应用概率统计 第二十四卷
第二期 2008年4月

Chinese Journal of Applied Probability
and Statistics Vol.24 No.2 Apr. 2008

多维局部平稳高斯过程最大值的联合渐近分布 *

杨春华[1,2]　　彭作祥[2]

([1]渝西学院数学系, 重庆, 402168; [2]西南师范大学数学系, 重庆, 400715)

摘 要

$\{(X_1(t),\cdots,X_p(t)),0\leq t\leq T\}$为$p$维局部平稳高斯过程, 具有渐近中心化的均值$m_k(t)$和常数的方差, $M_k(T)=\sup\{X_k(t),0\leq t\leq T\}$, $k=1,\cdots,p$, 当$T\to\infty$时, 本文在一定条件下获得了$M(T)=(M_1(T),\cdots,M_p(T))$的联合渐近分布.

关键词: 多维高斯过程, 局部平稳高斯过程, 最大值.

学科分类号: O211.4.

§1. 引言及主要定理

平稳高斯过程的最大值及上穿过点过程的渐近分布, 已经有很多学者进行了研究. Lindgren[1974]证明了有限个均方可微平稳高斯过程的上穿过点过程的渐近独立性. Leadbetter[1983]总结了前人的工作, 对平稳高斯过程的最大值、上穿过点过程的渐近分布给予了系统性的介绍和研究. 彭作祥[1996]进一步推广研究了多维非均方可微平稳高斯过程的ε-上穿过点过程的渐近分布, 同时得到了过程最大值的渐近分布, 本文把平稳高斯过程推广到局部平稳高斯过程, 获得了多维高斯过程的最大值的联合渐近分布.

$\{(X_1(t),\cdots,X_p(t)),0\leq t\leq T\}$为$p$维高斯过程, $\mathsf{E}X_k(t)=m_k(t)$, $DX_k(t)=1$, $0\leq t\leq T$, $k=1,\cdots,p$, 其交互相关系数和各自的相关系数分别记为: $r_{kk'}(t,s)=\mathsf{E}(X_k(t)-m_k(t))(X_{k'}(s)-m_{k'}(s))$, $r_k(t,s)=\mathsf{E}(X_k(t)-m_k(t))(X_k(s)-m_k(s))$, $k,k'=1,\cdots,p$, $k\neq k'$. 假定$s\to 0$时, $r_k(t,t+s)$满足

$$r_k(t,t+s)=1-c_k(t)\cdot|s|^\alpha+o(|s|^\alpha). \tag{1.1}$$

其中$0<\alpha\leq 2$, $c_k(t)$是连续的函数, 且满足$0<\inf\{c_k(t)|0\leq t\leq+\infty\}\leq\sup\{c_k(t)|0\leq t\leq+\infty\}<\infty$, $k=1,\cdots,p$. 当$\tau\to\infty$时,

$$\delta_k(\tau)\log\tau\to 0. \tag{1.2}$$

其中$\delta_k(\tau)=\sup\{|r_k(t,t+s)|:|s|>\tau\}$, $r_k(t,t+s)=1$当且仅当$s=0$, $k=1,\cdots,p$. 当$T'\to+\infty$时

$$A(T')=\max_{1\leq k,k'\leq p,k\neq k'}\sup\{|r_{kk'}(t,s)|\log|t-s|:|t-s|>T'\}\to 0, \tag{1.3}$$

$$\max_{1\leq k,k'\leq p,k\neq k'}\sup\{|r_{kk'}(t,s)|:t,s\geq 0\}=\delta<1. \tag{1.4}$$

*部分为国家自然科学基金资助(批准号: 70371061), 重庆市教委科学技术研究项目资助(批准号: KJ051203).
本文2004年11月12日收到, 2006年11月9日收到修改稿.

