Decentralized Adaptive Regulation for Nonlinear Systems with iISS Inverse Dynamics

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Abstract This paper considers the decentralized adaptive regulation via output-feedback for nonlinear systems with integral input-to-state stable (iISS) inverse dynamics, nonlinear uncertainties, and unknown control direction. It is shown that all the signals in the closed-loop system obtained are bounded, and the asymptotic regulation is achieved. A numerical example shows the effectiveness of the design.

Key words Large-scale systems, decentralized adaptive control, iISS, backstepping, output-feedback

The class of input-to-state stable (ISS) systems has been extensively investigated and has been playing an important role in the recent literature of nonlinear control theory. For instance, that cascades of ISS systems or ISS is widely used in stabilization, and the ISS small-gain theorem also becomes a popular tool to establish the stability of feedback interconnection of ISS systems. However, it is sometimes the case that feedback design does not render ISS behavior or that only a weaker than ISS property is verified in recursive design.

Such a weaker, but still very meaningful, property was given the name of integral ISS (iISS) in [1]. Sontag showed that iISS is, in general, strictly weaker than ISS, and he provided a very conservative Lyapunov-type sufficient condition[1]. Several foundational results were provided in [2], showing that the iISS property is the most natural one to be expected for well-behaved nonlinear systems, and admitting elegant Lyapunov-theoretic characterization. Stability criteria similar to the ISS small-gain theorem have been developed for interconnection involving iISS systems[3−7]. Pepe and Jiang further extended the ISS and the iISS theories to nonlinear time-delay systems[8]. Recently, Jiang et al. presented a unifying framework for the problem of robust global regulation via output feedback for nonlinear systems with iISS inverse dynamics[9]. Motivated by [9], this paper extends the framework to practically important classes of large-scale systems. Our main contributions are composed of two parts.

1) We accomplish variable separation from the input of iISS inverse dynamics. Moreover, a design function ψ_i is chosen to satisfy $\psi_i \geq 1$. All these bring about convenience to deal with the interaction terms effectively.

2) By combining Nussbaum-type gain approach, backstepping design technique, and a subtle analysis approach[10], we propose for the first time a decentralized adaptive control scheme for a class of large-scale systems in the presence of uncertain nonlinear functions, unmeasured iISS inverse dynamics and unknown direction control coefficients. It is shown that all the signals in the closed-loop system obtained are bounded, and asymptotic regulation is achieved. A numerical example demonstrates the effectiveness of the design.

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This paper is organized as follows. Section 1 begins with some mathematical preliminaries. Section 2 presents the corresponding output feedback control design procedure. An example is given in Section 3. Finally, the paper is concluded in Section 4.

1 Mathematical preliminaries

The following notations will be used throughout this paper. $\mathbf{R}_+$ denotes the set of all nonnegative real numbers. $\mathbf{R}^n$ denotes the real n-dimensional space. For a given vector or matrix X, X^{T} denotes its transpose; $|X|$ denotes the Euclidean norm of vector X; $\|X\|$ denotes the induced matrix norm of matrix X. A function $\gamma : \mathbf{R}_+ \mapsto \mathbf{R}_+$ is of class $\mathcal{K}$ if $\gamma(0) = 0$ and γ is continuous and (strictly) increasing; it is of class $\mathcal{K}_\infty$ if additionally it is unbounded; a function $\beta(s,t) : \mathbf{R}_+ \times \mathbf{R}_+ \mapsto \mathbf{R}_+$ is of class $\mathcal{KL}$ if it is of class $\mathcal{K}$ for each fixed t, and decreases to zero as $t \to \infty$ for each fixed s.

Consider a system with the form of

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}), \quad \boldsymbol{x} \in \mathbf{R}^n,\ \boldsymbol{u} \in \mathbf{R}^m,\ t \in \mathbf{R}_+ \tag{1}$$

where $\boldsymbol{f} : \mathbf{R}^n \times \mathbf{R}^m \mapsto \mathbf{R}^n$ is locally Lipschitz.

Definition 1. System (1) is iISS with respect to $\boldsymbol{u}$ if there exist functions $\alpha \in \mathcal{K}_\infty$, $\beta \in \mathcal{KL}$, and $\gamma \in \mathcal{K}$ such that for each initial condition $\boldsymbol{x}(0) \in \mathbf{R}^n$ and each measurable, locally essential bounded function $\boldsymbol{u} : \mathbf{R}_+ \mapsto \mathbf{R}^m$, the solution $\boldsymbol{x}(t)$ exists for each $t > 0$ and satisfies

$$\alpha(|\boldsymbol{x}(t)|) \leq \beta(|\boldsymbol{x}(0)|, t) + \int_0^t \gamma(|\boldsymbol{u}(\tau)|)\mathrm{d}\tau \tag{2}$$

In view of [1], iISS property can be equivalently characterized using the Lyapunov function.

