应用概率统计　第二十三卷
第二期　2007 年 5 月

Chinese Journal of Applied Probability
and Statistics Vol. 23 No. 2 May 2007

The Existence and the Structure of r-Truncated Annuity Distribution*

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Abstract

The primary objective of this paper is to define the r-truncated annuity distribution, which is a generalization of the annuity distribution in refs: [5], [6]. Conditions for the existence of these distributions are given. It is proved that under certain conditions, the r-truncated annuity distribution is the mixture of an annuity distribution and a special r-truncated annuity distribution.

Keywords: r-truncated annuity distribution, probability generating function, mixture.

AMS Subject Classification: 62P05, 91B30.

§ 1. Introduction

In the field of actuarial risk theory, a well known family of discrete distributions is the one which has probabilities satisfying the recursion

$$p_n = p_{n-1}\Big(a + \frac{b}{n}\Big), \qquad n = 1, 2, 3, \cdots, \tag{1.1}$$

where $p_0 \in (0,1)$ and $\sum\limits_{n=0}^{\infty} p_n = 1$, a and b are real constants. It is proved that (1.1) holds if and only if the discrete random variable N has one of Poisson ($a = 0$), the binomial ($a < 0$ and $b = -a(m+1)$ for some positive integer m) and the negative binomial ($0 < a < 1$) distributions (see Panjer [3, 4]). These distributions have been extensively used in the context of modelling the claim number process and the aggregate claims process (see Asmussen [1]).

In [5], Ramsay introduces a new real valued three-parameter (a, b, δ) family of discrete distributions, called annuity distributions. This family, which contains the family defined by equation (1.1) as a special case, has the probabilities satisfying the recursion

$$p_n = p_{n-1}\Big(a + \frac{b}{c_n}\Big), \qquad n = 1, 2, 3, \cdots, \tag{1.2}$$

*This research was supported by National Natural Science Foundation of China (Grant No. 10571092).

Received 2004. 2. 18. Revised 2006. 9. 14.

where c_n, $n=1,2,\cdots$, are continuous function of the parameter δ,

$$c_n = \begin{cases} \dfrac{1-e^{-n\delta}}{e^{\delta}-1}, & \delta \neq 0; \\ n, & \delta = 0. \end{cases} \tag{1.3}$$

In [6], Ramsay explores the properties of annuity distributions. In particular the conditions needed to ensure the existence of these distributions are given, the pgf and the cumulant of N^* (a discrete random variable which has an annuity distribution) are discussed. Some good conclusions are derived.

The motivation of this paper comes from the comments of Ramsay [6], and the problem of the parameter estimation in annuity distribution has already been solved in Liang [2]. Primary objective of this paper is to introduce r-truncated annuity distribution. The properties of these truncated annuity distributions are explored. Conditions for the existence of these distributions are given. In particular, it is proved that, under certain conditions, the pgf of N (a discrete random variable which has a r-truncated annuity distribution) satisfies a functional equation (or a differential equation). Using this functional equation, it is proved that N is unbounded, i.e., $p_n > 0$, for $n = r, r+1, r+2, \cdots$. Then the distribution of N is a mixture of an annuity distribution and a specific r-truncated annuity distribution.

§ 2. The Existence and the Structure of r-Truncated Annuity Distribution

2.1 Definitions

Definition 2.1 A discrete random variable N^* is said to have an annuity distribution, if its probabilities $p(N^*=n) \equiv p_n$, $n \geq 0$ satisfy the following conditions:

$$\begin{aligned} &\text{i)} \quad p_n = p_{n-1}(a + b/c_n), \ n = 1,2,3,\cdots, \\ &\text{ii)} \quad \sum_{n=0}^{\infty} p_n = 1, \\ &\text{iii)} \quad c_n = \begin{cases} \dfrac{1-e^{-n\delta}}{e^{\delta}-1}, & \delta \neq 0; \\ n, & \delta = 0, \end{cases} \end{aligned} \tag{2.1}$$

where a, b and δ are real constants.

From the following context we can see: the probabilities $\{p_n, n \geq 0\}$ of N^* are determined by the parameters a, b, and δ. So we call its distribution $AD(a,b,\delta)$, and denoted by $N^* \sim AD(a,b,\delta)$.

