From (3.3), we see that

$$\beta\|X-\widetilde{X}\|^2\leqslant[\mathscr{L}^{-1}(X-\widetilde{X}),X-\widetilde{X}]=[F(X)-F(\widetilde{X}),X-\widetilde{X}]=$$

$$h\sum_{i,j=1}^{k}d_{ij}<f_i(x_i)-f_i(\widetilde{x}_i),x_j-\widetilde{x}_j>+h^2\sum_{i,j=1}^{k}d_{ij}<g_i(x_i)-g_i(\widetilde{x}_i),x_{k+j}-\widetilde{x}_{k+j}>\leqslant$$

$$h^2\sum_{i,j=1}^{k}d_{ij}\|g_i(x_i)-g_i(\widetilde{x}_i)\|\cdot\|x_{k+j}-\widetilde{x}_{k+j}\|\leqslant h^2\sum_{i,j=1}^{k}d_{ij}\sigma\|x_i-\widetilde{x}_i\|\cdot\|x_{k+j}-\widetilde{x}_{k+j}\|\leqslant$$

$$\frac{h^2\sigma}{2}\sum_{i,j=1}^{k}d_{ij}(\|x_i-\widetilde{x}_i\|^2+\|x_{k+j}-\widetilde{x}_{k+j}\|^2)\leqslant\frac{h^2\sigma}{2}\|X-\widetilde{X}\|^2.$$

This, due to (3.2), implies that $x_j=\widetilde{x}_j$, $j=1,\cdots,2k$, and implies the proof of the theorem.

Remark Theorem 3.1 becomes Theorem 1.1 when $D=\mathrm{diag}\{d_1,d_2,\cdots,d_k\}$, $d_i>0$, $1\leqslant i\leqslant k$.

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多导块方法得到的非线性系统的 *LD*-suitability

仇 璘 匡蛟勋

提 要 *Suitability* 是指由隐式龙格库塔法解得的非线性方程组存在唯一解．研究了对于多导块方法的 *LD*-suitability，以前的研究结果成为了特例．

关键词 suitability；*LD*-suitability；多导块方法

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On the *LD*-suitability of the Nonlinear Systems in MDBMs

Qiu Lin Kuang Jiaoxun

Abstract The concept of suitability means that the nonlinear equation to be solved by implicit Runge-Kutta methods has a unique solution. In this paper, the authors introduce the concept of *LD*-suitability for multiderivative block methods and show that previous results are special cases of ours.

Keywords suitability; *LD*-suitability; multiderivative block methods

1 Introduction

We shall deal with the initial-value problem

$$y'(t) = f(t, y(t)),\ y(t_0) = y_0, \tag{1.1}$$

where $y_0 \in \mathbf{R}^s$, $f: \mathbf{R} \times \mathbf{R}^s \to \mathbf{R}^s$, is continuous. An approximate solution to (1.1) can be obtained by the multiderivative block method (MDBM) with second order derivatives:

$$y_{n+i} = y_n + h\sum_{j=1}^{k} a_{ij} f_{n+j} + h^2 \sum_{j=1}^{k} b_{ij} f_{n+j}^{(1)} + h\beta_{1i} + h^2 \beta_{2i} f_n^{(1)}, \tag{1.2}$$

where $i = 1, \cdots, k$, $y_n \in \mathbf{R}^s$, $f_n := f(t_n, y_n) \in \mathbf{R}^s$ and $f_n^{(1)} := \mathrm{d}f(t_n, y_n)/\mathrm{d}t \in \mathbf{R}^s$ are known vectors. It is proved that there exist a_{ij}, b_{ij}, β_{1i}, β_{2i}, $i, j = 1, 2, \cdots, k$ such that (1.2) converges with order $p = 2k + 2$ (see[1]), and is A-stable for $k \leqslant 5$ (see [5]). To compute the approximate solution $y_{n+j} \sim y(t_{n+j})$ requires the solution of the following nonlinear equations:

$$y_{n+i} = u_n + h\sum_{j=1}^{k} a_{ij} f_{n+j} + h^2 \sum_{j=1}^{k} b_{ij} f_{n+j}^{(1)},\ 1 \leqslant i \leqslant k, \tag{1.3}$$

where $u_n = y_n + h\beta_{1i} f_n + h^2 \beta_{2i} f_n^{(1)}$.