Proposition 1. System (1) is iISS if and only if there exists a positive definite and proper function V, called iISS-Lyapunov function, such that

$$\underline{\alpha}(|\boldsymbol{x}|) \leq V(t, \boldsymbol{x}) \leq \overline{\alpha}(|\boldsymbol{x}|) \tag{3}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u}) \leq -\alpha(|\boldsymbol{x}|) + \gamma(|\boldsymbol{u}|) \tag{4}$$

where α is a positive definite continuous function, $\underline{\alpha}$, $\overline{\alpha} \in \mathcal{K}_\infty$, and $\gamma \in \mathcal{K}$.

Motivated by [9], we give the following technical result, which will be used later. For simplicity, we use $\sigma_1(s) = O(\sigma_2(s))$ to mean that $\sigma_1(s) \leq c\sigma_2(s)$ for some constant $c > 0$ and all s in a small neighborhood of the origin.

Proposition 2. Consider an iISS system (1) with an iISS-Lyapunov function $V(t, \boldsymbol{x})$ satisfying (3) and (4), and take any smooth function ϕ with the following property

$$\phi^2(s) = O(\alpha(s)) \tag{5}$$

Moreover, when α is bounded, the following additional condition holds

$$\lim_{s\to\infty}\frac{\phi^2(s)}{\alpha(s)}<\infty \tag{6}$$

Then, there always exist a positive-definite function σ and class-$\mathcal{K}_\infty$ functions φ_i $(i=1,\cdots,m)$ such that

$$\int_0^t \phi^2(|\boldsymbol{x}(\tau)|)\mathrm{d}\tau \le \sigma(|\boldsymbol{x}(0)|)+\sum_{i=1}^m\int_0^t \varphi_i(|\boldsymbol{u}_i(\tau)|)\mathrm{d}\tau \tag{7}$$

Moreover, if the iISS-gain γ in (4) satisfies $\gamma(s)=O(s^2)$, so does φ_i.

Proof. For the proof of this proposition, the reader can refer to Proposition 2 of [9]. □

2 Output-feedback control design

Consider a class of large-scale nonlinear systems composed of N interconnected subsystems with relative degree ρ_i described by

$$\begin{aligned}\dot{\boldsymbol{\eta}}_i &= q_i(t,\boldsymbol{\eta}_i,y_i)+\sum_{k=1,k\ne i}^N \boldsymbol{f}_{ik}(t,y_k)\\ \dot{x}_{ij} &= x_{i,j+1}+g_{ij}(t,\boldsymbol{\eta}_i,y_i)+\sum_{k=1,k\ne i}^N h_{ik}^j(t,y_k)\\ \dot{x}_{i\rho_i} &= \beta_i u_i+g_{i\rho_i}(t,\boldsymbol{\eta}_i,y_i)+\sum_{k=1,k\ne i}^N h_{ik}^{\rho_i}(t,y_k)\\ y_i &= x_{i1},\quad 1\le i\le N,\quad 1\le j\le \rho_i-1\end{aligned} \tag{8}$$

where $\boldsymbol{\eta}_i\in\mathbf{R}^{n_i}$ and $\boldsymbol{x}_i=(x_{i1},\cdots,x_{i\rho_i})\in\mathbf{R}^{\rho_i}$ are the states, $u_i\in\mathbf{R}^1$ and $y_i\in\mathbf{R}^1$ are input and output of the ith subsystem, respectively; $\boldsymbol{f}_{ik}\in\mathbf{R}^{n_i}$ and $\boldsymbol{h}_{ik}\in\mathbf{R}^{\rho_i}$ denote the interactions from the kth subsystem to the ith subsystem. It is assumed that y_i is measurable and the uncertain functions q_i, g_{ij}, f_{ik}, and $h_{ik}^j(1\le j\le\rho_i)$ are locally Lipschitz. Here, β_i is an unknown nonzero constant with indefinite sign. In this section, the following hypotheses are made on system (8).

H1. The $\boldsymbol{\eta}_i$-subsystem of (8) is iISS with respect to $\boldsymbol{y}=(y_1,\cdots,y_N)^{\mathrm{T}}$ in the sense that there exists an iISS-Lyapunov function V_{i0} such that

$$\begin{aligned}&\underline{\alpha}_{i0}(|\boldsymbol{\eta}_i|)\le V_{i0}(t,\boldsymbol{\eta}_i)\le\overline{\alpha}_{i0}(|\boldsymbol{\eta}_i|)\\ &\frac{\partial V_{i0}}{\partial t}+\frac{\partial V_{i0}}{\partial\boldsymbol{\eta}_i}(t,\boldsymbol{\eta}_i)\Big(q_i(t,\boldsymbol{\eta}_i,y_i)+\sum_{k=1,k\ne i}^N\boldsymbol{f}_{ik}(t,y_k)\Big)\le\\ &-\alpha_{i0}(|\boldsymbol{\eta}_i|)+\gamma_{i0}(|\boldsymbol{y}|)\end{aligned} \tag{9}$$

where α_{i0} is a positive definite continuous function, $\underline{\alpha}_{i0}$, $\overline{\alpha}_{i0}\in\mathcal{K}_\infty$, and $\gamma_{i0}\in\mathcal{K}$.