$m_k(t)$是$[0,\infty)$上的有界函数且当$t\to\infty$时

$$m_k(t)\sqrt{2\log t}\to\gamma_k<\infty,\qquad k=1,\cdots,p. \tag{1.5}$$

其中相关系数$r_k(s,t)$满足(1.1)的高斯过程, 我们称为局部平稳高斯过程, 见Berman[1974]、Hüsler[1990]、Piterbarg[1996]. Hüsler[1990]研究了一维局部平稳高斯过程的极值, 本文把结果推广到多维情形.

首先给出主要定理

定理 1.1 高斯过程$\{(X_1(t),\cdots,X_p(t)),0\le t\le T\}$的$\mathsf{E}X_k(t)=0$, $DX_k(t)=1$, $k=1,\cdots,p$, $0\le t\le T$, 相关系数和交互相关系数满足条件(1.1)、(1.2)、(1.3)、(1.4), 水平$u_{k,T}$为

$$\begin{aligned} u_{k,T} &= \frac{-\log\tau_k}{(2\log T)^{1/2}}+(2\log T)^{1/2}+(2\log T)^{-1/2}(((2-\alpha)/2\alpha)\log\log T\\ &\quad +\log(C^*_{k,T}H_\alpha 2^{(2-\alpha)/2\alpha}(2\pi)^{-1/2}))\qquad k=1,\cdots,p. \end{aligned} \tag{1.6}$$

其中$C^*_{k,T}=\displaystyle\int_0^T c_k^{1/\alpha}(t)\mathrm{d}t\Big/T$, $\tau_k>0$, H_α为常数, 定义见Piterbarg[1996], 则$T\to\infty$时

$$\mathsf{P}\{X_k(t)\le u_{k,T},0\le t\le T,k=1,\cdots,p\}\to\exp\Big\{-\sum_{k=1}^p\tau_k\Big\}.$$

定理 1.2 高斯过程$\{(X_1(t),\cdots,X_p(t)),0\le t\le T\}$的$\mathsf{E}X_k(t)=m_k(t)$, $DX_k(t)=1$, $k=1,\cdots,p$, 相关系数和交互相关系数满足条件(1.1)、(1.2)、(1.3)、(1.4), $m_k(t)$满足条件(1.5), 水平$u_{k,T}$如(1.6)定义, 则当$T\to\infty$时

$$\mathsf{P}\{X_k(t)\le u_{k,T},0\le t\le T,k=1,\cdots,p\}\to\exp\Big\{-\sum_{k=1}^p\tau_k\exp\{\gamma_k\}\Big\}.$$

§2. 主要引理

首先将$[0,T]$分成$n=[T/h_T]$等份, 当$T\to\infty$时, 每一段的长度为$h_T\to0$, 且满足$h_T(u_T^2/\log u_T)^{1/\alpha}\to\infty$ (其中$u_T=\min\{u_{k,T}:k=1,\cdots,p\}$), 每个小区间$((l-1)h_T,lh_T]$又分成两部分$I_l=((l-1)h_T,lh_T-\varepsilon_T]$, $I_l^*=(lh_T-\varepsilon_T,lh_T]$且满足条件$\varepsilon_T/h_T\to0$, $\varepsilon_T(u_T^2/\log u_T)^{1/\alpha}\to\infty$, 在每个小区间$I_l$上添加分点, 等分的分点的长度为$q=q_0/(u_T)^{2/\alpha}$, 当$T\to\infty$时, q_0充分慢地趋于0 (见Leadbtter[1983]引理12.2.7). 记$\psi(x)=\phi(x)x^{2/\alpha-1}$, $\phi(x)$为标准正态分布函数, 指定K为可变常数.