Let y_{n+j} be denoted by y_j, $f(t_{n+j}, y_{n+j})$ by $f_j(y_j)$, and $f^{(1)}(t_{n+j}, y+j)$ by $g_j(y_j)$. Then (1.3) becomes

$$y_i = u_n + h\sum_{j=1}^{k} a_{ij} f_j(y_j) + h^2 \sum_{j=1}^{k} b_{ij} g_j(y_j),\ 1 \leqslant i \leqslant k. \tag{1.4}$$

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First author Qiu Lin, female, Department of Mathematics, Shanghai Teachers' University, Shanghai, 200234

There exists a unique soluton to (1.4) if and only if the following nonlinear equations have a unique solution:

$$y_i = h\sum_{j=1}^{k} a_{ij} f_j(y_j) + h^2 \sum_{j=1}^{k} b_{ij} g_j(y_j),\ 1 \leqslant i \leqslant k. \tag{1.5}$$

In [2], conditions under which system (1.2) has a unique solution have been considered. The following results (see [2]) will be important in this paper.

Theorem 1.1 Suppose that the function $f(t, u)$ satisfies the following conditions:

$$< f_j(u) - f_j(v),\ u - v > \leqslant 0,\ \forall\ u, v \in \mathbf{R}^s,\ 1 \leqslant j \leqslant k, \tag{1.6}$$

$$\| g_j(u) - g_j(v) \| \leqslant \sigma \| u - v \|,\ \forall\ t \in \mathbf{R},\ \forall\ u, v \in \mathbf{R}^s, \tag{1.7}$$

and there exists a positive definite diagonal matrix $\boldsymbol{D}$. *Let*

$$\widetilde{\boldsymbol{D}} = \begin{pmatrix} \boldsymbol{D} & 0 \\ 0 & \boldsymbol{D} \end{pmatrix}, \qquad \boldsymbol{L} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ 0 & \boldsymbol{I}_k \end{pmatrix},$$

where $\boldsymbol{A} = (a_{ij})_{k\times k}$, $\boldsymbol{B} = (b_{ij})_{k\times k}$, $\boldsymbol{I}_k$ is a $k \times k$ unit matrix, such that the matrix $\widetilde{\boldsymbol{D}}\boldsymbol{L} + \boldsymbol{L}^T\widetilde{\boldsymbol{D}}$ is positive definite and $h < \sqrt{\dfrac{2\beta}{\sigma}}$ is fulfilled. Then the system (1.5) has a unique solution for suitable $\beta > 0$.

We point out that though equation (1.5) has a unique solution for a matrix $\boldsymbol{A}$ associated with multiderivative block methods, there may not be a positive definite diagonal matrix $\boldsymbol{D}$ such that $\widetilde{\boldsymbol{D}}\boldsymbol{L} + \boldsymbol{L}^T\widetilde{\boldsymbol{D}}$ is positive definite, and therefore it is necessary to find more useful and weaker conditions to detect the existence and uniqueness of the solution of (1.5). In the next solution, we will introduce ***LD***-suitability and some useful results.

2 *LD*-suitability

Definition 2.1 A matrix $\boldsymbol{A}$ is called suitable if and only if the system (1.5) has a unique solution $y = (y_1, y_2, \cdots, y_k)^T \in \mathbf{R}^{ks}$, whenever $f_j: \mathbf{R}^s \to \mathbf{R}^s$ are continuous $(1 \leqslant j \leqslant k)$.

