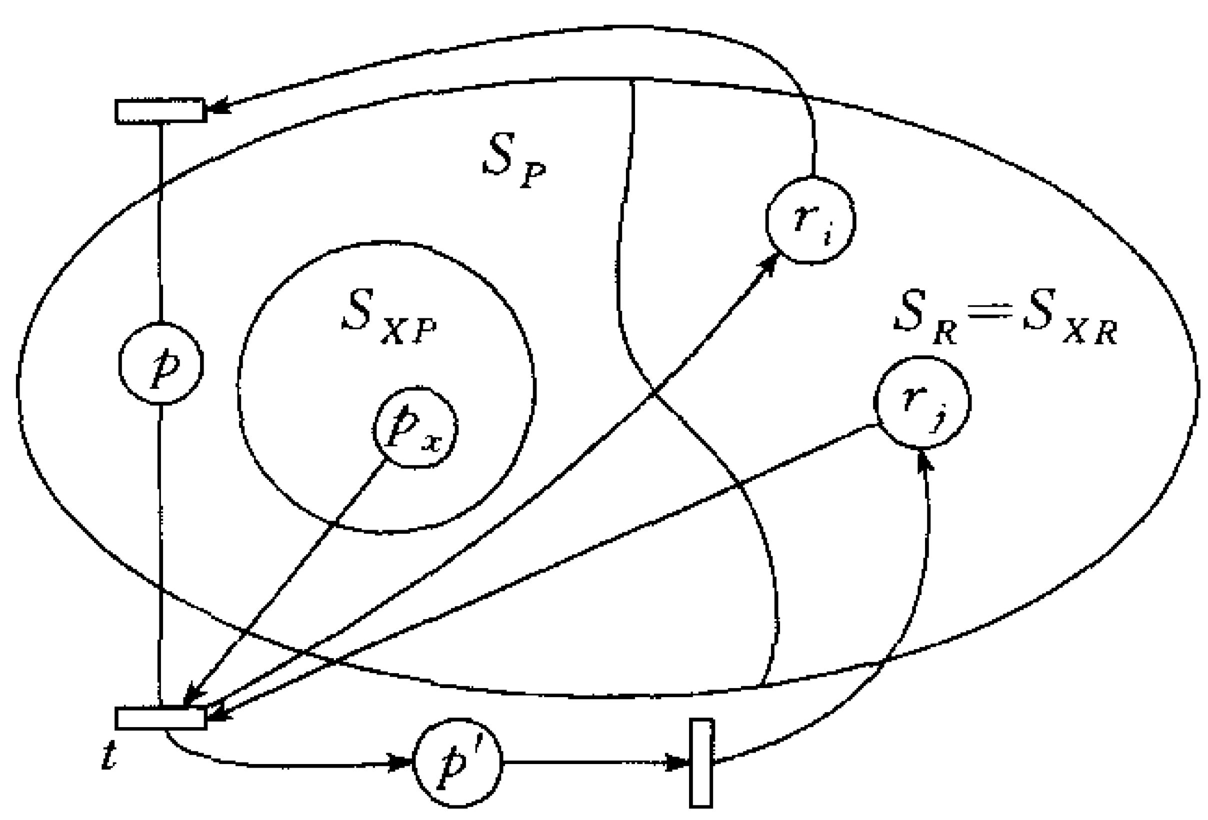

Fig. 1 The case of $S_{XR}=S_R$, $S_{XP}\subset S_P$

Consequently, $S=\{r_1,r_2,\cdots,r_m\}\cup\{p\mid p\in\bigcup_{r\in L}H(r)\wedge(p^{\bullet\bullet}\cap P)\nsubseteq\bigcup_{r\in L}H(r)\}$ is a minimal siphon in S^{3}PR. And if S does not contain the support of any P-invariant, S is an SMS accordingly. □

Definition 6. Let L_1 and L_2 be two resource circuits. We say that L_1 and L_2 are connected if $\exists r\in P_R$, $r\in L_1\cap L_2$.

Definition 7. Let $L_1, L_2,\cdots,$and $L_n(n\in IN/\{0,1\})$ be resource circuits. We call $L_1\cup L_2\cup\cdots\cup L_n$ a resource chain if $\forall i\in\{1,2,\cdots,n\}$, $\exists j\in\{1,2,\cdots,n\}\backslash\{i\}$ such that L_i and L_j

are connected.