H2. For each $1\le j\le\rho_i$, there exist two unknown positive constants, p_{ij1} and p_{ij2}, and two known positive semidefinite, smooth functions ϕ_{ij1} and ϕ_{ij2} such that

$$|g_{ij}(t,\boldsymbol{\eta}_i,y_i)|\le p_{ij1}\phi_{ij1}(|y_i|)+p_{ij2}\phi_{ij2}(|\boldsymbol{\eta}_i|) \tag{10}$$

H3. $\boldsymbol{f}_{ik}(t,y_k)$ and $\boldsymbol{h}_{ik}(t,y_k)$ satisfy

$$|\boldsymbol{f}_{ik}(t,y_k)|\le r_{ik1}|y_k|,\ \ \|\boldsymbol{h}_{ik}(t,y_k)\|\le r_{ik2}|y_k| \tag{11}$$

where r_{ik1} and r_{ik2} are unknown constants denoting the strengths of interactions.

Remark 1. The linear-growth condition in hypothesis H3 is made only for simplifying the presentation and highlighting the main contribution in this paper. In the spirit of [12−14], it can actually be relaxed by a nonlinear-growth condition.

The control objective is to design a decentralized adaptive controller for each subsystem so that all the signals of the closed-loop system are bounded over $[0,\infty)$, and all the states, inputs and outputs can be regulated to zero.

2.1 Adaptive backstepping controller design

First, the following filters are introduced to rebuild the unmeasured partial-states $(x_{i2},\cdots,x_{i\rho_i})$,

$$\begin{aligned}\dot{\hat{\xi}}_{ij} &= \hat{\xi}_{i,j+1}-L_{ij}\hat{\xi}_{i1},\quad 1\le j\le\rho_i-1\\ \dot{\hat{\xi}}_{i\rho_i} &= u_i-L_{i\rho_i}\hat{\xi}_{i1}\end{aligned} \tag{12}$$

where $\boldsymbol{L}_i=(L_{i1},\cdots,L_{i\rho_i})^{\mathrm{T}}$ is chosen such that $A_i=\left(-\boldsymbol{L}_i\ ,\ \begin{matrix}I_{\rho_i-1}\\ 0,\cdots,0\end{matrix}\right)$ is asymptotically stable. For each $1\le j\le\rho_i$, by denoting

$$\xi_{ij}=\frac{1}{\beta_i}x_{ij} \tag{13}$$

$$e_{ij}=\frac{\xi_{ij}-\hat{\xi}_{ij}}{p_i^*} \tag{14}$$

with $\boldsymbol{p}_i^*=\max\left\{\frac{1}{|\beta_i|},\ \frac{p_{ij1}}{|\beta_i|},\ \frac{p_{ij2}}{|\beta_i|},\ p_{i12}^2\mid\forall\ 1\le j\le\rho_i\right\}$, it follows that

$$\dot{\boldsymbol{e}}_i=A_i\boldsymbol{e}_i+\frac{1}{p_i^*}\boldsymbol{G}_i(t,\boldsymbol{\eta}_i,y_i)+\frac{1}{p_i^*\beta_i}\sum_{k=1,k\ne i}^N\boldsymbol{h}_{ik}(t,y_k) \tag{15}$$

with $\boldsymbol{G}_i(t,\eta_i,y_i)=\mathrm{col}((g_{i1}(t,\eta_i,y_i)/\beta_i+L_{i1}y_i/\beta_i),\cdots,(g_{i\rho_i}(t,\eta_i,y_i)/\beta_i+L_{i\rho_i}y_i/\beta_i))$. Because A_i is asymptotically stable, there exists a $P_i=P_i^{\mathrm{T}}>0$ such that

$$P_iA_i+A_i^{\mathrm{T}}P_i=-2I_{\rho_i} \tag{16}$$

Along the solutions of (15), differentiating the quadratic function $V_{\boldsymbol{e}_i}=\boldsymbol{e}_i^{\mathrm{T}}P_i\boldsymbol{e}_i$ yields

$$\begin{aligned}\dot{V}_{\boldsymbol{e}_i}\ \le\ &-|\boldsymbol{e}_i|^2+4\|P_i\|^2\sum_{j=1}^{\rho_i}(\phi_{ij1}(|y_i|)+L_{ij}|y_i|)^2+\\ &4\|P_i\|^2\sum_{j=1}^{\rho_i}\phi_{ij2}^2(|\boldsymbol{\eta}_i|)+2(N-1)\times\\ &\|P_i\|^2\sum_{k=1,k\ne i}^N r_{ik2}^2y_k^2\psi_k\end{aligned} \tag{17}$$