For the following discussion, we introduce the the set formed by the group of continuous functons $(f_1, f_2, \cdots, f_k)$, say $\mathscr{F} = \{(f_1, f_2, \cdots, f_k): f_j, 1 \leqslant j \leqslant k$ are continuous functions from $\mathbf{R}^s$ to $\mathbf{R}^s$,

$$\| g_j(u) - g_j(v) \| \leqslant \sigma \| u - v \|,\ \forall\ u, v \in \mathbf{R}^s,\ \sigma > 0, \tag{2.1}$$

$$\sum_{i,j=1}^{k} d_{ij} < f_i(u_i) - f_i(v_i),\ u_j - v_j > \leqslant 0,\ \forall\ u_i, u_j, v_i, v_j \in \mathbf{R}^s, \tag{2.2}$$

for some positive definite matrix $\boldsymbol{D} = (d_{ij})_{k\times k}$, where $d_{ij} \leqslant 0,\ i \neq j,\ d_{ii} > 0 (1 \leqslant i, j \leqslant k)\}$

Definition 2.2 A matrix $\boldsymbol{L}$ is called ***LD***-suitable if the system (1.5) has a unique soluition $y = (y_1, y_2, \cdots, y_k)^T \in \mathbf{R}^{ks}$ whenever the group of functions $(f_1, f_2, \cdots, f_k)$ in the set $\mathscr{F}$.

From Definition 2.1 and 2.2, one can see that if $\boldsymbol{L}$ is suitable, of course, it is ***LD***-suitable. In fact, if $f_i (1 \leqslant i \leqslant k)$ satisfy (1.6), (1.7), then $(f_1, f_2, \cdots, f_k) \in \mathscr{F}$ simply with $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \cdots, d_k)$, where $d_i > 0$ for $i = 1, 2, \cdots, k$.

3 Sufficient Conditions For *LD*-Suitability Of Matrix *A*

In this section, we will have some sufficient conditions for *LD*-suitability of the matrix *A*. Denote $A=(a_{ij})_{k\times k}$, $B=(b_{ij})_{s\times s}$, then kronecker's product of A and B is defined by

$$A\otimes B=\begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{1k}B \\ a_{21}B & a_{22}B & \cdots & a_{2k}B \\ & \cdots & & \\ a_{k1}B & a_{k2}B & \cdots & a_{kk}B \end{pmatrix},$$

where

$$a_{ij}B=\begin{pmatrix} a_{ij}b_{11} & a_{ij}b_{12} & \cdots & a_{ij}b_{1s} \\ a_{ij}b_{21} & a_{ij}b_{22} & \cdots & a_{ij}b_{2s} \\ & \cdots & & \\ a_{ij}b_{s1} & a_{ij}b_{s2} & \cdots & a_{ij}b_{ss} \end{pmatrix}, 1\leqslant i,\ j\leqslant k.$$

Lemma 3.1 Let L and D be $m\times m$ matrices, and I_s be the $s\times s$ unit matrix. Then $(LD)\otimes I_s=(L\otimes I_s)(D\otimes I_s)$. Furthermore, if $DL+L^TD$ and D are positive definite, then $(DL+L^TD)\otimes I_s$ and $D\otimes I_s$ are positive definite.

Let

$$L=\begin{pmatrix} A & B \\ 0 & I_k \end{pmatrix},\ \mathscr{L}=L\otimes I_s=\begin{pmatrix} A\otimes I_s & B\otimes I_s \\ 0 & I_k\otimes I_s \end{pmatrix},$$

where $A=(a_{ij})_{k\times k}$, $B=(b_{ij})_{k\times k}$.

Denote

$$y=(y_1^T,\ y_2^T,\ \cdots,\ y_k^T)^T,\ f(y)=(f_1(y_1)^T,\ f_2(y_2)^T,\ \cdots,\ f_k(y_k)^T)^T,$$

$$g(y)=(g_1(y_1)^T,\ g_2(y_2)^t,\ \cdots,\ g_k(y_k)^T)^T,$$

$$Y_g=\begin{pmatrix} y \\ h^2g(y) \end{pmatrix},\ F(Y)=\begin{pmatrix} hf(y) \\ h^2g(y) \end{pmatrix},$$

where

$$Y_g=(y_1^T,\ y_2^T,\ \cdots,\ y_k^T,\ h^2g_1(y_1)^T,\ h^2g_2(y_2)^T,\ \cdots,\ h^2g_k(y_k)^T)^T,$$

$$F(Y)=(hf_1(y_1)^T,\ \cdots,\ hf_k(y_k)^T,\ h^2g_1(y_1)^T,\ \cdots,\ h^2g_k(y_k)^T)^T,$$

$$y_i=(y_{i1},\ y_{i2},\ \cdots,\ y_{is})^T,\ f_i(y_i)=(f_{i1}(y_i),\ f_{i2}(y_i),\ \cdots,\ f_{is}(y_i))^T$$

$$g_i(y_i)=(g_{i1}(y_i),\ g_{i2}(y_i),\ \cdots,\ g_{is}(y_i))^T,\ 1\leqslant i\leqslant k.$$