Definition 2.2 Let r be a positive integer number. A discrete random variable N is said to have a r-truncated annuity distribution if its probabilities $p(N=n) \equiv p_n$, $n \geq 0$ satisfy the following conditions:

$$\begin{aligned} &\text{i)} \quad p_n = p_{n-1}(a + b/c_n), \ n = r, r+1, r+2, \cdots, \\ &\text{ii)} \quad \sum_{n=0}^{\infty} p_n = 1, \end{aligned} \tag{2.2}$$

iii) $c_n = \begin{cases} \dfrac{1-e^{-n\delta}}{e^{\delta}-1}, & \delta \neq 0; \\ n, & \delta = 0, \end{cases}$

where a, b and δ are real constants. When $p_n = 0$ for $n \geq r$, we call it has a degenerate distribution.

From the following context we can see that the probabilities $\{p_n, n \geq r-1\}$ are determined by the parameter a, b, δ and the probabilities $\{p_i, 0 \leq i \leq r-2\}$. So we call its distribution $AD(p_i, 0 \leq i \leq r-1; a, b, \delta)$, and denoted by $N \sim AD_r(a,b,\delta)$. It is obvious that $AD_1(a,b,\delta) = AD(a,b,\delta)$.

Evidently $\lim\limits_{n\to\infty} 1/c_n = 0$, when $\delta \leq 0$. However, if $\delta > 0$, $\lim\limits_{n\to\infty} 1/c_n = e^{\delta} - 1$.

Definition 2.3 A distribution F is called the mixture of distributions F_k $(k = 1, 2, \cdots, n)$, if the following equation holds:

$$p(z) = \sum_{k=1}^{n} a_k p_k(z), \tag{2.3}$$

where $p(z)$ is the pgf of the distribution F, $p_k(z)$ is the pgf of the distribution F_k, and a_k is a positive constant for $k = 1, 2, \cdots, n$, and $\sum\limits_{k=1}^{n} a_k = 1$.

2.2 The existence and the structure of $AD_2(a,b,\delta)$

2.2.1 The existence of $AD_2(a,b,\delta)$

From equation (2.1) and (2.2), the probability p_1 is given by

$$p_1 = \frac{1-p_0}{\widetilde{R}(a,b,\delta)}, \tag{2.4}$$

where

$$\widetilde{R}(a,b,\delta) = 1 + \sum_{n=2}^{\infty} \prod_{k=2}^{n} \left(a + \frac{b}{c_k}\right). \tag{2.5}$$

Theorem 2.1 For $1 > p_0 \geq 0$, a non-degenerate $N \sim AD_2(a,b,\delta)$ exists if the parameters a, b and δ satisfy one of the following conditions:

i)

$$a + \frac{b}{c_2} > 0, \tag{2.6}$$

and

$$0 \leq \lim_{n\to\infty} \left(a + \frac{b}{c_n}\right) < 1. \tag{2.7}$$

ii) $a < 0$ and there exists a positive integer $m > 1$ such that

$$b = -ac_{m+1}. \tag{2.8}$$

Only condition ii) results in a bounded support for N.

Proof Since $1 > p_0 \geq 0$, from equation (2.4), we can deduce $p_1 > 0$. For $p_2 = p_1(a + b/c_2)$, in order to avoid degeneracy, equation (2.6) must hold. $\widetilde{R}(a,b,\delta)$ exists if inequality (2.7) holds. The sequence $a + b/c_n$ are monotone and bounded, condition i) implies $a + b/c_n > 0$ for $n = 2, 3, \cdots$. Thus the probabilities are all non-negative.

The sufficiency of condition ii) can easily be established as follows: once equation (2.8) holds, it must be the case that $p_k = 0$ for $k \geq m+1$, making $\widetilde{R}(a,b,\delta)$ the sum of a finite number of terms. Because c_n is a monotone increasing function of n, the equation (2.8) jointly implies that $a + b/c_n = a(1 - c_{n+1}/c_n) > 0$ for $2 \leq n \leq m$. Since $p_n = p_{n-1}(a + b/c_n)$ for $n = 2, 3, \cdots$, and $p_1 > 0$, then $p_n = p_{n-1}(a + b/c_n) > 0$ for $2 \leq n \leq m$. #

Throughout this section it is assumed that the condition i) of Theorem 2.1 holds. This implies that N is an unbounded random variable.

