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Some results for the perturbation of the W-weighted Drazin inverse

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Abstract: Given an $m \times n$ matrix A and its perturbation matrix E, some new properties of the W-weighted Drazin inverse $A_{d,W}$ and $B_{d,W}$ of A and B are obtained, where $B = A + E$. Under certain conditions, the Banach – type perturbation theorem for the W-weighted Drazin inverses of A and B are established, and the perturbation bounds of $\| B_{d,W} \|$ and $\| B_{d,W} - A_{d,W} \| / \| A_{d,W} \|$ are presented. When A and B are square matrices and W is an identity matrix, some known results in the literature related to the Drazin inverse and the group inverse are reduced by the results in this paper as special cases.

Key words: perturbation; W-weighted Drazin inverse; index; core-rank; condition number

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1 Introduction

It is well known that the Drazin inverse of matrix is very useful because of its various applications. The perturbation theory of the Drazin inverse has been widely discussed and can be found in the literature[1,2,4-8]. In the important paper[3], Cline and Greville extended the Drazin inverse of a square matrix to a rectangular matrix and introduced the notion of the W-weighted Drazin inverse. Many properties and applications of the W-weighted Drazin inverse have been discussed later in [1,2,9,10,11]. In this paper, we study some properties of the W-weighted Drazin inverse, establish a Banach – type perturbation theorem for this inverse, and give the perturbation bounds for $\| B_{d,W} \|$ and $\| B_{d,W} - A_{d,W} \| / \| A_{d,W} \|$. Some results in [4,6,7] are special cases of our results.

2 Preliminaries

Let $A \in C^{n \times n}$. The smallest nonnegative integer k such that $\mathrm{rank}(A^k) = \mathrm{rank}(A^{k+1})$ is called the index of A, and denoted by $k = \mathrm{Ind}(A)$.

Let $A \in C^{m \times n}$ and $W \in C^{n \times m}$. A matrix X is called the W-weighted Drazin inverse of A if

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$$(AW)^{k+1}XW = (AW)^k,\ XWAWX = X,\ AWX = XWA, \tag{2.1}$$

for some integer $k > 0$. In this case, X is denoted by $X = A_{d,W}$. In particular, when A is square and $W = I$, the W-weighted Drazin inverse is called the Drazin inverse and denoted by $X = A_d$. Furthermore, if $k = 1$, the Drazin inverse is reduced to the group inverse and denoted by A_g .

Let $A \in C^{n\times n}$ with $\mathrm{Ind}(A) = k$. $\mathrm{rank}(A^k)$ is called the core-rank of A and written as core-rank(A).

Throughout this paper, all the notations are the same as those in [2]. And the following lemmas will be used.

Lemma 2.1[1,2] Let L and M be complementary subspaces of C^n . Let $A \in C^{n\times n}$. Then

(a) $P_{L,M}A = A$ if and only if $R(A) \subset L$;

(b) $AP_{L,M} = A$ if and only if $N(A) \supset M$.

Lemma 2.2[1,2] Let $A \in C^{n\times n}$. Then

(a) $\mathrm{Ind}(A) = k$ if and only if $R(A^k) \oplus N(A^k) = C^n$;

(b) If A has index k, A^l has index 1 and $(A^l)_g = (A_d)^l$ for $l \geqslant k$;

(c) If A has index k, $AA_d = A^l(A^l)_g$ and $A_d = (A^l)_g A^{l-1}$ for $l \geqslant k$.

Lemma 2.3[2,3] Let $A \in C^{m\times n}$, $W \in C^{n\times m}$. Then A has the unique W-weighted Drazin inverse $A_{d,W}$ which satisfies:

(a) $A_{d,W} = A(WA)_d^2 = (AW)_d^2 A$;

(b) $A_{d,W}W = (AW)_d$;

(c) $WA_{d,W} = (WA)_d$.