Then (1.5) can be written as

$$Y_g=\mathscr{L}F(Y) \qquad (3.1)$$

or

$$\begin{pmatrix} y \\ h^2g(y) \end{pmatrix}=\begin{pmatrix} A\otimes I_s & B\otimes I_s \\ 0 & I_k\otimes I_s \end{pmatrix}\begin{pmatrix} hf(y) \\ h^2g(y) \end{pmatrix}.$$

Let

$$\widetilde{D}=\begin{pmatrix} D & 0 \\ 0 & D \end{pmatrix}, \qquad \mathscr{D}=\widetilde{D}\otimes I_s=\begin{pmatrix} D\otimes I_s & 0 \\ 0 & D\otimes I_s \end{pmatrix},$$

where D is a $k \times k$ matrix. From Lemma 3.1, if $\widetilde{D}$ and $\widetilde{D}L + L^T\widetilde{D}$ are positive definite, the $\mathscr{D}$ and $\mathscr{D}\mathscr{L} + \mathscr{L}^T\mathscr{D}$ are positive definite.

Define the inner-product in R^{2ks} as

$$[X, Y] := \sum_{i,j=1}^{k} d_{ij} < x_i, y_j > + \sum_{i,j=1}^{k} d_{ij} < x_{k+i}, y_{k+j} > = X^T \mathscr{D} Y,$$

where

$$X = (x_1, x_2, \cdots, x_{2k})^T, \ Y = (y_1, y_2, \cdots, y_{2k})^T,$$

$$\widetilde{D} = \begin{pmatrix} D & 0 \\ 0 & D \end{pmatrix}, \qquad \mathscr{D} = \begin{pmatrix} D \otimes I_s & 0 \\ 0 & D \otimes I_s \end{pmatrix},$$

$D = (d_{ij})_{k \times k}$ is a positive definite matrix.

In the new inner product space, the new norm of X is defined by

$$\| X \| = [X, X]^{\frac{1}{2}}.$$

Lemma 3.2 Let D be a $k \times k$ positive definite matrix, and $d_{ij} \leqslant 0 (i \neq j)$, then

$$\sum_{i,j=1}^{k} d_{ij} \| x_i \|^2 + \sum_{j=1}^{k} d_{ij} \| x_{k+i} \|^2 \leqslant \| X \|^2, \text{ where } X = (x_1, x_2, \cdots, x_{2k})^T.$$

Proof Using the definition of [* , *], we have

$$\| X \|^2 = [X, X] = \sum_{i,j=1}^{k} d_{ij} < x_i, x_j > + \sum_{i,j=1}^{k} d_{ij} < x_{k+i}, x_{k+j} >.$$

From the symmetrical property of D, we have

$$\sum_{i,j=1}^{k} d_{ij} \| x_i \|^2 + \sum_{i,j=1}^{k} d_{ij} \| x_{k+i} \|^2 - \| X \|^2 =$$

$$\sum_{i,j=1}^{k} d_{ij} \| x_i \|^2 + \sum_{i,j=1}^{k} d_{ij} \| x_{k+i} \|^2 - \sum_{i,j=1}^{k} d_{ij} < x_i, x_j > - \sum_{i,j=1}^{k} d_{ij} < x_{k+i}, x_{k+j} > =$$

$$\sum_{i,j=1}^{k} d_{ij} < x_i, x_i - x_j > + \sum_{i,j=1}^{k} d_{ij} < x_{k+i}, x_{k+i} - x_{k+j} > =$$

$$\sum_{\substack{i,j=1 \\ i<j}}^{k} d_{ij} < x_i - x_j, x_i - x_j > + \sum_{\substack{i,j=1 \\ i<j}}^{k} d_{ij} < x_{k+i} - x_{k+j}, x_{k+i} - x_{k+j} > =$$

$$\sum_{\substack{i,j=1 \\ i<j}}^{k} d_{ij} \| x_i - x_j \|^2 + \sum_{\substack{i,j=1 \\ i<j}}^{k} d_{ij} \| x_{k+i} - x_{k+j} \|^2 \leqslant 0.$$

Therefore,

$$\sum_{i,j=1}^{k} d_{ij} \| x_i \|^2 + \sum_{i,j=1}^{k} d_{ij} \| x_{k+i} \|^2 \leqslant \| X \|^2.$$

This completes the lemma.