2.2.2 The pgf of $AD_2(a,b,\delta)$

Having established the conditions for the existence of the $AD_2(a,b,\delta)$, their pgfs will now be investigated. Let $p(z)$ be the pgf of $AD_2(a,b,\delta)$, i.e.,

$$p(z) = \sum_{n=0}^{\infty} p_n z^n, \qquad 0 \leq z \leq 1,$$

where p_n is defined in equation (2.2). In addition, define the parameters θ, β, γ and c as:

$$\begin{cases} \theta = a + b(e^{\delta} - 1), \\ \beta = p_1 - p_0(a + be^{\delta}), \\ \gamma = z(1 - e^{-\delta})\beta, \\ c = p_1 - p_0(a + b). \end{cases} \tag{2.9}$$

It will now be proved that the pgf of $AD_2(a,b,\delta)$ satisfies a functional equation when $\delta \neq 0$. However, when $\delta = 0$, the pgf satisfies a differential equation. This functional equation is used to establish the properties of $AD_2(a,b,\delta)$.

Theorem 2.2 i) If $\delta \neq 0$, then $p(z)$ satisfies the functional equation:

$$(1 - \theta z)p(z) = (1 - aze^{-\delta})p(ze^{-\delta}) + \gamma. \tag{2.10}$$

ii) If $\delta = 0$, then $p(z)$ satisfies a differential equation:

$$(1 - az)p'(z) = (a + b)p(z) + c. \tag{2.11}$$

Proof From equation (2.2),

$$p_n c_n = p_{n-1}(ac_n + b).$$

If $\delta \neq 0$, multiplying both sides by z^n and summing yields

$$\sum_{n=2}^{\infty} z^n p_n \Big(\frac{1 - e^{-n\delta}}{e^{\delta} - 1}\Big) = \sum_{n=2}^{\infty} z^n p_{n-1} \Big[a\Big(\frac{1 - e^{-n\delta}}{e^{\delta} - 1}\Big) + b\Big].$$

The left-hand side (LHS) of this equation reduces to

$$\frac{1}{e^{\delta} - 1}[p(z) - p(ze^{-\delta}) - p_1 z(1 - e^{-\delta})],$$

while its right-hand side (RHS) reduces to

$$\frac{az}{e^{\delta}-1}\{[p(z)-e^{-\delta}p(ze^{-\delta})]-p_0(1-e^{-\delta})\}+bzp(z)-bzp_0.$$

Equating the LHS and RHS expressions gives

$$\begin{aligned}&p(z)-p(ze^{-\delta})-p_1z(1-e^{-\delta})\\=\;&azp(z)-aze^{-\delta}p(ze^{-\delta})-azp_0(1-e^{-\delta})+[bzp(z)-bzp_0](e^{\delta}-1),\end{aligned}$$

and equation (2.10) results. In a similar manner, equation (2.11) can be derived by noting that $c_n=n$ when $\delta=0$.　#

2.2.3　Solutions to equations (2.10) and (2.11)

I) solution to equation (2.11), i.e., the case of $\delta=0$

From inequality (2.7), we know $0\le a<1$ when $\delta=0$.

Case A: $a=0$

Define the parameter $\widetilde{c}$ as

$$\widetilde{c}=p_1-bp_0. \tag{2.12}$$

By using the ordinary method, we can see that the solution to differential equation

$$\begin{cases}p'(z)=bp(z)+\widetilde{c},\\p(1)=1\end{cases}$$

is

$$p(z)=e^{b(z-1)}\left(\frac{\widetilde{c}}{b}+1\right)-\frac{\widetilde{c}}{b}. \tag{2.13}$$