Lemma 2.4[2] Let $A \in C^{m\times n}$, $W \in C^{n\times m}$, $\mathrm{Ind}(AW) = k_1$ and $\mathrm{Ind}(WA) = k_2$. Then

(a) $R(A_{d,W}) = R((AW)^{k_1}) = R((AW)_d)$;

(b) $N(A_{d,W}) = N((WA)^{k_2}) = N((WA)_d)$;

(c) $WAWA_{d,W} = (WA)(WA)_d = P_{R((WA)^{k_2}),N((WA)^{k_2})}$;

(d) $A_{d,W}WAW = (AW)_d(AW) = P_{R((AW)^{k_1}),N((AW)^{k_1})}$;

(e) $R(A_{d,W}W) \oplus N(A_{d,W}W) = C^m$;

(f) $R(WA_{d,W}) \oplus N(WA_{d,W}) = C^n$.

Lemma 2.5[1,2] Suppose that $\| F \| < 1$. Then $I + F$ is nonsingular and

$$\| (I - F)^{-1} \| \leqslant 1/(1 - \| F \|). \tag{2.2}$$

3 Main Results

In this section, we prove some new properties of $A_{d,W}$ and $B_{d,W}$, establish the Banach - type theorem , and give the perturbation bounds for $\| B_{d,W} \|$ and $\| B_{d,W} - A_{d,W} \| / \| A_{d,W} \|$.

Theorem 3.1 Let $A, E \in C^{m\times n}$ and $W \in C^{n\times m}$. Let $B = A + E$, $\mathrm{Ind}(AW) = k_1$, $\mathrm{Ind}(WA) = k_2$, $\mathrm{Ind}(BW) = j_1$, $\mathrm{Ind}(WB) = j_2$, $l_1 = max\{k_1, j_1\}$ and $l_2 = max\{k_2, j_2\}$. And let $(EW)(l_1) = (BW)^{l_1} - (AW)^{l_1}$ and $(WE)(l_2) = (WB)^{l_2} - (WA)^{l_2}$. If $\| WEWA_{d,W} \| < 1$ and $\| A_{d,W}WEW \| < 1$, then

$$B_{d,W} = (I + A_{d,W}WEW)^{-1}A_{d,W} = A_{d,W}(I + WEWA_{d,W})^{-1} \tag{3.1}$$

if and only if

$$\text{core-rank}(AW) = \text{core-rank}(BW); \tag{3.2a}$$

$$\text{core-rank}(WA) = \text{core-rank}(WB); \tag{3.2b}$$

$$A_{d,W}WAW(EW)(l_1) = (EW)(l_1) = (EW)(l_1)A_{d,W}WAW; \tag{3.2c}$$

$$WAWA_{d,W}(WE)(l_2) = (WE)(l_2) = (WE)(l_2)WAWA_{d,W}. \tag{3.2d}$$

Proof ($\Rightarrow$). Suppose that (3.1) holds. Then it is easy to see that

$$R(B_{d,W}) = R(A_{d,W}) \text{ and } N(B_{d,W}) = N(A_{d,W}).$$

By Lemma2.4, we obtain

$$R((AW)^{k_1}) = R((BW)^{j_1}) \text{ and } N((WA)^{k_2}) = N((WB)^{j_2}).$$

Hence

$$\text{core-rank}(AW) = \text{core-rank}(BW),$$
$$\text{core-rank}(WA) = \text{core-rank}(WB).$$

Furthermore, by using Lemma 2.1 and Lemma 2.4, we have

$$A_{d,W}WAW(EW)(l_1) = P_{R((AW)^{k_1}),N((AW)^{k_1})}((BW)^{l_1} - (AW)^{l_1}) = (BW)^{l_1} - (AW)^{l_1} = (EW)(l_1),$$

and

$$(WE)(l_2)WAWA_{d,W} = (WE)(l_2).$$

Similarly, we can establish the other two equalities in (3.2c) and (3.2d).