Let LC denote a resource chain. Note that a set of resources may be both a resource circuit and a resource chain. In that case, we preferentially treat it as a resource circuit.

Theorem 4. Let $LC=\{r_1, r_2, \cdots, r_m\}$ be a resource chain of an S^{3}PR. $S=\{r_1, r_2, \cdots, r_m\} \cup \{p \mid p \in \bigcup_{r\in LC} H(r) \wedge (p^{\cdot\cdot} \cap P) \not\subseteq \bigcup_{r\in LC} H(r)\}$ is a siphon. If S neither contains the support of any P-invariant nor is a superset of any siphon, then S is an SMS.

Proof. Similar to the proof of Theorem 3.

Theorem 5. Let $S=S_R \cup S_P$ be a siphon of an S^{3}PR, where $S_P=\{p \mid p \in (\bigcup_{r\in L} H(r)) \wedge (p^{\cdot\cdot} \cap P) \not\subseteq (\bigcup_{r\in L} H(r))\}$ and $S_R=\{r_1, r_2, \cdots, r_m\}$. We can conclude S_R is an L or an LC.

Proof. By induction.

First we claim the case of $|S_R|=2$, i.e., $S_R=\{r_i, r_j\}$ (It is proved in [4] that $|S_R|>1$, where S is a siphon in an S^{3}PR.). From definitions for SMS in S^{3}PR, there exists a place $p\in H(r_i)$ such that $p\notin S_P$. If $\{t\}=p^{\cdot}\cap{}^{\cdot}r_i$ we have $\exists r\in P_R$, $\{t\}=r^{\cdot}\cap{}^{\cdot}r_i$. Hence one can get $r=r_j$. Otherwise, we have $t\notin S^{\cdot}$ and S is not a siphon. As a result $\{t\}=r_j^{\cdot}\cap{}^{\cdot}r_i$, i.e., ${}^{\cdot}r_j\cap r_i^{\cdot}\neq\emptyset$ holds. Similarly, we can prove that ${}^{\cdot}r_j\cap r_i^{\cdot}\neq\emptyset$ holds as well. Therefore $\{r_i, r_j\}$ is a resource circuit. The case $|S_R|=2$ is proved.

Assume that when $|S_R|=m$, $\{r_1, r_2, \cdots, r_m\}$ is a resource circuit or chain. The case $|S_R|=m+1$ is proved as follows. If ${}^{\cdot}r_m\cap r_{m+1}^{\cdot}=\emptyset$ holds, we have $p\in S$, $\forall p\in H(r_{m+1})$. Otherwise, there exists a place $p\neg\in S$, $\{t\}=p^{\cdot}\cap{}^{\cdot}r_{m+1}\wedge t\notin S^{\cdot}$. This means $\{r_{m+1}\}\cup H(r_{m+1})$ is included in S. While $\{r_{m+1}\}\cup H(r_{m+1})$ is the support of a P-invariant, we can see that S is thus not a strict siphon. Consequently, ${}^{\cdot}r_m\cap r_{m+1}^{\cdot}=\emptyset$ is not possible. Assume that $\{t\}={}^{\cdot}r_m\cap r_{m+1}^{\cdot}$. We have $\{t\}=p'^{\cdot}\cap{}^{\cdot}r_m$, where $p'\in H(r_m)$. Obviously, $p'\notin S$ holds. Furthermore, assume $\{t\}={}^{\cdot}p\cap r_{m+1}^{\cdot}$, where $p\in H(r_{m+1})$. One can get $p\in S$. Moreover, there exists a directed path between p_1 and p_2, and $p_1\notin S$, where $p_1\in H(r_{m+1})$ and $p_2\in H(r_m)$. Otherwise, S will contain $\{r_{m+1}\}\cup H(r_{m+1})$. Let $\{t'\}=p_1^{\cdot}\cap{}^{\cdot}r_{m+1}$. Due to $p_1\notin S$, $t'\in r_m^{\cdot}$ holds. Otherwise, $t\in{}^{\cdot}r_{m+1}$. By $t\in S^{\cdot}$, we have ${}^{\cdot}r_{m+1}\cap r_m^{\cdot}=\{t'\}\neq\emptyset$. One can see that $\{r_1, r_2, \cdots, r_m\}$ and $\{r_{m+1}\}$ are connected. $\{r_1, r_2, \cdots, r_m, r_{m+1}\}$ is either an L or an LC. □