To summarize, the complete system can be expressed as

$$\begin{aligned}\dot{\boldsymbol{\eta}}_i &= \boldsymbol{q}_i(t,\boldsymbol{\eta}_i,y_i)+\sum_{k=1,k\ne i}^N\boldsymbol{f}_{ik}(t,y_k)\\ \dot{\boldsymbol{e}}_i &= A_i\boldsymbol{e}_i+\frac{1}{p_i^*}\boldsymbol{G}_i(t,\boldsymbol{\eta}_i,y_i)+\frac{1}{p_i^*\beta_i}\sum_{k=1,k\ne i}^N\boldsymbol{h}_{ik}(t,y_k)\\ \dot{y}_i &= \beta_i\hat{\xi}_{i2}+\beta_ip_i^*e_{i2}+g_{i1}(t,\boldsymbol{\eta}_i,y_i)+\sum_{k=1,k\ne i}^N h_{ik}^1(t,y_k)\\ \dot{\hat{\xi}}_{ij} &= \hat{\xi}_{i,j+1}-L_{ij}\hat{\xi}_{i1},\quad 1\le j\le\rho_i-1\\ \dot{\hat{\xi}}_{i\rho_i} &= u_i-L_{i\rho_i}\hat{\xi}_{i1}\end{aligned} \tag{18}$$

The next task is to design a controller for (18).

Step 1. Begin with the y_i-subsystem of (18) and consider $\hat{\xi}_{i2}$ as the virtual control input. One can choose for the ith subsystem the following virtual control law

$$\alpha_{i1} = c_i N_0(k_i)\psi_i(y_i)y_i, \quad \dot{k}_i = \Gamma_i \psi_i(y_i) y_i^2 \tag{19}$$

where $N_0(\cdot)$ is a smooth Nussbaum-type function introduced to counteract the lack of a priori knowledge of the high-frequency-gain sign ($\mathrm{sgn}(\beta_i)$), c_i, $\Gamma_i > 0$ are design constants, and design function $\psi_i \geq 1$. A Nussbaum-type function $N_0(\cdot)$ possesses the properties $\lim_{k\to\infty}\sup \frac{1}{k}\int_0^k N_0(s)\mathrm{d}s = \infty$, $\lim_{k\to\infty}\inf \frac{1}{k}\int_0^k N_0(s)\mathrm{d}s = -\infty$. In this paper, we choose $N_0 : s \mapsto s^2\cos(s)$. Setting $z_{i1} = \hat{\xi}_{i2} - \alpha_{i1}(k_i, y_i)$, one obtains

$$\dot{z}_{i1} = \upsilon_{i2} - \frac{\partial \alpha_{i1}}{\partial y_i}\left(\beta_i\hat{\xi}_{i2} + \beta_i p_i^* e_{i2} + g_{i1} + \sum_{k=1,k\neq i}^{N} h_{ik}^1\right) \tag{20}$$

where $\upsilon_{i2} = \hat{\xi}_{i3} - L_{i2}\hat{\xi}_{i1} - \frac{\partial \alpha_{i1}}{\partial k_i}\Gamma_i\psi_i(y_i)y_i^2$. Denote $V_{i1} = \frac{1}{2}y_i^2 + \frac{1}{2\lambda_i}(\hat{p}_i - p_i)^2$, where $\lambda_i > 0$ is a design parameter, $\hat{p}_i$ is the estimate of $p_i = \max\left\{\frac{|\beta_i|}{2},\ \frac{\beta_i^2}{4},\ p_i^* + \frac{(\beta_i p_i^*)^2}{4\varepsilon_{i1}} + \frac{N-1}{4}\right\}$, $0 < \varepsilon_{i1} < \rho_i^{-1}$ is a small design parameter.

Step 2. Consider the augmented system composed of the y_i-subsystem and (20) in which υ_{i2} (or equivalently $\hat{\xi}_{i3}$) is viewed as the virtual control input. The derivative of the Lyapunov function $V_{i2} = V_{i1} + \frac{1}{2}z_{i1}^2$ along the solutions of (18) satisfies

$$\begin{aligned}\dot{V}_{i2} =\ & c_i\beta_i N_0(k_i)\psi_i(y_i)y_i^2 + \beta_i y_i z_{i1} + \beta_i p_i^* e_{i2} y_i + g_{i1} y_i + \\ & \sum_{k=1,k\neq i}^{N} h_{ik}^1 y_i + z_{i1}\Bigg[\upsilon_{i2} - \frac{\partial \alpha_{i1}}{\partial y_i}\Bigg(\beta_i(\alpha_{i1} + z_{i1}) + g_{i1} + \\ & \beta_i p_i^* e_{i2} + \sum_{k=1,k\neq i}^{N} h_{ik}^1\Bigg)\Bigg] + \frac{1}{\lambda_i}(\hat{p}_i - p_i)\dot{\hat{p}}_i\end{aligned} \tag{21}$$