($\Leftarrow$). Conversely, suppose that all the four equations in (3.2) hold. From Lemma 2.2 and Lemma 2.3, we have

$$(BW)^{l_1} = (AW)^{l_1} + (EW)(l_1) = (AW)^{l_1} + A_{d,W}WAW(EW)(l_1) = (AW)^{l_1} + (AW)^{l_1}((AW)^{l_1})_g(EW)(l_1) = (AW)^{l_1}(I + ((AW)^{l_1})_g(EW)(l_1)).$$

Notice that core-rank(AW) = core-rank(BW). Then $R((AW)^{k_1}) = R((BW)^{j_1})$. Similarly, notice that core-rank(WA) = core-rank(WB). We have $(WB)^{l_2} = (I + (WE)(l_2)((WA)^{l_2})_g)(WA)^{l_2}$ and $N((WA)^{k_2}) = N((WB)^{j_2})$. Therefore, we have

$$R(A_{d,W}) = R(B_{d,W}) = R((AW)^{k_1}),\ N(A_{d,W}) = N(B_{d,W}) = N((WA)^{k_2}).$$

By computation, we obtain

$$B_{d,W} - A_{d,W} = -B_{d,W}WEWA_{d,W} + B_{d,W} - A_{d,W} + B_{d,W}W(B-A)WA_{d,W} = -B_{d,W}WEWA_{d,W} + (B_{d,W} - B_{d,W}P_{R((WA)^{k_2}),N((WA)^{k_2})}) - (A_{d,W} - P_{R((BW)^{j_1}),N((BW)^{j_1})}A_{d,W}) = -B_{d,W}WEWA_{d,W}.$$

Therefore

$$B_{d,W}(I + WEWA_{d,W}) = A_{d,W}.$$

Because of the assumption $\| WEWA_{d,W} \| < 1$, $I + WEWA_{d,W}$ is nonsingular. Thus

$$B_{d,W} = A_{d,W}(I + WEWA_{d,W})^{-1}.$$

Similarly, by the method described above, we can obtain the other equality in (3.1). This completes the proof.

From this Theorem, we can easily obtain the following corollary.

Corollary 3.1 Let $A, E \in C^{m\times n}$ and $W \in C^{n\times m}$. Let $B = A + E$, $\text{Ind}(AW) = k_1$, $\text{Ind}(WA) = k_2$, $\text{Ind}(BW) = j_1$, $\text{Ind}(WB) = j_2$, $l_1 = max\{k_1, j_1\}$ and $l_2 = max\{k_2, j_2\}$. And let $(EW)(l_1) = (BW)^{l_1} - (AW)^{l_1}$, $(WE)(l_2) = (WB)^{l_2} - (WA)^{l_2}$, $\| WEWA_{d,W} \| < 1$ and $\| A_{d,W}WEW \| < 1$. If Eqs. (3.2) hold, then

$$WAWA_{d,W} = WBWB_{d,W} \text{ and } A_{d,W}WAW = B_{d,W}WBW.$$

Theorem 3.2 Let $A, E \in C^{m\times n}$ and $W \in C^{n\times m}$. Let $B = A + E$, $\text{Ind}(AW) = k_1$, $\text{Ind}(WA) = k_2$, $\text{Ind}(BW) = j_1$, and $\text{Ind}(WB) = j_2$. Suppose that

$$A_{d,W}WAWEW = EW = EWA_{d,W}WAW, \tag{3.3a}$$

and

$$WEWAWA_{d,W} = WE = WAWA_{d,W}WE. \tag{3.3b}$$

Then the matrices

$$I + WA_{d,W}WE,\ I + EWA_{d,W}W,\ I + WEWA_{d,W},\ I + A_{d,W}WEW, \tag{3.4}$$

are nonsingular if and only if

$$N((BW)^i) = N((AW)^i),\ R((BW)^i) = R((AW)^i),\ i = 1,\cdots,k_1 \tag{3.5a}$$

and

$$N((WB)^j) = N((WA)^j),\ R((WB)^j) = R((WA)^j),\ j = 1,\cdots,k_2. \tag{3.5b}$$

Proof ($\Leftarrow$). Suppose that Eqs. (3.5) hold. We note that $\text{Ind}(BW) = \text{Ind}(AW) = k_1$ and $\text{Ind}(WB) = \text{Ind}(WA) = k_2$. By Lemma 2.2, we have some facts which will be used in this proof:

$$R((AW)^{k_1}) \oplus N((AW)^{k_1})) = C^m,\ R((BW)^{k_1}) \oplus N((BW)^{k_1})) = C^m,$$
$$R((WA)^{k_2}) \oplus N((WA)^{k_2})) = C^n,\ R((WB)^{k_2}) \oplus N((WB)^{k_2})) = C^n,$$

and

$$WAWA_{d,W} = WBWB_{d,W},\ A_{d,W}WAW = B_{d,W}WBW. \tag{3.6}$$

We shall prove that all the four matrices in (3.4) are nonsingular through the following four steps.

(a) Suppose that for some vector $x \in C^n$, $(I + WEWA_{d,W})x = 0$. From the assumption of Theorem3.2, we have

$$x = -WEWA_{d,W}x = -WAWA_{d,W}WEWA_{d,W}x = -P_{R((WA)^{k_2}),N((WA)^{k_2})}WEWA_{d,W}x.$$

This tell us that $x \in R((WA)^{k_2})$.

On the other hand, it holds that $B_{d,W}(I + WEWA_{d,W})x = 0$. Furthermore, we see from (3.6) that

$$B_{d,W}(I + WEWA_{d,W}) = B_{d,W} + B_{d,W}WEWA_{d,W} = B_{d,W} + B_{d,W}WBWA_{d,W} - B_{d,W}WBWB_{d,W} = A_{d,W}WAWA_{d,W} = A_{d,W}.$$

Then we have $A_{d,W}x = 0$, i. e., $x \in N(A_{d,W}) = N((WA)^{k_2})$. Hence

$$x \in R((WA)^{k_2}) \cap N((WA)^{k_2}) = \{0\}.$$

Therefore, we can conclude that $I + WEWA_{d,W}$ is nonsingular.

(b) Suppose that for some vector $x \in C^m$, $(I + A_{d,W}WEW)x = 0$. Then $x = -A_{d,W}WEWx \in R((AW)^{k_1})$. On the other hand, we note that

$$x = -A_{d,W}W(B - A)Wx = -A_{d,W}WBWx + A_{d,W}WAWx$$

and from Lemma2.4(d), we obtain

$$A_{d,W}WAWx = P_{R((AW)^{k_1}),N((AW)^{k_1})}x = x. \tag{3.7}$$

Hence $A_{d,W}WBWx = 0$. This tell us that $WBWx \in N(A_{d,W}) = N(B_{d,W})$, i. e, $B_{d,W}WBWx = 0$. From the fact (3.6), we have $A_{d,W}WAWx = 0$. By (3.7), we obtain $x = 0$. Therefore, $I + A_{d,W}WEW$ is nonsingular.

(c) Suppose that $x \in C^m$ and $(I + EWA_{d,W}W)x = 0$. We notice that

$$x + EWA_{d,W}Wx = 0. \tag{3.8}$$

Then $(I + WEWA_{d,W})Wx = 0$. Thus $Wx = 0$ from the non-singularity of $I + WEWA_{d,W}$. By (3.8), we obtain $x = 0$. This tell us that $I + EWA_{d,W}W$ is nonsingular.

(d) By an analogous argument like that in (c), we can prove that $I + WA_{d,W}WE$ is also nonsingular.

($\Rightarrow$). We shall prove this part of Theorem3.2 by induction.