引理 2.1 假设高斯过程$X(t)$的均值为0, 方差为1且满足条件(1.1)、(1.2), 水平u_T为:

$$\begin{aligned} u_T &= \frac{-\log\tau}{(2\log T)^{1/2}}+(2\log T)^{1/2}\\ &\quad+(2\log T)^{-1/2}(((2-\alpha)/2\alpha)\log\log T+\log(C^*_T H_\alpha 2^{(2-\alpha)/2\alpha}(2\pi)^{-1/2})). \end{aligned} \tag{2.1}$$

其中$C_T^*=\int_0^T c^{1/\alpha}(t)\mathrm{d}t\Big/T$, $\tau>0$. 当$T\to\infty$时, 有

(2.1.1) $\mu(l):=\mathsf{P}\Big\{\max\limits_{t\in I_l}X(t)>u_T\Big\}\sim H_\alpha c^{1/\alpha}(lh)\psi(u_T)h_T$;

(2.1.2) $\mathsf{P}\{X(jq)\le u_T, jq\in I_l\}-\mathsf{P}\{X(t)\le u_T, t\in I_l\}=o(\mu(l))$, $l=1,2,\cdots,n$;

(2.1.3) $\mathsf{P}\{X(t)\le u_T, 0\le t\le T\}=\exp\Big\{-\sum\limits_{l=1}^n\mu(l)+o(1)\Big\}=\exp\Big\{-H_\alpha\int_0^T c^{1/\alpha}(t)$ $\cdot\psi(u_T)\mathrm{d}t+o(1)\Big\}\to\exp\{-\tau\}$.

证明: 见Hüsler[1990]定理2.2, 引理3.2和定理4.2. □

引理 2.2 假设高斯过程$X(t)$的均值为0, 方差为1且满足条件(1.1)、(1.2), 水平u_T如(2.1)定义, 当$T\to\infty$时, 有

(2.2.1) $\sum\limits_{l=1}^n(\mathsf{P}\{X(t)\le u_T, t\in I_l\}-\mathsf{P}\{X(t)\le u_T, t\in I_l\cup I_l^*\})\to 0$;

(2.2.2) $\sum\limits_{jq\in I_l, j'q\in I_{l'}, l\ne l'}|r(jq,j'q)|\exp\{-u_T^2/[1+r(jq,j'q)]\}\to 0$.

证明: 见Hüsler[1990]引理3.1和引理3.4. □

引理 2.3 高斯过程$\{(X_1(t),\cdots,X_p(t)), 0\le t\le T\}$的$\mathsf{E}X_k(t)=0$, $DX_k(t)=1$, $0\le t\le T$, $k=1,\cdots,p$, 相关系数和交互相关系数满足(1.1)、(1.2)、(1.3)、(1.4), 水平$u_{k,T}$如(1.6)定义, 当$T\to\infty$时有

(2.3.1) $0\le\mathsf{P}\{X_k(jq)\le u_{k,T}, jq\in I_l, l\le n, k=1,\cdots,p\}-\mathsf{P}\{X_k(t)\le u_{k,T}, 0\le t\le T, k=1,\cdots,p\}\to 0$;

(2.3.2) $0\le\prod\limits_{l=1}^n\mathsf{P}\{X_k(jq)\le u_{k,T}, jq\in I_l, k=1,\cdots,p\}-\prod\limits_{l=1}^n\mathsf{P}\{X_k(t)\le u_{k,T}, t\in I_l, k=1,\cdots,p\}\to 0$;

(2.3.3) $\Big|\mathsf{P}\{X_k(jq)\le u_{k,T}, jq\in I_l, l\le n, k=1,\cdots,p\}-\prod\limits_{l=1}^n\mathsf{P}\{X_k(jq)\le u_{k,T}, jq\in I_l, k=1,\cdots,p\}\Big|\to 0$.

证明: 由引理(2.2.1)知

$$\begin{aligned}0 &\le \mathsf{P}\{X_k(t)\le u_{k,T}, t\in I_l, l\le n, k=1,\cdots,p\}\\ &\quad -\mathsf{P}\{X_k(t)\le u_{k,T}, 0\le t\le T, k=1,\cdots,p\}\\ &\le \sum_{k=1}^p\sum_{l=1}^n(\mathsf{P}\{X_k(t)\le u_{k,T}, t\in I_l\}-\mathsf{P}\{X_k(t)\le u_{k,T}, t\in I_l\cup I_l^*\})\to 0.\end{aligned}$$

由引理(2.1.2)知

$$\begin{aligned}0 &\le \mathsf{P}\{X_k(jq)\le u_{k,T}, jq\in I_l, l\le n, k=1,\cdots,p\}\\ &\quad -\mathsf{P}\{X_k(t)\le u_{k,T}, t\in I_l, l\le n, k=1,\cdots,p\}\\ &\le \sum_{k=1}^p(\mathsf{P}\{X_k(jq)\le u_{k,T}, jq\in I_l, l\le n\}-\mathsf{P}\{X_k(t)\le u_{k,T}, t\in I_l, l\le n\})\to 0.\end{aligned}$$

故(2.3.1)式成立.