The sufficient and necessary conditions under which a set of places of an S^{3}PR net is an SMS are given by Theorems 3, 4, and 5. Accordingly, we can use Theorems 3 and 4 to calculate SMS. The outline of our algorithm is: 1) Find all Ls of the net; 2) Find all LCs of the net; 3) Calculate all strict siphons and siphons of the net based on Ls and LCs, respectively; and 4) Eliminate each strict siphon which contains the support of any P-invariant and each non-minimal siphon which is a superset of any siphon.

For example, Fig. 2 is the Petri net model of an FMS and it belongs to S^{3}PR class. There are three resource circuits, namely $L_1=\{$M1, R1$\}$, $L_2=\{$M2, R1$\}$, and $L_3=\{$M3, R1, M4, R2$\}$ and four resource chains, namely $LC_1=\{$M1, R1, M2$\}$, $LC_2=\{$M1, M3, R1, M4, R2$\}$, $LC_3=\{$M2, M3, R1, M4, R2$\}$, and $LC_4=\{$M1, M2, M3, R1, M4, R2$\}$. From these resource circuits and chains, one can get seven siphons, $Q_1=\{p_2, p_5, p_{11}, p_{13}$, M1, R1$\}$, $Q_2=\{p_2, p_5, p_{13}$, M2, R1$\}$, $Q_3=\{p_2, p_7, p_{11}, p_{13}$, M3, R1, M4, R2$\}$, $Q_4=\{p_5, p_{13}$, M1, R1, M2$\}$, $Q_5=\{p_2, p_7, p_{11}, p_{13}$, M1, M3, R1, M4, R2$\}$, $Q_6=\{p_2, p_7, p_{13}$, M2, M3, R1, M4, R2$\}$, and $Q_7=\{p_7, p_{13}$, M1, M2, R1, M3, R2, M4$\}$. It can be verified that five of them are SMS, namely $S_1=\{p_2, p_5, p_{13}$, M2, R1$\}$, $S_2=\{p_2, p_7, p_{11}, p_{13}$, M3, R1, M4, R2$\}$, $S_3=\{p_5, p_{13}$, M1, R1, M2$\}$, $S_4=\{p_2, p_7, p_{13}$, M2, M3, R1, M4, R2$\}$, $S_5=\{p_7, p_{13}$, M1, M2, R1, M3, R2, M4$\}$.

In the following a method to compute elementary siphons is proposed.

Definitioin 8. Let $N=(P, T, F)$ be a net with $|P|=m$, $|T|=n$ and we assume N has k SMS, S_1, S_2, $\cdots$, and S_k, m, n, $k\in IN$. Let λ_{S_i} (η_{S_i}) be the characteristic P(T)-

vector of SMS S_i. We define $[\lambda]_{k\times m}=[\boldsymbol{\lambda}_{S1}^{T}|\boldsymbol{\lambda}_{S1}^{T}|\cdots|\boldsymbol{\lambda}_{Sk}^{T}|]^{T}$ and $[\eta]_{k\times n}=[\lambda]_{k\times m}\times C_{m\times n}=[\boldsymbol{\eta}_{S1}^{T}|\boldsymbol{\eta}_{S1}^{T}|\cdots|\boldsymbol{\eta}_{Sk}^{T}|]^{T}$. $[\lambda]([\eta])$ is called the characteristic P(T)-vector matrix of the SMS of N.