Suppose that the matrices in (3.4) are nonsingular. Firstly, with the assumption, it is evident that

$$BW = AW + EW = AW + EWA_{d,W}WAW = (I + EWA_{d,W}W)AW, \tag{3.9}$$

and

$$BW = AW(I + A_{d,W}WEW). \tag{3.10}$$

By using the non - singularity of the matrices in (3.4), we see that $R(AW) = R(BW)$ and $N(AW) = N(BW)$. Secondly, suppose that

$$N((BW)^i) = N((AW)^i),\ R((BW)^i) = R((AW)^i),\ i = 1,\cdots,k_1 - 1. \tag{3.11}$$

Then we have

$$\begin{aligned}\text{rank}((BW)^{k_1}) &= \text{rank}((I + EWA_{d,W}W)AW(BW)^{k_1-1}) = \\ &\text{rank}(AW(BW)^{k_1-1}) = \dim(R(AW(BW)^{k_1-1})) = \\ &\dim(AWR((BW)^{k_1-1})) = \dim(R(AW)^{k_1}) = \\ &\text{rank}((AW)^{k_1}).\end{aligned} \tag{3.12}$$

From (3.9), (3.10), (3.11), and (3.12), it follows that

$$N((BW)^{k_1}) = N((AW)^{k_1}),\ R((BW)^{k_1}) = R((AW)^{k_1}).$$

This completes the proof of (3.5a) by induction. Similarly, we can prove that (3.5b) also holds. With the above work, the proof is completed.

Combining Theorem 3.1 and Theorem 3.2, when both A and E are square matrices and W is an identity matrix, we come to the following results immediately.

Corollary 3.2[7] Let $B = A + E$ with $\text{Ind}(A) = k$ and $\text{Ind}(B) = j$. Let $l = max\{k, l\}$ and $E(l) = B^l - A^l$. If $\| EA_d \| < 1$. Then

$$B_d = (I + A_dE)^{-1}A_d = A_d(I + EA_d)^{-1},$$

if and only if

$$\text{core-rank}(B) = \text{core-rank}(A) \text{ and } AA_dE(l) = E(l) = E(l)AA_d.$$

Corollary 3.3 [7] Let $B = A + E$with $\text{Ind}(A) = k$. Suppose that $AA_dE = E = EAA_d$. Then $I + A_dE$ is invertible if and only if

$$R(B^i) = R(A^i) \text{ and } N(B^i) = N(A^i),\ i = 1,\cdots,k.$$

Furthermore, $AA_dE(k) = E(k) = E(k)AA_d$, where $E(k) = B^k - A^k$.

Now we show the Banach - type perturbation theorem for the weighted Drazin inverse.

Theorem 3.3 Let $B = A + E \in C^{m\times n}$, $W \in C^{n\times m}$, $\text{Ind}(AW) = k_1$, $\text{Ind}(WA) = k_2$, $\text{Ind}(BW) = j_1$, $\text{Ind}(WB) = j_2$, $l_1 = max\{k_1, j_1\}$, $l_2 = max\{k_2, j_2\}$, $(EW)(l_1) = (BW)^{l_1} - (AW)^{l_1}$ and $(WE)(l_2) = (WB)^{l_2} - (WA)^{l_2}$. Suppose that (3.2) holds. If $\| WEWA_{d,W} \| < 1$ and $\| A_{d,W}WEW \| < 1$, then

$$\frac{\| A_{d,W} \|}{1 + \| WEWA_{d,W} \|} \leqslant \| B_{d,W} \| \leqslant \frac{\| A_{d,W} \|}{1 - \| WEWA_{d,W} \|}, \tag{3.13}$$

and

$$\frac{\| WEWA_{d,W} \|}{|K_{d,W}}(A)(1 + \| WEW \| \ \| A_{d,W} \|) \leqslant \frac{\| B_{d,W} - A_{d,W} \|}{\| A_{d,W} \|}$$

$$\leqslant \frac{K_{d,W}(A)\ \|WEW\| / \|WAW\|}{1 - K_{d,W}(A)\ \|WEW\| / \|WAW\|}, \tag{3.14}$$

where $K_{d,W}(A) = \|WAW\|\ \|A_{d,W}\|$ is defined as the condition number of the W-weighted Drazin inverse $A_{d,W}$, and $\|\cdot\|$ indicates any consistent matrix norm with $\|I\| = 1$.