Case B: $0<a<1$

In a similar manner, the solution to differential equation

$$\begin{cases}(1-az)p'(z)=(a+b)p(z)+c,\\p(1)=1\end{cases}$$

is

$$p(z)=\left(1+\frac{c}{a+b}\right)e^{\lambda(v(z)-1)}-\frac{c}{a+b}, \tag{2.14}$$

where

$$\lambda=-\frac{a+b}{a}\ln(1-a),\qquad v(z)=\frac{\ln(1-az)}{\ln(1-a)}.$$

II) Solution to equation (2.10), i.e., the case of $\delta\neq0$

Let

$$\nu=e^{-|\delta|},\qquad 0<\nu<1.$$

Case A′: $\delta>0$

From equation (2.10), we know

$$p(zv^j)(1-\theta zv^j) = (1-azv^{j+1})p(zv^{j+1}) + v^j\gamma.$$

Since $zv^j \to 0$ as $j \to 0$, and $p(0) = p_0$, then by successive substitutions the following result is derived:

$$p(z) = a_1^+ p_1^+(z) + a_2^+ p_2^+(z), \tag{2.15}$$

where

$$\begin{cases} a_1^+ = p_0 \prod\limits_{j=0}^{\infty}\Big(\dfrac{1-av^{j+1}}{1-\theta v^j}\Big), \\ a_2^+ = 1 - a_1^+, \\ p_1^+(z) = \prod\limits_{j=0}^{\infty}\Big(\dfrac{1-\theta v^j}{1-av^{j+1}}\Big)\Big(\dfrac{1-azv^{j+1}}{1-\theta zv^j}\Big), \\ p_2^+(z) = \dfrac{\widetilde{A}(z)}{\widetilde{A}(1)}, \end{cases}$$

where

$$\widetilde{A}(z) = \frac{1}{1-\theta z}\gamma + \sum_{j=0}^{\infty}\Big[\prod_{k=0}^{j}\Big(\frac{1-azv^{k+1}}{1-\theta zv^k}\Big)\Big]\frac{v^{j+1}}{1-\theta zv^{j+1}}\gamma.$$

Case B′: $\delta < 0$

In a similar manner the following result is derived:

$$p(z) = a_1^- p_1^-(z) + a_2^- p_2^-(z), \tag{2.16}$$

where

$$\begin{cases} a_1^- = p_0 \prod\limits_{j=0}^{\infty}\Big(\dfrac{1-\theta v^{j+1}}{1-av^j}\Big), \\ a_2^- = 1 - a_1^-, \\ p_1^-(z) = \prod\limits_{j=0}^{\infty}\Big(\dfrac{1-av^j}{1-\theta v^{j+1}}\Big)\Big(\dfrac{1-\theta zv^{j+1}}{1-azv^j}\Big), \\ p_2^-(z) = \dfrac{A(z)}{A(1)}, \end{cases}$$

where

$$A(z) = \frac{1}{1-az}\widetilde{\gamma} + \sum_{j=0}^{\infty}\Big[\prod_{k=0}^{j}\Big(\frac{1-\theta zv^{k+1}}{1-azv^k}\Big)\Big]\frac{v^{j+1}}{1-azv^{j+1}}\widetilde{\gamma},$$

where

$$\widetilde{\gamma} = -e^{\delta}\gamma.$$

2.2.4 The structure of $AD_2(a, b, \delta)$

Let

$$R(a,b,\delta) = 1 + \sum_{n=1}^{\infty}\prod_{j=1}^{n}\Big(a + \frac{b}{c_j}\Big),$$

$$\widetilde{R}(0,b,0) = 1 + \sum_{n=2}^{\infty}\prod_{k=2}^{n}\Big(\frac{b}{k}\Big),$$

$$\widetilde{R}(a,b,0) = 1 + \sum_{n=2}^{\infty}\prod_{k=2}^{n}\Big(a + \frac{b}{k}\Big),$$

and condition A):

$$a + be^{\delta} > 0, \tag{2.17}$$

condition B):

$$p_0 < \frac{1}{R(a,b,\delta)}. \tag{2.18}$$

From the solutions (2.13), (2.14), (2.15) and (2.16), we can see that, under certain conditions, $AD_2(a,b,\delta)$ is the mixture of the other two distributions.

From equations (2.13), (2.14) and Definition 2.3, we have the following results:

Theorem 2.3 When $\delta = 0$,

i) If $a = 0$ and $p_0 > [1 + b\widetilde{R}(0,b,0)]^{-1}$, then pgf $p(z)$ is the mixture of zero and a Poisson distribution.

ii) If $0 < a < 1$ and $p_0 > [1 + (a+b)\widetilde{R}(a,b,0)]^{-1}$, then pgf $p(z)$ is the mixture of zero and a negative binomial distribution.