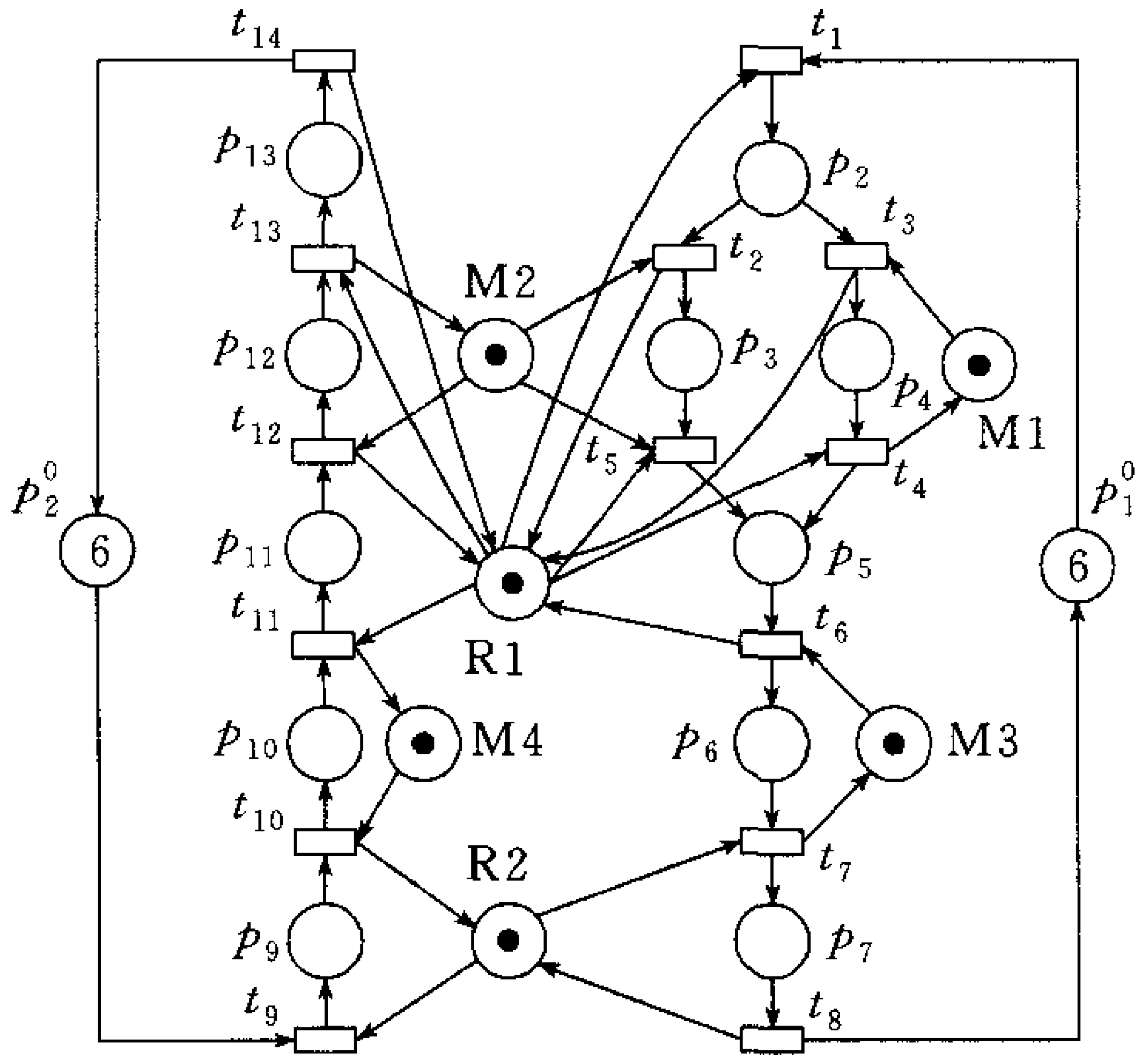

Fig. 2 The Petri net model of an FMS

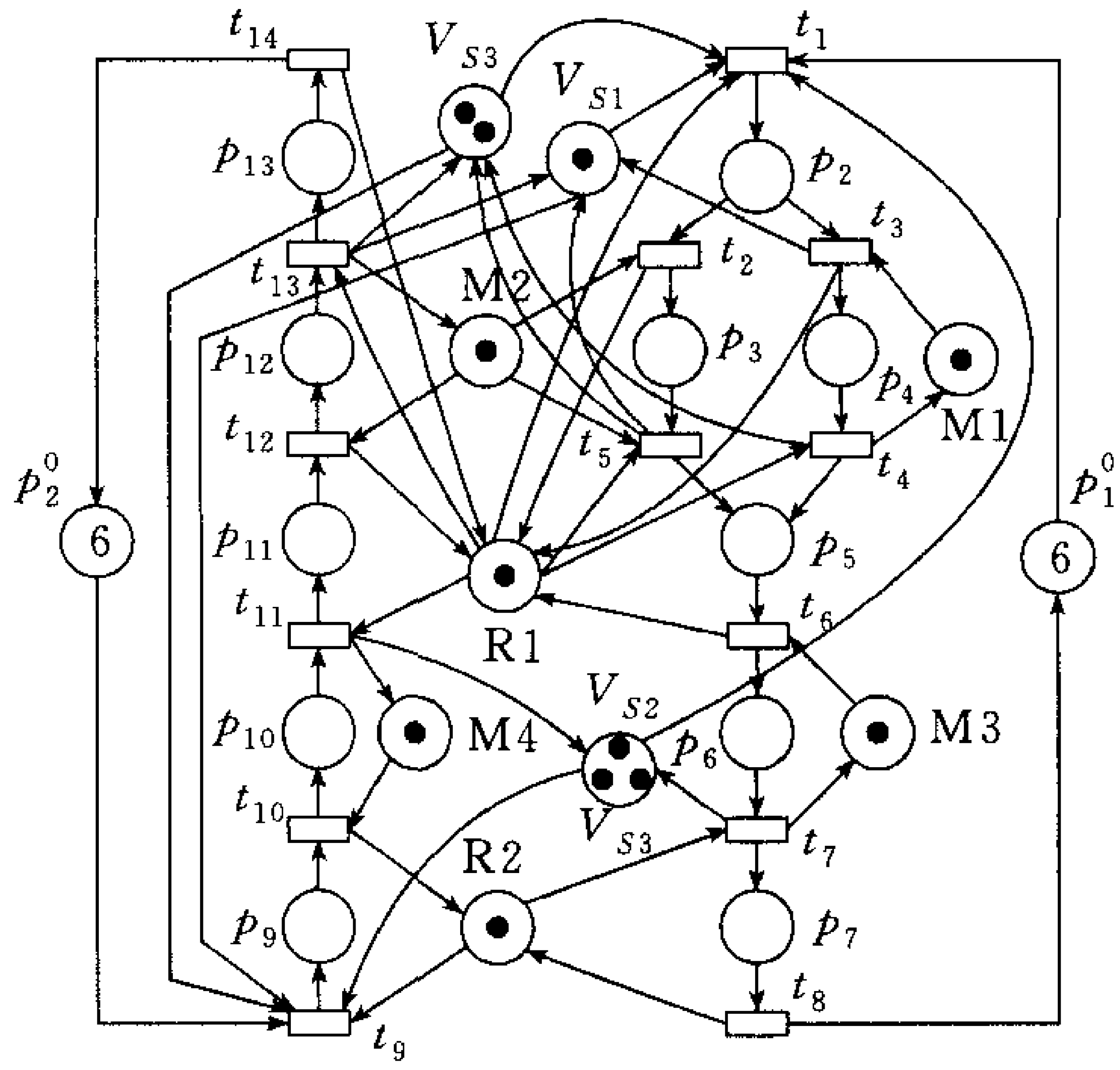

Fig. 3 The live Petri net with 3 additional places

Theorem 6. Let N be a net and $[\eta]$ be the characteristic T-vector matrix of the SMS of it. The number of elementary siphons in N is equal to the rank of $[\eta]$.

Proof. Assume N has k SMS and k' elementary siphons ($k\geqslant k'$). Obviously, there are k-k' redundant SMS in N. By Definition 3, $\boldsymbol{\eta}_{Si}$ ($i=k'+1,k'+2,\cdots,k$) can be linearly represented by $\boldsymbol{\eta}_{Sj}$ ($j=1, 2,\cdots,k'$). According to the definition of the rank of a matrix, we have the rank of $[\eta]_{k\times n}$ is k'. □

In Fig. 2, it is easy to verify that $\boldsymbol{\eta}_4=\boldsymbol{\eta}_1+\boldsymbol{\eta}_2$ and $\boldsymbol{\eta}_5=\boldsymbol{\eta}_2+\boldsymbol{\eta}_3$. Hence, the rank of $[\eta]$ is 3. Thus S_1, S_2 and S_3 are elementary siphons and S_4 and S_5 are redundant siphons.

4 Deadlock prevention policy

In [4], to keep all SMS being marked at any reachable marking, a place and several arcs are added to the original net system for each SMS, which leads to a more complex Petri net controller. However, only a small number of SMS are considered in our method. All SMS of a net system can possibly be marked if its elementary siphons are controlled. For the case that there exists an emptiable siphon when all elementary siphons are controlled, we have to add a place to control it using the control approach developed in [4]. Note that in [4] it is guranteed that no new siphon is generated owing to the addition of control places. This research investigates conditions under which an SMS in a Petri net system can always be marked by controlling its elementary siphons.