Theorem 3.1 Let $D \in \mathbf{R}^{k \times k}$ be a positive definite matrix, $d_{ij} \leqslant 0 (i \neq j)$, $d_{ii} > 0$ $(i = 1, \cdots, k)$, and $(f_1, f_2, \cdots, f_k) \in \mathscr{F}$. If $\widetilde{D}L + L^T\widetilde{D}$ is positive definite and

$$h < \sqrt{2\beta/\sigma}, \tag{3.2}$$

is fulfilled, then L is LD-suitable for suitable $\beta > 0$.

Proof We only need to prove that equation (3.1) has a unique solution. Let $\omega \in \mathbf{R}^{2ks}, \omega \neq 0$. Then

$$[\mathscr{L}\omega,\omega]=\omega^T\mathscr{L}^T\mathscr{D}\omega,\qquad [\mathscr{L}\omega,\omega]=[\omega,\mathscr{L}\omega]=\omega^T\mathscr{D}\mathscr{L}\omega.$$

Hence

$$[\mathscr{L}\omega,\omega]=\frac{1}{2}\omega^T\{\mathscr{D}\mathscr{L}+\mathscr{L}^T\mathscr{D}\}\omega.$$

If $\widetilde{D}L+L^T\widetilde{D}$ is positive definite, then so is $\mathscr{D}\mathscr{L}+\mathscr{L}^T\mathscr{D}$ by Lemma 2.1. Therefore,

$$[\mathscr{L}\omega,\omega]>0,\forall\ \omega\in R^{2ks},\ \omega\neq 0,$$

and this implies that $\mathscr{L}$ is regular and there exisits $\mathscr{L}^{-1}$, and that $[\mathscr{L}^{-1}\omega,\omega]>0$ for all $\omega\in\mathbf{R}^{2ks}$, $\omega\neq 0$. From the properties of finite dimensional spaces, we see that

$$\min_{\|\omega\|=1}[\mathscr{L}^{-1}\omega,\ \omega]=\beta>0,$$

and then

$$[\mathscr{L}^{-1}\omega,\omega]\geqslant\beta\|\omega\|^2,\ \omega\in\mathbf{R}^{2ks},\ \omega\neq 0 \tag{3.3}$$

Next we prove the existence of solution to (3.1). Let $G(X)=\mathscr{L}^{-1}(X)-F(X)$, $X\in\mathbf{R}^{2ks}$. Then

$$G(X)-G(0)=\mathscr{L}^{-1}X-F(X)+F(0).$$

Let X be denoted by $(v_1^T,v_2^T)^T$ with $v_1=(x_1,\cdots,x_k)^T$, $v_2=(x_{k+1},\cdots,x_{2k})^T$. Since $(f_1,\cdots,f_k)\in\mathscr{F}$ and by using Schwartz' inequality and Lemma 3.2, we have

$$[G(X)-G(0),\ X]=[\mathscr{L}^{-1}X,\ X]-[F(X)-F(0),\ X-0]\geqslant$$

$$\beta\|X\|^2-h\sum_{i,j=1}^{k}d_{ij}<f_i(x_i)-f_i(0),\ x_j-0>-h^2\sum_{i,j=1}^{k}d_{ij}<g_i(x_i)-g_i(0),\ x_{k+j}-0>\geqslant$$

$$\beta\|X\|^2-h^2\sum_{i,j=1}^{k}d_{ij}\|g_i(x_i)-g_i(0)\|\cdot\|x_{k+j}\|\geqslant$$

$$\beta\|X\|^2-\frac{h^2\sigma}{2}\sum_{i,j=1}^{k}d_{ij}(\|x_i\|^2+\|x_{k+j}\|^2)=\beta_1\|X\|^2,$$

where $\beta_1=\beta-\frac{h^2\sigma}{2}$. Therefore, we have $[G(X)-G(0),\ X-0]\geqslant\beta_1\|X\|^2$.