Theorem 2.4 When $\delta \neq 0$, if condition A) holds, then $p_1^+(z)$ and $p_1^-(z)$ are pgfs of $AD(a,b,\delta)$.

Proof See, for example, the equation (13) and Lemma 1 (i) in [6], the proof is obvious. #

Theorem 2.5 When $\delta \neq 0$, if conditions A) and B) hold, then $p_2^+(z)$ and $P_2^-(z)$ are pgfs of $AD_2(a,b,\delta)$ with $p_0 = 0$, we call it $AD_2'(a,b,\delta)$.

Proof We only discuss the case of $\delta > 0$ (in a similar manner, the case of $\delta < 0$ can be derived).

Clearly, $P_2^+(1) = 1$. Let

$$p_1^+(z) = \sum_{n=0}^{\infty} p_n^{(1)} z^n.$$

From Theorem 2.4, we know that $\{p_n^{(1)}, n = 0, 1, \cdots\}$ are the probabilities of $AD(a,b,\delta)$ which satisfy the equation (1.2).

From equation (2.15), we know that

$$a_2^+ p_2^+(z) = p(z) - a_1^+ p_1^+(z) = \sum_{n=0}^{\infty} (p_n - a_1^+ p_n^{(1)}) z^n.$$

From condition B), we can see $0 < a_1^+, a_2^+ < 1$, so

$$p_2^+(z) = \sum_{n=0}^{\infty} \left(\frac{p_n - a_1^+ p_n^{(1)}}{a_2^+} \right) z^n.$$

Let

$$p_n^{(2)} = \frac{p_n - a_1^+ p_n^{(1)}}{a_2^+}, \qquad n = 0, 1, 2, \cdots,$$

then

$$p_2^+(z) = \sum_{n=0}^{\infty} p_n^{(2)} z^n,$$

where

$$\begin{cases} p_0^{(2)} = 0, \\ p_1^{(2)} = \dfrac{1}{\widetilde{R}(a,b,\delta)}, \\ p_2^{(2)} = \dfrac{1}{\widetilde{R}(a,b,\delta)}\Big(a+\dfrac{b}{c_2}\Big) = p_1^{(2)}\Big(a+\dfrac{b}{c_2}\Big), \\ \quad \vdots \\ p_n^{(2)} = p_{n-1}^{(2)}\Big(a+\dfrac{b}{c_n}\Big), \qquad n=2,3,4,\cdots. \end{cases}$$

From equation (2.6) and inequality (2.7), the probabilities $\{p_n^{(2)}, n=1,2,\cdots\}$ are all positive. #

From Theorem 2.4 and Theorem 2.5, the following result is easily derived:

Theorem 2.6 When $\delta \neq 0$, if condition A) and B) hold, then $AD_2(a,b,\delta)$ is the mixture of $AD(a,b,\delta)$ and $AD_2'(a,b,\delta)$.

The results of $AD_2(a,b,\delta)$ can easily be extended to $AD_r(a,b,\delta)$.

2.3 The existence and the structure of $AD_r(a,b,\delta)$

From equation (2.2), the probability p_{r-1} is given by

$$p_{r-1} = \frac{1-M_r}{R_r(a,b,\delta)}, \tag{2.19}$$

where

$$M_r = \sum_{n=0}^{r-2} p_n,$$

and

$$R_r(a,b,\delta) = 1 + \sum_{n=r}^{\infty}\prod_{j=r}^{n}\Big(a+\frac{b}{c_j}\Big).$$

From the Theorem 2.1, we can easily get the following conditions for the existence of $AD_r(a, b,\delta)$.

Theorem 2.7 For $p_n \geq 0$, $n=0,1,2,\cdots,r-2$ and $M_r<1$, a non-degenerate $N \sim AD_r(a,b,\delta)$ exists if the parameters a, b, r and δ satisfy one of the following conditions:

i)

$$a+\frac{b}{c_r} > 0, \tag{2.20}$$

and

$$0 \leq \lim_{n\to\infty}\Big(a+\frac{b}{c_n}\Big) < 1.$$

ii) $a<0$ and there exists a positive integer $m \geq r$ such that

$$b = -ac_{m+1}.$$

Similarly, only condition ii) results in a bounded support for N. Throughout this section it is assumed that the condition i) of Theorem 2.7 holds.