As stated in Definition 3, we use RS to denote a redundant siphon, $RS_R=RS\cap P_R$ to denote the set of resources of RS, and $C_{RS}=(\bigcup\{H(r),r\in RS_R\})\backslash RS$ to denote the complementary set of RS. Elementary siphons can be similarly described.

Definition 9. Let (N, M_0) be a Petri net system and RS be a redundant siphon of N. We say RS is a controllable redundant siphon by elementary siphons if elementary siphons of N are controlled means that RS is controlled.

Let (N,M_0) be an S^3PR. Assume that there are m elementary siphons $S_1,S_2,\cdots$, and S_m in the net system. By the prevention deadlock method in [4], we add m control places $V_{S1},V_{S2},\cdots$, and V_{Sm} to control $S_1,S_2,\cdots$, and S_m, respectively. The new net system is denoted as (N_A,M_{0A}). By the siphon control method in [4], three control places V_{S1},V_{S2}, and V_{S3} are added to the net model in Fig. 2. The resultant Petri net which is live is shown in Fig. 3. This can be verified by the following procedures. Next we discuss the conditions under which a redundant siphon RS of (N, M_0) can not be emptied in (N_A, M_{0A}).

An empty redundant siphon RS at any marking $M_A \in R(N_A, M_{0A})$ must satisfies conditions Cond_1, Cond_2, and Cond_3 as follows. That means if one of these conditions deos not hold for RS, it will always be marked at M_{0A}. For a siphon S, P_S and C_S are defined in [4].

Cond_1. By the definition of C_{RS}, $C_{RS} \cup RS$ is the support of a P-invariant, which is the union of RS_R and $(\bigcup\{H(r), r \in RS_R\})$. Note that $RS_R = RS \cap P_R$ is the set of resources of RS and $\bigcup\{H(r), r \in RS_R\}$ is the union of holders of those resources. If RS is unmarked at marking M_A, we have $\sum M_A(p \mid p \in C_{RS}) = \sum M_{0A}(p \mid p \in RS) = \sum M_{0A}(p \mid p \in RS_R)$ due to $\forall p \in RS_P$, $M_{0A}(p) = 0$. Note that the union of the set consisting of a resource of RS and the holders of the resource is the support of a P-invariant. According to the properties of P-invariants, $\forall r \in RS_R, M_A(r) + \sum M_A(p \mid p \in H(r)) = M_A(r) + \sum M_A(p \mid p \in C_{RS} \cap H(r)) + \sum M_A(p \mid p \in RS \cap H(r)) = M_{0A}(r)$ trivially holds, i. e., $\forall r \in RS_R$, $\sum M_A(p \mid p \in C_{RS} \cap H(r)) = M_{0A}(r)$. This equality means that when RS is umarked, all tokens in each resource of RS are "stolen" by the holders of the resource, which are included in RS.

Cond_2. Suppose that m control places are added for m elementary siphons in N. From the definition of P_S, $C_S \subseteq P_S$ clearly holds. Due to the properties of P-invariants, we can have the following relationship when an RS is unmarked: $\forall j \in \{1, 2, \cdots, m\}, \sum M_{0A}(p \mid p \in S_j) - 1 = M_{0A}(V_{S_j}) = M_A(V_{S_j}) + \sum M_A(p \mid p \in P_{S_j}) \geqslant M_A(V_{S_j}) + \sum M_A(p \mid p \in (C_{RS} \cap P_{S_j})) \geqslant \sum M_A(p \mid p \in (C_{RS} \cap P_{S_j}))$, i. e., $\forall j \in \{1, 2, \cdots, m\}, \sum M_{0A}(p \mid p \in S_j) - 1 \geqslant \sum M_A(p \mid p \in (C_{RS} \cap P_{S_j}))$.