Proof From Theorem 3.1, we see that $B_{d,W} = A_{d,W}(I + WEWA_{d,W})^{-1}$. Notice that $\|\cdot\|$ is a consistent matrix norm with $\|I\| = 1$. By Lemma 2.5, we obtain (3.13) immediately

$$\frac{\|A_{d,W}\|}{1 + \|WEWA_{d,W}\|} \leqslant \|B_{d,W}\| \leqslant \frac{\|A_{d,W}\|}{1 - \|WEWA_{d,W}\|}.$$

From the fact that $B_{d,W} - A_{d,W} = B_{d,W}WEWA_{d,W}$ and (3.13), we have

$$\frac{\|B_{d,W} - A_{d,W}\|}{\|A_{d,W}\|} \leqslant \frac{\|B_{d,W}\|\ \|WEWA_{d,W}\|}{\|A_{d,W}\|} \leqslant \frac{\|WEWA_{d,W}\|}{1 - \|WEWA_{d,W}\|} \leqslant$$
$$\frac{\|A_{d,W}\|\ \|WAW\|\ \|WEW\| / \|WAW\|}{1 - \|A_{d,W}\|\ \|WAW\|\ \|WEW\| / \|WAW\|} =$$
$$\frac{K_{d,W}(A)\ \|WEW\| / \|WAW\|}{1 - K_{d,W}(A)\ \|WEW\| / \|WAW\|}. \tag{3.15}$$

This is the upper bound of (3.14). Next we deal with the lower bound of (3.14).

By Corollary 3.1, we see that

$$WAWA_{d,W} = WBWB_{d,W} \text{ and } A_{d,W}WAW = B_{d,W}WBW.$$

Then

$$WEWA_{d,W} = W(B - A)WA_{d,W} = WBWA_{d,W} - WAWA_{d,W} =$$
$$WBWA_{d,W} - WBWB_{d,W} = WBW(A_{d,W} - B_{d,W}) =$$
$$(WAW + WEW)(A_{d,W} - B_{d,W}).$$

Thus

$$\|B_{d,W} - A_{d,W}\| \geqslant \frac{\|WEWA_{d,W}\|}{\|WAW\| + \|WEW\|}.$$

Notice the fact

$$\|WEW\|\ \|A_{d,W}\| = \|WEW\|\ \|A_{d,W}WAWA_{d,W}\|$$
$$\leqslant \|WEW\|\ \|A_{d,W}\|\ K_{d,W}(A).$$

We have

$$\frac{\|B_{d,W} - A_{d,W}\|}{\|A_{d,W}\|} \geqslant \frac{\|WEWA_{d,W}\|}{\|WAW\|\ \|A_{d,W}\| + \|WEW\|\ \|A_{d,W}\|}$$
$$\Leftrightarrow \frac{\|WEWA_{d,W}\|}{K_{d,W}(A) + \|WEW\|\ \|A_{d,W}\|}$$
$$\Leftrightarrow \frac{\|WEWA_{d,W}\|}{K_{d,W}(A)(1 + \|WEW\|\ \|A_{d,W}\|)}. \tag{3.16}$$

Therefore, (3.14) holds from (3.15) and (3.16). This completes the proof.

Moreover, by Theorem3.3, when both A and E are square and $W = I$, we can directly get some results in [4,6,7] which are omitted here.

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关于加权 Drazin 逆扰动的几个结果

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摘要：设 A 是一个 $m \times n$ 矩阵，E 是 A 的扰动矩阵并且 $B = A + E$. 给出了 A 和 B 的加权 Drazin 逆 $A_{d,W}$ 和 $B_{d,W}$ 的新的性质，在一定的条件下，建立了加权 Drazin 逆 $A_{d,W}$ 和 $B_{d,W}$ 的 Banach-型扰动定理，得到了 $\| B_{d,W} \|$ 和 $\| B_{d,W} - A_{d,W} \| / \| A_{d,W} \|$ 的上下界估计，推广了有关 Drazin 逆和群逆的文献中的相应结果.

关键词：扰动；加权 Drazin 逆；指标；核秩；条件数