由引理(2.2.1)知

$$
\begin{aligned}
&\Big|\prod_{l=1}^{n}\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}-\prod_{l=1}^{n}\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l\cup I_l^*\}\Big|\\
\le\ &\sum_{l=1}^{n}|\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}\\
&-\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l\cup I_l^*,k=1,\cdots,p\}|\\
\le\ &\sum_{k=1}^{p}\sum_{l=1}^{n}|\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l\}-\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l\cup I_l^*\}|\to 0.
\end{aligned}
$$

由引理(2.1.2)知

$$
\begin{aligned}
&\Big|\prod_{l=1}^{n}\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,k=1,\cdots,p\}\\
&-\prod_{l=1}^{n}\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}\Big|\\
\le\ &\sum_{k=1}^{p}\sum_{l=1}^{n}|\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l\}-\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l\}|\to 0.
\end{aligned}
$$

故(2.3.2)式成立.

根据正态比较引理知:

$$
\begin{aligned}
&\Big|\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,l\le n,k=1,\cdots,p\}\\
&-\prod_{l=1}^{n}\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,k=1,\cdots,p\}\Big|\\
\le\ &K\sum_{k=1}^{p}\sum_{jq\in I_l,j'q\in I_{l'},l\ne l'}|r_k(jq,j'q)|\exp\Big\{-\frac{u_{k,T}}{1+r_k(jq,j'q)}\Big\}\\
&+K\sum_{1\le k\ne k'\le p}\sum_{jq\in I_l,j'q\in I_{l'},l\ne l'}|r_{kk'}(jq,j'q)|\exp\Big\{-\frac{1}{2}\frac{u_{k,T}^2+u_{k'T}^2}{1+r_{kk'}(jq,j'q)}\Big\}\\
=\ &I+II.
\end{aligned}
$$

由引理(2.2.2)知: $T\to\infty$时, $I\to 0$. 由条件(1.3)、(1.4), 与Leadbetter(1983)的引理(12.3.1)的证明完全相似知: $T\to\infty$时, $II\to 0$, 故(2.3.3)式成立. □

引理 2.4　高斯过程$\{(X_1(t),\cdots,X_p(t)),0\le t\le T\}$的$\mathsf{E}X_k(t)=0$, $DX_k(t)=1$, $0\le t\le T$, $k=1,\cdots,p$, 相关系数和交互相关系数满足条件(1.1)、(1.4), 水平$u_{k,T}$如(1.6)定义, 则对任意的$l\le n$, $k\ne k'$, $T\to\infty$时

(2.4.1) $\mathsf{P}\{M_k(I_l)>u_{k,T},M_{k'}(I_l)>u_{k',T}\}=o\Big(\sum_{k=1}^{p}\mu_k(l)\Big)$;

(2.4.2) $\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}=1-\sum_{k=1}^{p}\mathsf{P}\{M_k(I_l)>u_{k,T}\}+o\Big(\sum_{k=1}^{p}\mu_k(l)\Big)$.

其中$\mu_k(l):=\mathsf{P}\{M_k(I_l)>u_{k,T}\}$, $M_k(I_l)=\max\{X_k(t),t\in I_l\}$.

