QCD Factorization for Semi-inclusive Deep-Inelastic Scattering at Low Transverse Momentum*

JI Xiang-Dong[1] MA Jian-Ping[2] YUAN Feng[1]

1 (Department of Physics, University of Maryland, College Park, Maryland 20742, USA)
2 (Institute of Theoretical Physics, CAS, Beijing 100080, China)

Abstract We argue a factorization formula for semi-inclusive deep-inelastic scattering with hadrons in the current fragmentation region detected at low transverse momentum. To facilitate the factorization, we introduce the transverse-momentum dependent parton distributions and fragmentation functions with gauge links slightly off the light-cone, and with soft-gluon radiations subtracted. We verify the factorization to one-loop order in perturbative quantum chromodynamics and argue that it is valid to all orders in perturbation theory.

Key words semi-inclusive deep-inelastic scattering, quantum chromodynamics, factorization

In recent years, semi-inclusive deep-inelastic (SIDIS) lepton-nucleon scattering has emerged as an important tool to learn various aspects of perturbative and non-perturbative quantum chromodynamics (pQCD), the internal structure of the nucleon, in particular. The European Muon Collaboration experiment at CERN has provided us with valuable information about the flavor dependence of quark fragmentation functions. The H1 and ZEUS collaborations at the DESY HERA collider have measured the topology of the hadron final states in great detail and have compared them with the predictions of perturbative QCD. In the area of polarized semi-inclusive DIS, the Spin Muon Collaboration, and recently the HERMES collaboration at DESY, have extracted the sea quark distributions and the polarized gluon distribution with controlled accuracy. More recently, the target single-spin asymmetry measured by HERMES in semi-inclusive DIS is a new observable sensitive, for example, to the quark transversity distribution through the transverse momentum dependence of the produced hadron.

In the semi-inclusive production of DIS, both the longitudinal momentum fraction z and the transverse-momentum $P_{\rm h\perp}$ of the hadron yield can be measured. When the transverse momentum is integrated over or when it is comparable to the hard photon-mass scale, $P_{\rm h\perp} \sim Q$, the cross sections can be calculated from the standard pQCD formalism similar to inclusive DIS and Feynman parton distributions. In these cases, the theoretical tool has been well tested against experimental data with notable successes. When the transverse-momentum is much smaller than Q, but is still hard, $P_{\rm h\perp} \gg \Lambda_{\rm QCD}$, the cross section can be calculated again with integrated parton distributions augmented by small non-perturbative QCD corrections. The hard part contains the large double logarithms of the type $\alpha_{\rm s} \ln^2 P_{\rm h\perp}/Q^2$. To make reliable predictions, these large logarithms must be summed. An adequate formalism was developed by Collins and Soper in the case of e^+e^- annihilation, and shortly thereafter applied to the Drell-Yan process by Collins, Soper and Sterman (CSS). A first application of the CSS approach to SIDIS was made by Meng, Olness, and Soper. Recently, a quantitative comparison between

Received 30 March 2007
* Talk given by J.P. Ma at the Xining Conference

this theory and data from HERA collider has been made by Nadolsky, Stump and Yuan.

In this paper, we are interested in a special kinematic regime in SIDIS where $P_{\mathrm{h}\perp}$ is soft, i.e. on the order of Λ_{QCD}, and Q^2 is not too large, for example, on the order of tens or hundreds of GeV^2. When Q^2 is large, the soft gluon radiations become important and can easily generate a large transverse momentum $\gg \Lambda_{\mathrm{QCD}}$. Then the cross section for the hadron yield with $P_{\mathrm{h}\perp} \sim \Lambda_{\mathrm{QCD}}$ is exponentially suppressed. To have a significant fraction of events with $P_{\mathrm{h}\perp} \sim \Lambda_{\mathrm{QCD}}$, fixed-target experiments with lepton beam energies on the order of tens to hundreds of GeV are preferred. The above kinematic regime is in fact ideal for studying transverse-momentum dependent (TMD) parton distributions in the nucleon and the related quark fragmentation functions. Recent interest in this subject has been stimulated by Collins's observation that semi-inclusive DIS at low-$P_\perp$ provides a tool to measure the quark transversity distribution. The physics potential has been reinforced by the rediscovery of Siver's effect by Brodsky, Hwang, and Schimdt.