Since $[G(0),\ X]\geqslant-\|G(0)\|\cdot\|X\|$, we get

$$[G(X),\ X]\geqslant\beta_1\|X\|^2-\|G(0)\|\cdot\|X\|=\|X\|(\beta_1\|X\|-\|G(0)\|),$$

and this implies that $[G(X),\ X]\neq 0$ for all $X\in\mathbf{R}^{2ks}$ with $\|X\|\geqslant\|G(0)\|/\beta_1$. Let $0\neq X\in\mathbf{R}^{2ks}$, $\lambda>1$. Define $H(X)=X-G(X)$. Then

$$[\lambda X-H(X),\ X]=[(\lambda-1)X+G(X),\ X]\geqslant[(\lambda-1)X,\ X]>0.$$

Hence $H(X)\neq\lambda X$ for all X with $\|X\|\geqslant\|G(0)\|/\beta_1$. By Schauder's fixed point theorem (see [4,6]) and (3.3), $H(X)$ has a fixed

$$X^*=(v_1^*,\ v_2^*)^T\in\mathbf{R}^{2ks},\ v_1^*,\ v_2^*\in\mathbf{R}^{ks}$$

with $\|X^*\|<\frac{\|G(0)\|}{\beta_1}$ and

$$X^*=H(X^*)=X^*-G(X^*),\ \text{or}\ G(X^*)=0.$$

Hence $X^*=\mathscr{L}F(X^*)$.

Last we prove the uniqueness of the solution to (3.1).

Assume that there exist two solutions X and $\widetilde{X}$ satisfying (3.1), that is

$$0=X-\mathscr{L}F(X)=\widetilde{X}-\mathscr{L}F(\widetilde{X}).$$

From (3.3), we see that

$$\beta \| X - \widetilde{X} \|^2 \leqslant [\mathscr{L}^{-1}(X - \widetilde{X}), X - \widetilde{X}] = [F(X) - F(\widetilde{X}), X - \widetilde{X}] =$$

$$h \sum_{i,j=1}^{k} d_{ij} < f_i(x_i) - f_i(\widetilde{x}_i), x_j - \widetilde{x}_j > + h^2 \sum_{i,j=1}^{k} d_{ij} < g_i(x_i) - g_i(\widetilde{x}_i), x_{k+j} - \widetilde{x}_{k+j} > \leqslant$$

$$h^2 \sum_{i,j=1}^{k} d_{ij} \| g_i(x_i) - g_i(\widetilde{x}_i) \| \cdot \| x_{k+j} - \widetilde{x}_{k+j} \| \leqslant h^2 \sum_{i,j=1}^{k} d_{ij} \sigma \| x_i - \widetilde{x}_i \| \cdot \| x_{k+j} - \widetilde{x}_{k+j} \| \leqslant$$

$$\frac{h^2 \sigma}{2} \sum_{i,j=1}^{k} d_{ij} (\| x_i - \widetilde{x}_i \|^2 + \| x_{k+j} - \widetilde{x}_{k+j} \|^2) \leqslant \frac{h^2 \sigma}{2} \| X - \widetilde{X} \|^2.$$

This, due to (3.2), implies that $x_j = \widetilde{x}_j$, $j = 1, \cdots, 2k$, and implies the proof of the theorem.

Remark　Theorem 3.1 becomes Theorem 1.1 when $D = \mathrm{diag}\{d_1, d_2, \cdots, d_k\}$, $d_i > 0$, $1 \leqslant i \leqslant k$.

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多导块方法得到的非线性系统的 *LD*-suitability

仇　璘　　匡蛟勋

提　要　*Suitability* 是指由隐式龙格库塔法解得的非线性方程组存在唯一解．研究了对于多导块方法的 *LD*-suitability，以前的研究结果成为了特例．

关键词　suitability; *LD*-suitability; 多导块方法

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