By H2 and H3, with Young's inequality, one obtains

$$\begin{aligned}&\beta_i y_i z_{i1} - \beta_i\frac{\partial \alpha_{i1}}{\partial y_i}z_{i1}(\alpha_{i1} + z_{i1}) \leq \\ &\beta_i^2 y_i^2\psi_i + \frac{1 + \left(c_i N_0\psi_i\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2}{2\psi_i}z_{i1}^2 + \\ &\frac{|\beta_i|}{2}\left(1 + \left(\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2\right)z_{i1}^2\end{aligned} \tag{22}$$

$$\begin{aligned}&- z_{i1}\frac{\partial \alpha_{i1}}{\partial y_i}(\beta_i p_i^* e_{i2} + g_{i1}) \leq \\ &\varepsilon_{i1}e_{i2}^2 + \left(p_i^* + \frac{(\beta_i p_i^*)^2}{4\varepsilon_{i1}}\right)\left(\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2 z_{i1}^2 + \frac{1}{4p_i^*}g_{i1}^2\end{aligned} \tag{23}$$

$$\begin{aligned}&\sum_{k=1,k\neq i}^{N} h_{ik}^1 y_i - z_{i1}\frac{\partial \alpha_{i1}}{\partial y_i}\sum_{k=1,k\neq i}^{N} h_{ik}^1 \leq \\ &\sum_{k=1}^{N} r_{i1}^k y_k^2\psi_k + \frac{N-1}{4}\left(\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2 z_{i1}^2\end{aligned} \tag{24}$$

By setting

$$\begin{aligned}\Phi_{i1}(t, e_{i2}, \eta_i, y_i) = \ & \varepsilon_{i1}e_{i2}^2 + \beta_i p_i^* e_{i2} y_i + g_{i1}(t, \eta_i, y_i)y_i + \\ & \beta_i^2 y_i^2 \psi_i(y_i) + \frac{1}{4p_i^*}g_{i1}^2,\end{aligned} \tag{25}$$

$$\begin{aligned}\alpha_{i2} = \ & -c_{i1}z_{i1} + L_{i2}\hat{\xi}_{i1} + \frac{\partial \alpha_{i1}}{\partial k_i}\Gamma_i\psi_i(y_i)y_i^2 - \\ & \frac{1 + \left(c_i N_0\psi_i\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2}{2\psi_i}z_{i1} - \\ & \hat{p}_i\left(1 + 2\left(\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2\right)z_{i1}\end{aligned} \tag{26}$$

$$z_{i2} = \hat{\xi}_{i3} - \alpha_{i2}, \tag{27}$$

$$\tau_{i1} = \lambda_i\left(1 + 2\left(\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2\right)z_{i1}^2 \tag{28}$$

where $c_{i1} > \rho_i - 2$ is a design parameter. (21) becomes

$$\begin{aligned}\dot{V}_{i2} \leq \ & c_i\beta_i N_0(k_i)\psi_i(y_i)y_i^2 + \Phi_{i1} + z_{i1}z_{i2} - c_{i1}z_{i1}^2 + \\ & \frac{1}{\lambda_i}(\hat{p}_i - p_i)\left(\dot{\hat{p}}_i - \tau_{i1}\right) + \sum_{k=1}^{N} r_{i1}^k y_k^2\psi_k\end{aligned} \tag{29}$$

and z_{i2} satisfies

$$\begin{aligned}\dot{z}_{i2} = \ & \upsilon_{i3} - \frac{\partial \alpha_{i2}}{\partial \hat{p}_i}\dot{\hat{p}}_i - \frac{\partial \alpha_{i2}}{\partial y_i}\Bigg(\beta_i\hat{\xi}_{i2} + \beta_i p_i^* e_{i2} + g_{i1} + \\ & \sum_{k=1,k\neq i}^{N} h_{ik}^1\Bigg)\end{aligned} \tag{30}$$

where $\upsilon_{i3} = \hat{\xi}_{i4} - L_{i3}\hat{\xi}_{i1} - \frac{\partial \alpha_{i2}}{\partial k_i}\Gamma_i\psi_i(y_i)y_i^2 - \sum_{s=1}^{2}\frac{\partial \alpha_{i2}}{\partial \hat{\xi}_{is}}(\hat{\xi}_{i,s+1} - L_{is}\hat{\xi}_{i1})$.