证明: 由引理(2.1.2)知, 只需证$q\to 0$, $qu^{2/\alpha}=q_0\to 0$的速度充分慢的情形下, 有

$$\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_{k,T},\max_{jq\in I_l}X_{k'}(jq)>u_{k',T}\Big\}=o(\mu_k(l)+\mu_{k'}(l)).$$

由引理(2.1.1)和引理(2.1.2)知

$$\begin{aligned}&\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_k\Big\}\sim\mu_k(l),\\&\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_{k,T}\Big\}\mathsf{P}\Big\{\max_{jq\in I_l}X_{k'}(jq)>u_{k',T}\Big\}=o(\mu_k(l)+\mu_{k'}(l)).\end{aligned}$$

于是只需证明

$$\begin{aligned}&\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_{k,T}\Big\}\mathsf{P}\Big\{\max_{jq\in I_l}X_{k'}(jq)>u_{k',T}\Big\}\\&-\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_{k,T},\max_{jq\in I_l}X_{k'}(jq)>u_{k',T}\Big\}=o(\mu_k(l)+\mu_{k'}(l)).\end{aligned}$$

根据正态比较引理知:

$$\begin{aligned}&\Big|\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_{k,T}\Big\}\mathsf{P}\Big\{\max_{jq\in I_l}X_{k'}(jq)>u_{k',T}\Big\}\\&-\mathsf{P}\Big\{\max_{jq\in I_l}X_k(jq)>u_{k,T},\max_{jq\in I_l}X_{k'}(jq)>u_{k',T}\Big\}\Big|\\\le\;& K\sum_{jq,j'q\in I_l}|r_{kk'}(jq,j'q)|\exp\Big\{-\frac{1}{2}\frac{u_{k,T}^2+u_{k',T}^2}{1+r_{kk'}(jq,j'q)}\Big\}\\\le\;& K\sum_{jq,j'q\in I_l}\exp\Big\{-\frac{u_T^2}{1+\delta}\Big\}\le K\frac{h_T^2}{q^2}\exp\Big\{-\frac{u_T^2}{1+\delta}\Big\}\\=\;& K\frac{h_T^2}{q^2}\frac{\exp\{-u_T^2/2\}}{u_T^{1-2/\alpha}}\exp\Big\{-\frac{1-\delta}{2(1+\delta)}u_T^2\Big\}u_T^{1-2/\alpha}\\=\;& K\frac{h_T^2}{q_0^2}u_T^{1+2/\alpha}\exp\Big\{-\frac{1-\delta}{2(1+\delta)}u_T^2\Big\}O(\mu_k(l)+\mu_{k'}(l))\\=\;& o\Big(\sum_{k=1}^{p}\mu_k(l)\Big).\end{aligned}$$

故(2.4.1)式成立. 而

$$\begin{aligned}&\sum_{k=1}^{p}\mathsf{P}\{M_k(I_l)>u_{k,T}\}-\sum_{1\le k\ne k'\le p}\mathsf{P}\{M_k(I_l)>u_{k,T},M_{k'}(I_l)>u_{k',T}\}\\\le\;&\mathsf{P}\Big\{\bigcup_{k=1}^{p}(M_k(I_l)>u_{k,T})\Big\}\le\sum_{k=1}^{p}\mathsf{P}\{M_k(I_l)>u_{k,T}\}.\end{aligned}$$

由(2.4.1)式知, (2.4.2)式成立. □

§3. 定理的证明

定理 1.1 的证明: 由引理(2.3.1), (2.3.2), (2.3.3)知:

$$\begin{aligned}
&\Big|\mathsf{P}\{X_k(t)\le u_{k,T},0\le t\le T,k=1,\cdots,p\}\\
&-\prod_{l=1}^{n}\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}\Big|\\
\le\ &|\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,l\le n,k=1,\cdots,p\}\\
&-\mathsf{P}\{X_k(t)\le u_{k,T},0\le t\le T,k=1,\cdots,p\}|\\
&+\Big|\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,l\le n,k=1,\cdots,p\}\\
&-\prod_{l=1}^{n}\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,k=1,\cdots,p\}\Big|\\
&+\Big|\prod_{l=1}^{n}\mathsf{P}\{X_k(jq)\le u_{k,T},jq\in I_l,l\le n,k=1,\cdots,p\}\\
&-\prod_{l=1}^{n}\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}\Big|\to 0.
\end{aligned}$$