Cond_3. Assume that an S^3PR is composed by k S^2P $(N_i = (P_i \cup \{p_i^0\}, T_i, F_i))$ via shared resources. There are thus k P-invariants due to k S^2P, whose supports are $\| I_i \| = P_i \cup \{p_i^0\}$, $i \in \{1, 2, \cdots, k\}$. Whether $\| I_i \| \cap C_{RS} \neq \varnothing$, we always have, by the definition of P-invariants, the following relationship: $\forall i \in \{1, 2, .., k\}, \sum M_A(p \mid p \in C_{RS} \cap \| I_i \|) \leqslant \sum M_{0A}(p \mid p \in \| I_i \|) = M_{0A}(p \in \{p_i^0\})$.

For an RS, by solving the set of inequalities derived from Cond_1, Cond_2, and Cond_3, we can find the conditions under which it is emptied. Otherwsie, RS can always be marked if there does not exist a feasible solution. In other words, RS is controlled by its elementary siphons.

Note that if, for an S^2PR, the initial number of tokens in $M_{0A}(p_i^0)$ is larger than or equal to the sum of tokens initially marked at the resources possessed by the S^2PR, the inequalities derived from Cond_3 must hold. Therefore, nothing but the inequalities derived from Cond_1 and Cond_2 need to be solved. When a redundant siphon is not controllable by the above methods, an additional control place is added by the method proposed in [4]. If all siphons in the target net system are controlled, the augmented net system is live by the results of [4].

Let us examine if the redundant siphons in Fig. 2 are controllable after all elementary siphons are controlled. First we deal with $S_4 = \{p_2, p_7, p_{13}, M2, M3, R1, M4, R2\}$. For a redundant siphon RS, we have $C_{RS} = (\bigcup\{(H(r), r \in RS_R)\}) \backslash RS$. Thus $C_{RS4} = (H(M2) \cup H(M3) \cup H(R1) \cup H(M4) \cup H(R2)) \backslash RS_4 = \{p_3, p_{12}, p_6, p_2, p_5, p_{11}, p_{13}, p_{10}, p_7, p_9\} \backslash RS_4 = \{p_3, p_{12}, p_6, p_5, p_{11}, p_{10}, p_9\}$. $C_{RS4} \cap H(M2) = \{p_3, p_{12}\}$, $C_{RS4} \cap H(M3) = \{p_6\}$, $C_{RS4} \cap H(R1) = \{p_5, p_{11}\}$, $C_{RS4} \cap H(M4) = \{p_{10}\}$, $C_{RS4} \cap H(R2) = \{p_9\}$.

We can obtain the following relationships (1)~(5), (6)~(8), and (9),(10) by Cond_1, Cond_2, and Cond_3, respectively.

$$\sum M_A(p \mid p \in C_{RS4} \cap H(\mathrm{M2})) = M_{0A}(\mathrm{M2}) = 1 \Rightarrow M_A(p_3) + M_A(p_{12}) = 1 \quad (1)$$

$$\sum M_A(p \mid p \in C_{RS4} \cap H(\mathrm{M3})) = M_{0A}(\mathrm{M3}) = 1 \Rightarrow M_A(p_6) = 1 \quad (2)$$

$$\sum M_A(p \mid p \in C_{RS4} \cap H(\mathrm{R1})) = M_{0A}(\mathrm{R1}) = 1 \Rightarrow M_A(p_5) + M_A(p_{11}) = 1 \quad (3)$$

$$\sum M_A(p \mid p \in C_{RS4} \cap H(\mathrm{M4})) = M_{0A}(\mathrm{M4}) = 1 \Rightarrow M_A(p_{10}) = 1 \quad (4)$$

$$\sum M_A(p \mid p \in C_{RS4} \cap H(\mathrm{R2})) = M_{0A}(\mathrm{R2}) = 1 \Rightarrow M_A(p_9) = 1 \quad (5)$$

$$M_A(p_3) + M_A(p_9) + M_A(p_{10}) + M_A(p_{11}) + M_A(p_{12}) \leqslant 1 \text{ (for } S_1) \quad (6)$$

$$M_A(p_3) + M_A(p_5) + M_A(p_6) + M_A(p_9) + M_A(p_{10}) \leqslant 3 \text{ (for } S_2) \quad (7)$$

$$M_A(p_3) + M_A(p_9) + M_A(p_{10}) + M_A(p_{11}) + M_A(p_{12}) \leqslant 2 \text{ (for } S_3) \quad (8)$$