Step $\boldsymbol{l = 3, \cdots, \rho_i}$. It is easy to obtain the following conclusion by induction that the time derivative of the augmented function $V_{il} = V_{i,l-1} + \frac{1}{2}z_{i,l-1}^2$ satisfies

$$\begin{aligned}&\dot{V}_{il} \leq c_i\beta_i N_0(k_i)\psi_i(y_i)y_i^2 + \Phi_{i,l-1} + z_{i,l-1}z_{il} - (c_{i1} - l + \\ &2)\times z_{i1}^2 - \textstyle\sum_{s=2}^{l-1}c_{is}z_{is}^2 + \frac{1}{\lambda_i}\left(\hat{p}_i - p_i - \sum_{s=1}^{l-1}\lambda_i z_{is}\frac{\partial \alpha_{is}}{\partial \hat{p}_i}\right)\times \\ &\left(\dot{\hat{p}}_i - \tau_{i,l-1}\right) + \textstyle\sum_{k=1}^{N} r_{i,l-1}^k y_k^2\psi_k\end{aligned} \tag{31}$$

for any $l = 3, \cdots, \rho_i$, where

$$\Phi_{i,l-1} = \Phi_{i,l-2} + \beta_i^2 y_i^2\psi_i(y_i) + \varepsilon_{i1}e_{i2}^2 + \frac{1}{4p_i^*}g_{i1}^2 \tag{32}$$

$$\begin{aligned}\alpha_{il} = \ & -c_{i,l-1}z_{i,l-1} + L_{il}\hat{\xi}_{i1} + \frac{\partial \alpha_{i,l-1}}{\partial k_i}\Gamma_i\psi_i(y_i)y_i^2 + \\ & \sum_{s=1}^{l-1}\frac{\partial \alpha_{i,l-1}}{\partial \hat{\xi}_{is}}\left(\hat{\xi}_{i,s+1} - L_{is}\hat{\xi}_{i1}\right) - z_{i,l-2} + \\ & \frac{\partial \alpha_{i,l-1}}{\partial \hat{p}_i}\tau_{i,l-1} - \frac{\left(c_i N_0\frac{\partial \alpha_{i1}}{\partial y_i}\right)^2\psi_i}{4}z_{i,l-1} + \\ & \sum_{s=1}^{l-2}2z_{is}\frac{\partial \alpha_{is}}{\partial \hat{p}_i}\lambda_i\left(\frac{\partial \alpha_{i,l-1}}{\partial y_i}\right)^2 z_{i,l-1} - \\ & 2\hat{p}_i\left(\frac{\partial \alpha_{i,l-1}}{\partial y_i}\right)^2 z_{i,l-1}, \quad 1 \leq s \leq l\end{aligned} \tag{33}$$

$$z_{is} = \hat{\xi}_{i,s+1} - \alpha_{is}\left(k_i, y_i, \hat{\xi}_{i1}, \cdots, \hat{\xi}_{is}, \hat{p}_i\right) \tag{34}$$

$$\tau_{i,l-1} = \tau_{i,l-2} + 2\lambda_i \left(\frac{\partial \alpha_{i,l-1}}{\partial y_i}\right)^2 z_{i,l-1} \tag{35}$$

with $c_{i,l-1} > 0$.

Therefore, at step ρ_i, one obtains the smooth dynamic output feedback law

$$\begin{aligned} &\dot{k}_i = \Gamma_i \psi_i(y_i) y_i^2, \quad \dot{\hat{p}}_i = \tau_{i,\rho_i-1} \\ &u_i = \alpha_{i\rho_i}\left(k_i, y_i, \hat{\xi}_{i1}, \cdots, \hat{\xi}_{i\rho_i}, \hat{p}_i\right) \end{aligned} \tag{36}$$

such that the time derivative of the function $V_{i\rho_i} = \frac{1}{2}y_i^2 + \sum_{s=1}^{\rho_i-1} \frac{1}{2}z_{is}^2 + \frac{1}{2\lambda_i}(\hat{p}_i - p_i)^2$ satisfies

$$\begin{aligned} \dot{V}_{i\rho_i} \le\ & c_i\beta_i N_0(k_i)\psi_i(y_i)y_i^2 + \Phi_{i,\rho_i-1} - (c_{i1} - \rho_i + 2)z_{i1}^2 - \\ & \sum_{s=2}^{\rho_i-1} c_{is} z_{is}^2 + \sum_{k=1}^{N} r_{i,\rho_i-1}^k y_k^2 \psi_k \end{aligned} \tag{37}$$

2.2 Main result

Now, we state the main theorem in this paper.