The main result of this paper is a QCD factorization theorem for the SIDIS cross section in the above kinematics region, accurate up to the power corrections $(P_{\mathrm{h}\perp}^2/Q^2)^n$ and to all orders in perturbation theory. This factorization has been conjectured by Collins, following the early work of Collins and Soper on $\mathrm{e}^+\mathrm{e}^-$ annihilation. However, an exact statement of the factorization theorem requires an adequate definition of the TMD parton distributions and fragmentation functions in QCD and a systematic factorization (and subtraction) of soft, collinear, and hard gluon contributions. In light of the recent development in this area, here we provide a first detailed examination of QCD radiative corrections in DIS.

The factorization theorem we propose for the leading spin-independent structure function is

$$F(x_{\mathrm{B}}, z_{\mathrm{h}}, P_{\mathrm{h}\perp}, Q^2) = \sum_{q=u,d,s,\ldots} \mathrm{e}_{\mathrm{q}}^2 \int \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}^2\boldsymbol{l}_\perp \times$$
$$q(x_{\mathrm{B}}, k_\perp, \mu^2, x_{\mathrm{B}}\zeta, \rho)\, \hat{q}_{\mathrm{T}}\left(z_{\mathrm{h}}, p_\perp, \mu^2, \hat{\zeta}/z_{\mathrm{h}}, \rho\right) \times$$
$$S(\boldsymbol{l}_\perp, \mu^2, \rho) H(Q^2, \mu^2, \rho)\, \delta^2(z_{\mathrm{h}}\boldsymbol{k}_\perp + \boldsymbol{p}_\perp + \boldsymbol{l}_\perp - \boldsymbol{P}_{\mathrm{h}\perp}), \tag{1}$$

where μ is a renormalization (and collinear factorization) scale; ρ is a gluon rapidity cut-off parameter; the μ and ρ dependence cancels among various factors. In a special system of coordinates in which $x_{\mathrm{B}}\zeta = \hat{\zeta}/z_{\mathrm{h}}$, one has $\zeta^2 x_{\mathrm{B}}^2 = \hat{\zeta}^2/z_{\mathrm{h}}^2 = Q^2\rho$. The physical interpretation of the factors is as follows: q is TMD quark distribution function depending on, among others, the Bjorken x_{B}; $\hat{q}$ is the TMD quark fragmentation function depending on, among others, the hadron momentum fraction z_{h}; H represents the contribution of parton hard scattering and is a perturbation series in α_{s}; and, finally, the soft factor S comes from soft gluon radiations and is defined by a matrix element of Wilson lines in QCD vacuum. The above result shows that the hadron transverse momentum is generated from the combined effects of transverse-momentum of the quarks in the nucleon, soft gluon radiation, and the transverse-momentum of the quark fragmentation.

There is no contribution from the TMD gluon distributions and fragmentation functions at the leading twist. For the gluons to contribute, one must introduce the soft quark lines.

The main steps to establish the above factorization is as follows. We introduce the TMD parton distribution and fragmentation function, and calculate them to one-loop order in perturbative QCD. The result contains both soft and collinear divergences and obeys the Collins and Soper evolution equation in the rapidity cut-off. We study the factorization of the TMD distributions by subtracting away the soft contributions. Then one-loop result for semi-inclusive DIS scattering is obtained, and the factorization is shown to be true on the diagram-by-diagram basis. After checking the one-loop case we generalize the one-loop result to all orders by identifying the leading regions for an arbitrary Feynman diagram using soft and collinear power counting. We then argue that a systematic factorization of the leading region leads to the general formula in Eq. (1). At the end, the large logarithms in the perturbative expression are summed through solving evolution equations. Detailed results and relevant references can be found in Ref. [1]. We also extend our work to Drell-Yan

processes[2] and to the case where gluon TMD distributions are involved[3].

References

1 JI X D, MA J P, YUAN F. Phys. Rev., 2005, **D71**: 034005. hep-ph/0404183

2 JI X D, MA J P, YUAN F. Phys. Lett., 2004, **B597**: 299—308. hep-ph/0405085

3 JI X D, MA J P, YUAN F. JHEP, 2005, **0507**: 020. hep-ph/0503015

小横动量半深度非弹性散射的量子色动力学因子化

季向东[1]　马建平[2]　袁锋[1]

1 (美国马里兰大学物理系　马里兰　20742　美国)
2 (中国科学院理论物理所　北京　100080)

摘要　论证了在小横动量区域内半深度非弹性散射量子色动力学因子化公式. 为完成因子化, 引进了依赖于横动量的部分子分布函数和碎裂函数, 并扣除了软胶子辐射. 在单圈图的情形下, 证明了因子化的成立, 同时论证了在微扰论的任意阶该因子化的正确性.

关键词　半深度非弹性散射　量子色动力学　因子化

2007 – 03 – 30 收稿