又由引理(2.1.1), 引理(2.1.3)和引理(2.4.2)知:

$$\begin{aligned}
&\prod_{l=1}^{n}\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\}\\
=\ &\exp\Big\{\sum_{l=1}^{n}\log(\mathsf{P}\{X_k(t)\le u_{k,T},t\in I_l,k=1,\cdots,p\})\Big\}\\
=\ &\exp\Big\{\sum_{l=1}^{n}\log\Big(1-\sum_{k=1}^{p}\mathsf{P}\{M_k(I_l)>u_{k,T}\}+o\Big(\sum_{k=1}^{p}\mu_k(l)\Big)\Big)\Big\}\\
=\ &\exp\Big\{-\sum_{l=1}^{n}\sum_{k=1}^{p}\mathsf{P}\{M_k(I_l)>u_{k,T}\}\Big\}+o(1)\\
\to\ &\exp\Big\{-\sum_{k=1}^{p}\tau_k\Big\}.
\end{aligned}$$

定理1.1证毕. □

定理 1.2 的证明: 不妨假定$\gamma_k\ne 0$, $\gamma_k=0$的情形完全相似的证明. 由条件(1.5)知: 对任意的$k=1,\cdots,p$, 存在m_0, $|m_k(t)|\le m_0$, 并且对充分大的T, 有$m_0<(1/8)\cdot(\log T)^{1/2}$

$$\begin{aligned}
&\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),0\le t\le T^{1/2}\}\\
\ge\ &\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_0,0\le t\le T^{1/2}\}\\
=\ &\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-u_k(T^{1/2})-m_0+u_k(T^{1/2}),0\le t\le T^{1/2}\}.
\end{aligned}$$

其中

$$C^*_{k,T^{1/2}}=\int_0^{T^{1/2}}c_k^{1/\alpha}(t)\mathrm{d}t\Big/T^{1/2},$$

$$\begin{aligned} u_k(T^{1/2}) &= (2\log T^{1/2})^{1/2} + (2\log T^{1/2})^{-1/2}(((2-\alpha)/2\alpha)\log\log T^{1/2} \\ &\quad + \log(C^*_{k,T^{1/2}} H_\alpha 2^{(2-\alpha)/2\alpha}(2\pi)^{-1/2})), \\ u_{k,T} - u_k(T^{1/2}) - m_0 &= (2\log T)^{1/2} - (\log T)^{1/2} - m_0 + o(1) \\ &= \frac{\log T}{(2\log T)^{1/2} + (\log T)^{1/2}} - m_0 + o(1) \geq \frac{1}{8}(\log T)^{1/2}. \end{aligned}$$

由引理(2.1.3)知$T\to\infty$

$$\mathsf{P}\{X_k(t) - m_k(t) \leq u_{k,T} - m_k(t), 0 \leq t \leq T^{1/2}\} \to 1.$$

令$\rho_T = (\log T - \log\log T)/\log T$

$$\begin{aligned} &\mathsf{P}\{X_k(t) - m_k(t) \leq u_{k,T} - m_k(t), 0 \leq t \leq T^{\rho_T}\} \\ = &\mathsf{P}\{X_k(t) - m_k(t) \leq u_{k,T} - m_k(t), T^{1/2} \leq t \leq T^{\rho_T}\} + o(1). \end{aligned}$$

因为$m_k(t)\sqrt{2\log t}\to\gamma_k$, 对充分大的$T$, 有$T^{1/2}\leq t\leq T^{\rho_T}$, $|m_k(t)\sqrt{2\log T}|\leq 3|\gamma_k|$, 故上式

$$\begin{aligned} &\geq \mathsf{P}\Big\{X_k(t) - m_k(t) \leq u_{k,T} - \frac{3|\gamma_k|}{\sqrt{2\log T}}, T^{1/2} \leq t \leq T^{\rho_T}\Big\} + o(1) \\ &= \mathsf{P}\Big\{X_k(t) - m_k(t) \leq u_k(T^{\rho_T}) + u_{k,T} - u_k(T^{\rho_T}) - \frac{3|\gamma_k|}{\sqrt{2\log T}}, 0 \leq t \leq T^{\rho_T}\Big\} + o(1). \end{aligned}$$