Theorem 1. Assume that the hypotheses H1 and H2 hold with the following properties

$$\phi_{ij2}^2(s) = O(\alpha_{i0}(s)), \quad 1 \le j \le \rho_i \tag{38}$$

and that in the case, where α_{i0} is bounded,

$$\lim_{s\to\infty} \sup \frac{\phi_{ij2}^2(s)}{\alpha_{i0}(s)} < \infty, \quad 1 \le j \le \rho_i \tag{39}$$

If $\gamma_{i0}(s) = O(s^2)$ in H1, then the solutions of (8) and (36) are well-defined and bounded over $[0, \infty)$ for appropriately chosen smooth function ψ_i. Furthermore,

$$\lim_{t\to\infty} (|\boldsymbol{x}_i(t)| + |\boldsymbol{\eta}_i(t)| + |u_i(t)|) = 0 \tag{40}$$

Proof. Consider the function as follows

$$V_{ic} = V_{e_i} + V_{i\rho_i} \tag{41}$$

With the help of H2, by (25) and (32), one has

$$\begin{aligned} |\Phi_{i,\rho_i-1}| \le\ & \rho_i\varepsilon_{i1}e_{i2}^2 + (\rho_i - 1)\beta_i^2 y_i^2 \psi_i(y_i) + \rho_i \phi_{i12}^2(|\boldsymbol{\eta}_i|) + \\ & \theta_{i0}\left(y_i^2 + \phi_{i11}(|y_i|)|y_i| + \phi_{i11}^2(|y_i|)\right) \end{aligned} \tag{42}$$

where $\theta_{i0} \ge \max\left\{\frac{(\beta_i p_i^*)^2}{4\varepsilon_{i1}} + \frac{p_{i12}^2}{4}, p_{i11}, \frac{p_{i11}^2}{2p_i^*}(\rho_i - 1)\right\}$. By H1 and Proposition 2,

$$\begin{aligned} &\int_0^t \phi_{ij2}^2(|\boldsymbol{\eta}_i(s)|)\mathrm{d}s \le \\ &\sigma_{ij0}(|\boldsymbol{\eta}_i(0)|) + \sum_{k=1}^{N}\int_0^t \varphi_{ijk}(|y_k(s)|)\mathrm{d}s \end{aligned} \tag{43}$$

holds, where σ_{ij0} $(1 \le j \le \rho_i)$ are positive definite, and φ_{ijk} $(1 \le j \le \rho_i,\ 1 \le k \le N)$ are of class $\mathcal{K}_\infty$ and quadratic near the origin. Take a smooth function $\psi_i \ge 1$ so that

$$\begin{aligned} \psi_i(y_i)y_i^2 \ge \max\Big\{& y_i^2 + |y_i|\phi_{i11}(|y_i|) + \phi_{i11}^2(|y_i|), \varphi_{mji}(|y_i|) \\ & (\phi_{ij1}(|y_i|) + L_{ij}|y_i|)^2, \gamma_{m0i}(|y_i|), \forall 1 \le j \le \rho_i \\ & 1 \le m \le N\Big\} \end{aligned} \tag{44}$$

Such a function ψ_i always exists because of the conditions of Theorem 1. Then, with (17), (37), (42), and (44), it follows from (41) that

$$\begin{aligned} \dot{V}_{ic} \le\ & c_i\beta_i N_0(k_i)\psi_i(y_i)y_i^2 - (c_{i1} - \rho_i + 2)z_{i1}^2 - \\ & \sum_{s=2}^{\rho_i-1} c_{is}z_{is}^2 - (1 - \rho_i\varepsilon_{i1})|e_i|^2 + \big[(\rho_i - 1)\beta_i^2 + \\ & \theta_{i0} + 4\rho_i\|P_i\|^2\big]y_i^2\psi_i(y_i) + \rho_i\phi_{i12}^2(|\boldsymbol{\eta}_i|) + 4\|P_i\|^2 \times \\ & \sum_{j=1}^{\rho_i} \phi_{ij2}^2(|\boldsymbol{\eta}_i|) + \sum_{k=1}^{N} r_{i\rho_i}^k y_k^2 \psi_k(y_k) \end{aligned} \tag{45}$$

Integrating both sides of (45), and using (19), (43), and (44), one has

$$V_{ic}(t) \le c_i\Gamma_i^{-1}\int_0^{k_i(t)} \beta_i N_0(s)\mathrm{d}s + \sum_{m=1}^{N} r_i^m \Gamma_m^{-1} k_m(t) + d_{i1} \tag{46}$$

where

$$r_i^m = \begin{cases} (\rho_i - 1)\beta_i^2 + \theta_{i0} + 8\rho_i\|P_i\|^2 + \rho_i + r_{i\rho_i}^i, & m = i \\ \rho_i + 4\rho_i\|P_i\|^2 + r_{i\rho_i}^m, & m \ne i \end{cases}$$

$$\begin{aligned} d_{i1} = V_{ic}(0) - & \sum_{m=1}^{N} \gamma_i^m \Gamma_m^{-1} k_m(0) - c_i\Gamma_i^{-1}\int_0^{k_i(0)} \beta_i N_0(s)\mathrm{d}s + \\ & \rho_i\sigma_{i10}(|\boldsymbol{\eta}_i(0)|) + 4\|P_i\|^2 \sum_{j=1}^{\rho_i} \sigma_{ij0}(|\boldsymbol{\eta}_i(0)|) \end{aligned}$$

For the proof of the first statement on the boundedness property and the second statement on the convergence property (40), the reader can refer to Theorem 1 of [9] and Section III-B of [10]. □

3 A simulation example

According to the design procedure given in Section 2, this section considers the control design for the following interconnected system consisting of two relative degree-two subsystems and illustrates the dynamics behaviors of all the closed-loop signals.