$$M_A(p_3) + M_A(p_5) + M_A(p_6) \leqslant 6 \quad (9)$$

$$M_A(p_9) + M_A(p_{10}) + M_A(p_{11}) + M_A(p_{12}) \leqslant 6 \quad (10)$$

The complementary sets of elementary siphons are $C_{S1} = \{p_3, p_{11}, p_{12}\}$, $C_{S2} = \{p_5, p_6, p_9, p_{10}\}$, and $C_{S3} = \{p_2, p_3, p_4, p_{11}, p_{12}\}$. Due to the definitions of P_S and C_S, where S is a siphon, we have $P_{S1} = \{p_2, p_3, p_9, p_{10}, p_{11}, p_{12}\}$, $P_{S2} = \{p_2, p_3, p_4, p_5, p_6, p_9, p_{10}\}$, $P_{S3} = \{p_2, p_3, p_4, p_9, p_{10}, p_{11}, p_{12}\}$. $C_{RS4} \cap P_{S1} = \{p_3, p_9, p_{10}, p_{11}, p_{12}\}$, $C_{RS4} \cap P_{S2} = \{p_3, p_5, p_6, p_9, p_{10}\}$, and $C_{RS4} \cap P_{S3} = \{p_3, p_9, p_{10}, p_{11}, p_{12}\}$. The supports of the P-invariants derived from S^2P in Fig. 2 are $\| I_1 \| = \{p_1^0, p_2, p_3, p_4, p_5, p_6, p_7\}$ and $\| I_2 \| = \{p_2^0, p_9, p_{10}, p_{11}, p_{12}, p_{13}\}$. Thus we have $C_{RS4} \cap \| I_1 \| = \{p_3, p_5, p_6\}$, $C_{RS4} \cap \| I_2 \| = \{p_9, p_{10}, p_{11}, p_{12}\}$. Obviously, (6) does not hold when (4) and (5) are taken into account. Thus RS_4 is controlled in Fig. 3. We can also verify that another redundant siphon of Fig. 2 is controlled, as well. Therefore, no more control place needs to be added and the Petri net shown in Fig. 3 is live. To make the Petri net in Fig. 2 live, three control places and 14 arcs are added when our method is employed. However, five control places and 22 arcs are added when the method in [4] is used for this example. The number of reachable markings generated is 88 by either method. Our case study reveals that the more complex a net, the better performance of our method. For the net in Fig. 8 of [4], our method uses 6 control places and 32 arcs to make the net system live while 18 control places and 106 arcs were added to the original net model to achieve the same purpose.

5 Discussions and conclusions

In this paper, we propose a new deadlock prevetion policy. By preventing a smaller number of SMS from being emptied, deadlocks can be prevented for ordinary Petri nets and liveness can be guaranteed for S^3PR, a special class of Petri nets proposed in [4]. The advantage of this method lies in the fact that the final Petri net model is of much less additional places and arcs. In addition, we propose a method to compute elementary siphons and SMS in an S^3PR system with less computation burden. The concept of elementary siphons looks promising in deadlock prevention problems arising in FMS context. Further research will apply this method to more general Petri net classes.

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基于 Petri 网的柔性制造系统一种预防死锁方法

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摘　要　基于 Petri 网的结构特性分析，研究了 FMS(柔性制造系统)一种预防死锁方法. 提出了 Petri 网的一种特殊拓扑结构——基本信标的概念. 在 Petri 网中基本信标的集合是 SMS(严格极小信标)集合的一个真子集. 尤其在大型 Petri 网系统中，基本信标的集合比 SMS 的集合要小得多. 对于 Petri 网的一个子类 S^3PR，只对每一个基本信标添加一个库所使其不被清空，就可实现预防死锁，也就是说无须控制 S^3PR 的所有 SMS 而达到无信标被清空的目的. 此外，对于 S^3PR，还提出了一种求取 SMS 和基本信标的方法. 相对于现在普遍采用的控制所有 SMS 来预防死锁的策略，其具三方面优势. 1)只需控制少量的 SMS 即所谓的基本信标. 相应地，添加少量的控制库所和连接弧，就可得到无死锁或活的 Petri 网. 2)不需要先行计算出极小信标的集合. 3)明显地，这种方法更适合大型 Petri 网系统. 我们通过穿插在文中的一个例子来说明这些方法.

关键词　Petri 网，预防死锁，基本信标，柔性制造系统

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