其中:

$$\begin{aligned} C^*_{k,T^{\rho_T}} &= \int_0^{T^{\rho_T}} c_k^{1/\alpha}(t)\mathrm{d}t \Big/ T^{\rho_T}, \\ u_k(T^{\rho_T}) &= (2\log T^{\rho_T})^{1/2} + (2\log T^{\rho_T})^{-1/2} \\ &\quad \cdot (((2-\alpha)/\alpha)\log\log T^{\rho_T} + \log(C^*_{k,T^{\rho_T}} H_\alpha 2^{(2-\alpha)/2\alpha}(2\pi)^{-1/2})), \\ &\Big[u_{k,T} - u_k(T^{\rho_T}) - \frac{3|\gamma_k|}{\sqrt{2\log T}}\Big]\sqrt{2\log T^{\rho_T}} \\ = &[(2\log T)^{1/2} - (2\log T - 2\log\log T)^{1/2} + (2\log T)^{-1/2}((2-\alpha)/2\alpha\log\log T) \\ &- (2\log T - 2\log\log T)^{-1/2}((2-\alpha)/\alpha(\log\log T + \log\rho_T)) + O((2\log T)^{-1/2})] \\ &\cdot\sqrt{2\log T - 2\log\log T} \\ = &\frac{2\log\log T}{(2\log T)^{1/2} + (2\log T - 2\log\log T)^{1/2}}\sqrt{2\log T - 2\log\log T} \\ &+ \Big[\Big(\frac{2\log T - 2\log\log T}{2\log T}\Big)^{1/2} - 1\Big](2-\alpha)/2\alpha\log\log T + O(1) \\ \geq &\frac{1}{8}\log\log T. \end{aligned}$$

对于任意$M>0$, 存在充分大的T, 有: $u_{k,T}-u_k(T^{\rho_T})-3|\gamma_k|/\sqrt{2\log T}>M/\sqrt{2\log T^{\rho_T}}$, 由引理(2.1.3)知:

$$\begin{aligned}&\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),0\le t\le T^{\rho_T}\}\\ \ge\ &\mathsf{P}\Big\{X_k(t)-m_k(t)\le u_k(T^{\rho_T})+\frac{M}{\sqrt{2\log T^{\rho_T}}},0\le t\le T^{\rho_T}\Big\}\\ \to\ &\exp\{-\exp\{-M\}\}.\end{aligned}$$

由M的任意性, 有

$$\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),0\le t\le T^{\rho_T}\}\to 1.$$

对于任意的ϵ, 对于充分大的T, $T^{\rho_T}\le t\le T$有

$$(\gamma_k-\epsilon)(1-\epsilon)\le m_k(t)\sqrt{2\log T}\le(\gamma_k+\epsilon)(1+\epsilon),$$

$$\begin{aligned}&\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),0\le t\le T\}\\ =\ &\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),T^{\rho_T}\le t\le T\}+o(1)\\ \ge\ &\mathsf{P}\Big\{X_k(t)-m_k(t)\le u_{k,T}-\frac{(\gamma_k+\epsilon)(1+\epsilon)}{\sqrt{2\log T}},0\le t\le T\Big\}+o(1)\\ \to\ &\exp\{-\tau_k\exp(\gamma_k+\epsilon)(1+\epsilon)\}.\end{aligned}$$