2.3.1 The other conclusions

Let $p(z)$ is the pgf of $AD_r(a,b,\delta)$, i.e.,

$$p(z)=\sum_{n=0}^{\infty}p_nz^n,\qquad 0\le z\le 1,$$

where p_n is defined in equation (2.2). In addition, define the parameters β_n,γ_n and φ_n as:

$$\beta_n=p_n-p_{n-1}(a+b/c_n),\qquad \gamma_n=z^n(1-e^{-n\delta})\beta_n,\qquad \varphi_n=p_n-p_{n-1}(a+b/n),$$

and condition C):

$$\begin{cases}0<p_0<\dfrac{1}{R(a,b,\delta)},\\ p_n\ge p_0\prod\limits_{j=1}^{n}(a+b/c_j),\qquad n=1,2,\cdots,r-2.\end{cases}$$

From Theorem 2.2, the following results is obvious:

Theorem 2.8 i) If $\delta\ne 0$, then $p(z)$ satisfies the functional equation:

$$(1-\theta z)p(z)=(1-aze^{-\delta})p(ze^{-\delta})+\sum_{n=1}^{r-1}\gamma_n.\tag{2.21}$$

ii) If $\delta=0$, then $p(z)$ satisfies a differential equation:

$$(1-az)p'(z)=(a+b)p(z)+\sum_{n=1}^{r-1}\varphi_n.\tag{2.22}$$

In the following context, only the case of $\delta>0$ is discussed (the case of $\delta\le 0$ can be derived by the same method).

From equation (2.15) and theorems 2.5, 2.6, the following results are easily derived:

Theorem 2.9 If $\delta>0$, then $p(z)$ can be given by

$$p(z)=a_1p_1(z)+a_2p_2(z),\tag{2.23}$$

where

$$\begin{cases}a_1=p_0\prod\limits_{j=0}^{\infty}\left(\dfrac{1-av^{j+1}}{1-\theta v^j}\right),\\ a_2=1-a_1,\\ p_1(z)=\prod\limits_{j=0}^{\infty}\left(\dfrac{1-\theta v^j}{1-av^{j+1}}\right)\left(\dfrac{1-azv^{j+1}}{1-\theta zv^j}\right),\\ p_2(z)=\dfrac{B(z)}{B(1)},\end{cases}$$

where

$$B(z)=\frac{1}{1-\theta z}\sum_{n=1}^{r-1}\gamma_n+\sum_{n=0}^{\infty}\Big[\prod_{j=0}^{n}\left(\frac{1-azv^{j+1}}{1-\theta zv^j}\right)\Big]\Big[\frac{1}{1-\theta zv^{n+1}}\sum_{k=1}^{r-1}(v^{n+1})^k\gamma_k\Big].$$

Theorem 2.10 When $\delta>0$, if conditions A) and C) hold, then $p_2(z)$ is the pgf of $AD_r(a,b,\delta)$, and its probabilities $\{p_n^{(2)},n=0,1,\cdots,r-2\}$ satisfy the following equations:

i) $p_0^{(2)}=0$,

ii) $p_n^{(2)}=\Big[p_n-p_0\prod\limits_{j=1}^{n}(a+b/c_j)\Big]\Big/a_2,\ n=1,2,\cdots,r-2.$

This is a specific $AD_r(a,b,\delta)$, we call it $AD'_r(a,b,\delta)$.

Theorem 2.11 When $\delta>0$, if conditions A) and C) hold, then $AD_r(a,b,\delta)$ is the mixture of $AD(a,b,\delta)$ and $AD'_r(a,b,\delta)$.

Acknowledgment The authors would like to thank the anonymous referees for their helpful comments on an earlier version of this paper, which led to a considerable improvement of the presentation of our work.

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r 尾年金分布的存在性和结构

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本文主要是在年金分布的基础上，推广、定义并研究了一类 r 尾年金分布的存在性和结构，给出了这类分布的存在性条件，证明了在一定条件下， r 尾年金分布是年金分布与某一特殊 r 尾年金分布的混合.

关 键 词： r 尾年金分布，概率母函数，混合.

学 科 分 类 号： O211.9.