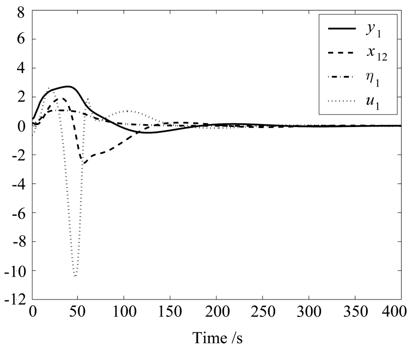
$$\begin{aligned} \dot{\eta}_1 &= -c_{01}\eta_1 + q_1(y_1)\eta_1 + f_1(y_2) \\ \dot{x}_{11} &= x_{12} + g_{11}(\eta_1, y_1) + h_1(y_2) \\ \dot{x}_{12} &= \beta_1 u_1 \\ y_1 &= x_{11} \\ \dot{\eta}_2 &= -c_{02}\eta_2 + q_2(y_2)\eta_2 + f_2(y_1) \\ \dot{x}_{21} &= x_{22} + g_{21}(\eta_1, y_1) + h_2(y_1) \\ \dot{x}_{22} &= \beta_2 u_2 \\ y_2 &= x_{21} \end{aligned} \tag{47}$$

Let uncertain functions and interconnections be as follows.

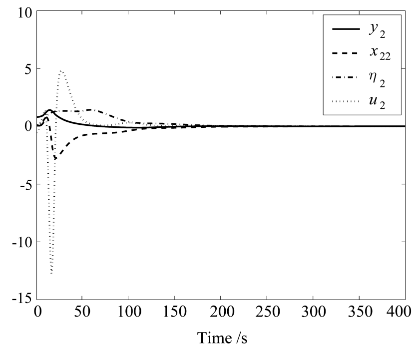
$$\begin{aligned} &q_1(y_1) = 0.1y_1^2,\ f_1(y_2) = y_2\sin(y_2) \\ &g_{11}(\eta_1, y_1) = \frac{4\eta_1}{1 + \eta_1^2}\cos(y_1),\ h_1(y_2) = y_2 \\ &q_2(y_2) = 0.1y_2^2,\ f_2(y_1) = y_1\sin(y_1) \\ &g_{21}(\eta_2, y_2) = \frac{\eta_2}{1 + \eta_2^2}\cos(y_2),\ h_2(y_1) = 0 \end{aligned}$$

For η_i-subsystem, one employs $V_{i0} = \ln(1 + \eta_i^2)$ $(i = 1, 2)$, $\rho_1(s) = 1$, and $\rho_2(s) = 2$. It follows from (43) and (44) that $\psi_i(y_i) = 4$ $(i = 1, 2)$. The design parameters are chosen as $c_{01} = c_{02} = \beta_1 = \beta_2 = 1$, $c_1 = c_2 = 0.25$, $L_{11} = L_{12} = L_{21} = L_{22} = 1$, $c_{11} = c_{21} = 0.5$, $\lambda_1 = \lambda_2 = 0.05$, $\Gamma_1 = 0.05$, $\Gamma_2 = 0.3$, and the initial conditions are $\hat{\xi}_{11}(0) = \hat{\xi}_{12}(0) = \hat{\xi}_{21}(0) = \hat{\xi}_{22}(0) = 1$, $\eta_1(0) = \eta_2(0) = 0.1$, $y_1(0) = 0.5$, $x_{12}(0) = 0.2$, $y_2(0) = 0.8$, $x_{22}(0) = 0.1$, $k_1(0) = k_2(0) = 0$, $\hat{p}_1(0) = 1$, $\hat{p}_2(0) = 0.9$.

From Fig. 1, one can see that for linear growth interconnections, the designed decentralized adaptive controllers are robust to the nonlinear unmodeled dynamics and can achieve good regulation performance.



(a)



(b)

Fig. 1 Responses of adaptive output feedback system

4 Conclusion

In this paper, a decentralized adaptive output regulation problem is addressed for a class of large-scale nonlinear systems with iISS inverse dynamics, nonlinear uncertainties, and unknown control direction. The main contributions are to deal with the interconnections tactfully by using variable separation technique and choosing proper design functions, and to propose a constructive decentralized adaptive control scheme by combining Nussbaum-type gain approach, backstepping design technique and the subtle analysis approach[10]. It is shown that all the signals in the closed-loop system obtained are bounded and the asymptotic regulation is achieved.

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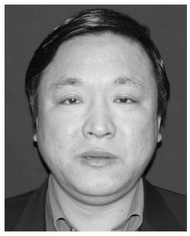

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