同理

$$\begin{aligned}&\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),0\le t\le T\}\\ \le\ &\mathsf{P}\Big\{X_k(t)-m_k(t)\le u_{k,T}-\frac{(\gamma_k-\epsilon)(1-\epsilon)}{\sqrt{2\log T}},0\le t\le T\Big\}+o(1)\\ \to\ &\exp\{-\tau_k\exp(\gamma_k-\epsilon)(1-\epsilon)\},\\ &\mathsf{P}\{x_k(t)-m_k(t)\le u_{k,T}-m_k(t),k=1,\cdots,p,0\le t\le T\}\\ =\ &\mathsf{P}\{X_k(t)-m_k(t)\le u_{k,T}-m_k(t),k=1,\cdots,p,T^{\rho_T}\le t\le T\}+o(1)\\ \le\ &\mathsf{P}\Big\{X_k(t)-m_k(t)\le u_{k,T}-\frac{(\gamma_k-\epsilon)(1-\epsilon)}{\sqrt{2\log T}},k=1,\cdots,p,0\le t\le T\Big\}+o(1)\\ \to\ &\exp\Big\{-\sum_{k=1}^{p}\tau_k\exp\{(\gamma_k-\epsilon)(1-\epsilon)\}\Big\}.\end{aligned}$$

同理

$$\begin{aligned}&\mathsf{P}\{x_k(t)-m_k(t)\le u_{k,T}-m_k(t),k=1,\cdots,p,0\le t\le T\}\\ \ge\ &\mathsf{P}\Big\{X_k(t)-m_k(t)\le u_{k,T}-\frac{(\gamma_k+\epsilon)(1+\epsilon)}{\sqrt{2\log T}},k=1,\cdots,p,0\le t\le T\Big\}+o(1)\\ \to\ &\exp\Big\{-\sum_{k=1}^{p}\tau_k\exp\{(\gamma_k+\epsilon)(1+\epsilon)\}\Big\}.\end{aligned}$$

由ϵ的任意性有:

$$\mathsf{P}\{x_k(t)-m_k(t)\le u_{k,T}-m_k(t),k=1,\cdots,p,0\le t\le T\}\to\exp\Big\{-\sum_{k=1}^{p}\tau_k\exp\gamma_k\Big\}.$$

定理1.2证毕. □

参 考 文 献

[1] Piterbarg, V.I., Asymptotic methods in the theory of Gaussian processes and fields, *Trans. Math. Monographs*, **148**(1996), Ams, Providence, RI. 97–118.

[2] Hüsler, J., Exteme values and high boundary crossings of locally stationary Gaussian process, *Ann. Probab.*, **18**(1990), 1141–1158.

[3] Lindgren, G., A note on the asymptotic independence of high level crossings for dependent Gaussian processes, *Ann. Probab.*, **2**(1974), 535–539.

[4] Berman, S.M., Sojourn and extremes of Gaussian process, *Ann. Probab.*, **2**(1974), 999–1026 (correction **8**(1980), 999).

[5] Pickands, J., Asymptotic properties of the maximam in a stationary Gaussian processes, *Trans. Amer. Math. Soc.*, **145**(1969), 75–86.

[6] Leadbetter, M.R., Lindgren, G., *Holger Rootźen Extremes and Related Properties of Random Seqences and Processes*, New York, Springer, 1983.

[7] 彭作祥, 一类向量高斯过程之上穿过点过程的渐近分布, 应用数学学报, **19(2)**(1996), 290–296.

Asymptotic Distribution of Maxima Multivariate Locally Stationary Gaussian Processes

YANG CHUNHUA[1,2] PENG ZUOXIANG[2]

([1]*Department of Mathematics, YUXi University, Chongqing, 402168*)

([2]*Department of Mathematics, Southwest Normal University, Chongqing, 400715*)

Let $\{(X_1(t),\cdots,X_p(t)),0\le t\le T\}$ be p dimensional locally stationary Gaussian processes with asymptotically centered mean $m_k(t)$, $k=1,\cdots,p$ and constant variance. $M_k(T)=\sup\{X_k(t),0\le t\le T\}$, $k=1,\cdots,p$. Under some conditions, the asymptotic distribution of $M(T)=(M_1(T),\cdots,M_p(T))$ as $T\to\infty$ is obtained.

Keywords: Multivariate Gaussian processes, locally stationary Gaussian processes, maxima.

AMS Subject Classification: 60G15.